OPTICAL WAVEGUIDE MODES

Prof. Alina Karabchevsky, www.alinakarabchevsky.com

Integrated Photonics Course 377-2-5599

School of ECE

Ben-Gurion University of the Negev, Israel

WHAT WAS STUDIED AT THE LAST LECTURE?

Introduction

- History

Advantages of Integrated Optics

- Comparison of Optical Fibers with Other Interconnectors
- Comparison of Optical Integrated Circuits with Electrical Integrated Circuits

Substrate Materials for Optical Integrated Circuits

- Hybrid Versus Monolithic Approach
- III–V and II–VI Ternary Systems
- Hybrid OIC's in Lithium niobate ($LiNbO_3$)

THIS LECTURE WILL COVER:

Introduction

Modes

- Modes in a planar waveguide

Boundary conditions

Transcendental equation

Cut off condition

- Symmetric waveguide
- Asymmetric waveguide

Experimental observation of waveguide modes

Numerical modeling

Ray optics approach

Goos-Hanchen shifts

EVANESCENT EXCITATION VS. NORMAL INCIDENCE

Variation of refractive indices:

Longitudinal

1. Refractive indices vary along the light propagation direction
2. Approach: transfer matrix method
3. Devices: Distributed Bragg Gratings, Anti-Reflective Coatings.

Transverse

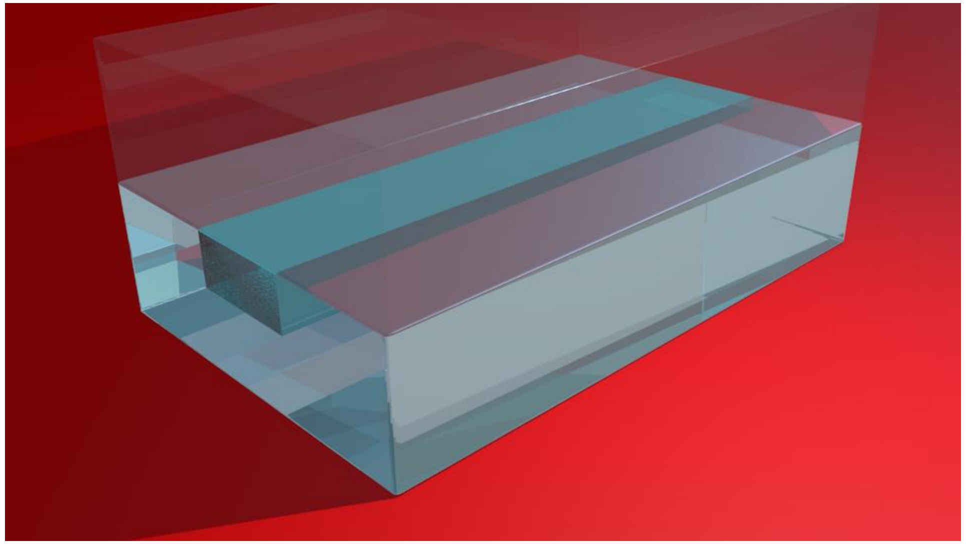
1. The index distribution is not a function of the light propagation direction
2. Approach: guided wave optics
3. Devices: fibers, planar waveguides.

A WAVEGUIDE

Thank you once again Joe, without your skills, this waveguide system would not have been ready in time...

A WAVEGUIDE

The optical waveguide is the fundamental element that interconnects the various devices of an optical integrated circuit, just as a metallic strip does in an electrical integrated circuit. However, unlike electrical current that flows through a metal strip according to Ohm's law, optical waves travel in the waveguide in distinct optical modes. A mode, in this sense, is a spatial distribution of optical energy in one or more dimensions that remains constant in time.



HOW DOES LIGHT PROPAGATE IN A WAVEGUIDE?

- A propagation mode of an ideal loss-less waveguide at a given λ preserves the cross-sectional shape in which the wave propagates.
- Waveguide mode profiles are wavelength dependent.
- Waveguide modes at any given λ are determined by the cross-sectional geometry.
- Waveguide modes at any given λ are determined by the refractive index profile of the waveguide.

CONCEPT OF TOTAL INTERNAL REFLECTION

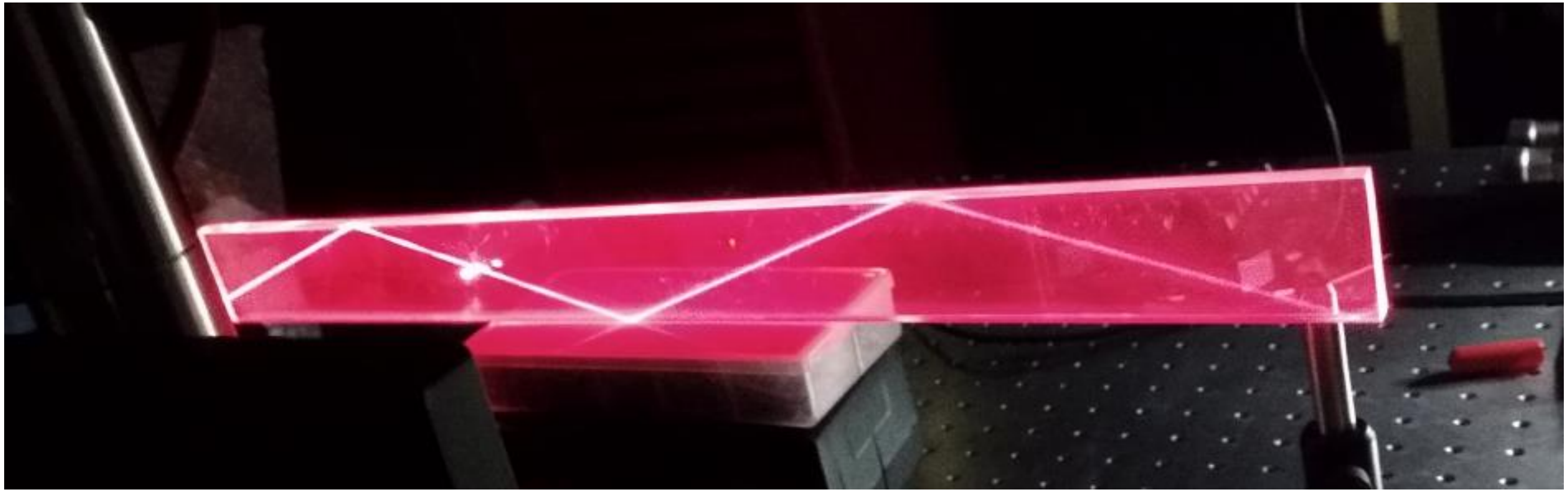


Figure 1: A laser beam through acrylic shows the concept of total internal reflection (the light doesn't continue straight through the edge of the glass but bounces back and forth until exiting at the end).

SURVEY: EVANESCENT WAVES

- EasyPolls:

What is an "evanescent field" ?

- ○ Side effect of TIR
- ○ Appears in the optically less dense medium
- ○ Characterized by its propagation in the x direction
- ○ Characterized by its exponential attenuation in the z direction
- ○ No energy flows across the boundary
- ○ The component of Poynting vector in the direction normal to the boundary is finite, but its time average vanishes

results vote

SNELL'S LAW OF REFRACTION

- When light hits the boundary between two materials, the light is reflected and refracted. In the transition from one medium to another medium, the propagation angle changes.
- In 1621, Snell discovered empirically the relationship between the indices of the materials and the propagation angles of the light.
- The refraction angle can be calculated by Snell's law of refraction which is defined as

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$$

where **1** is the incident medium, **2** is the transmitted medium, v is the velocity,

θ is the angle of the light in the medium and n is the refractive index.

REFLECTION AND REFRACTION

<u>Assumptions:</u>

- Plane wave propagation.
- Linear medium.
- Isotropic medium.
- Smooth planar optical interface.

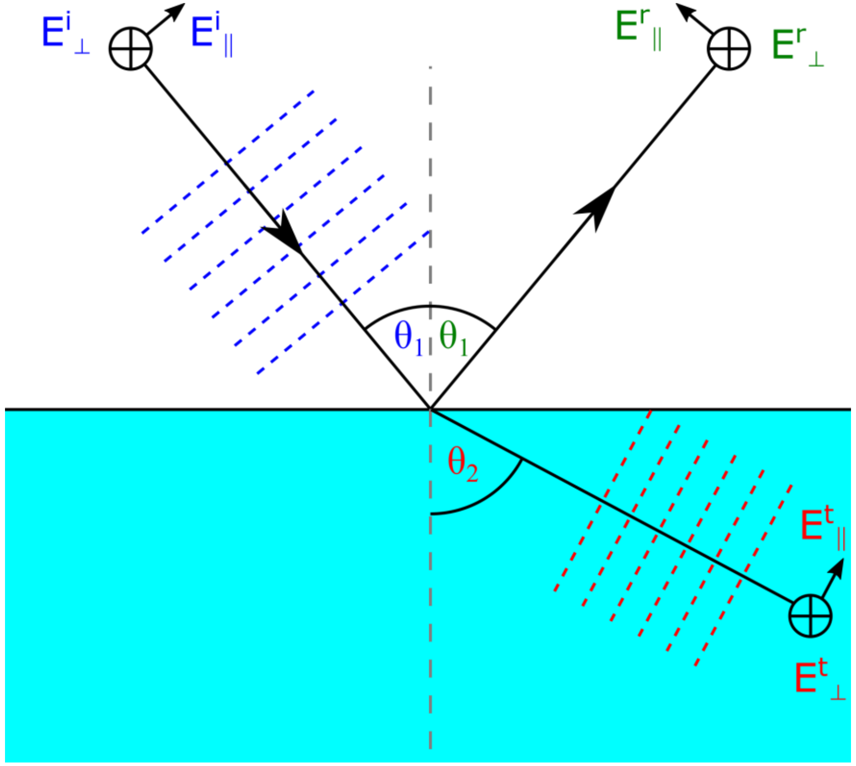
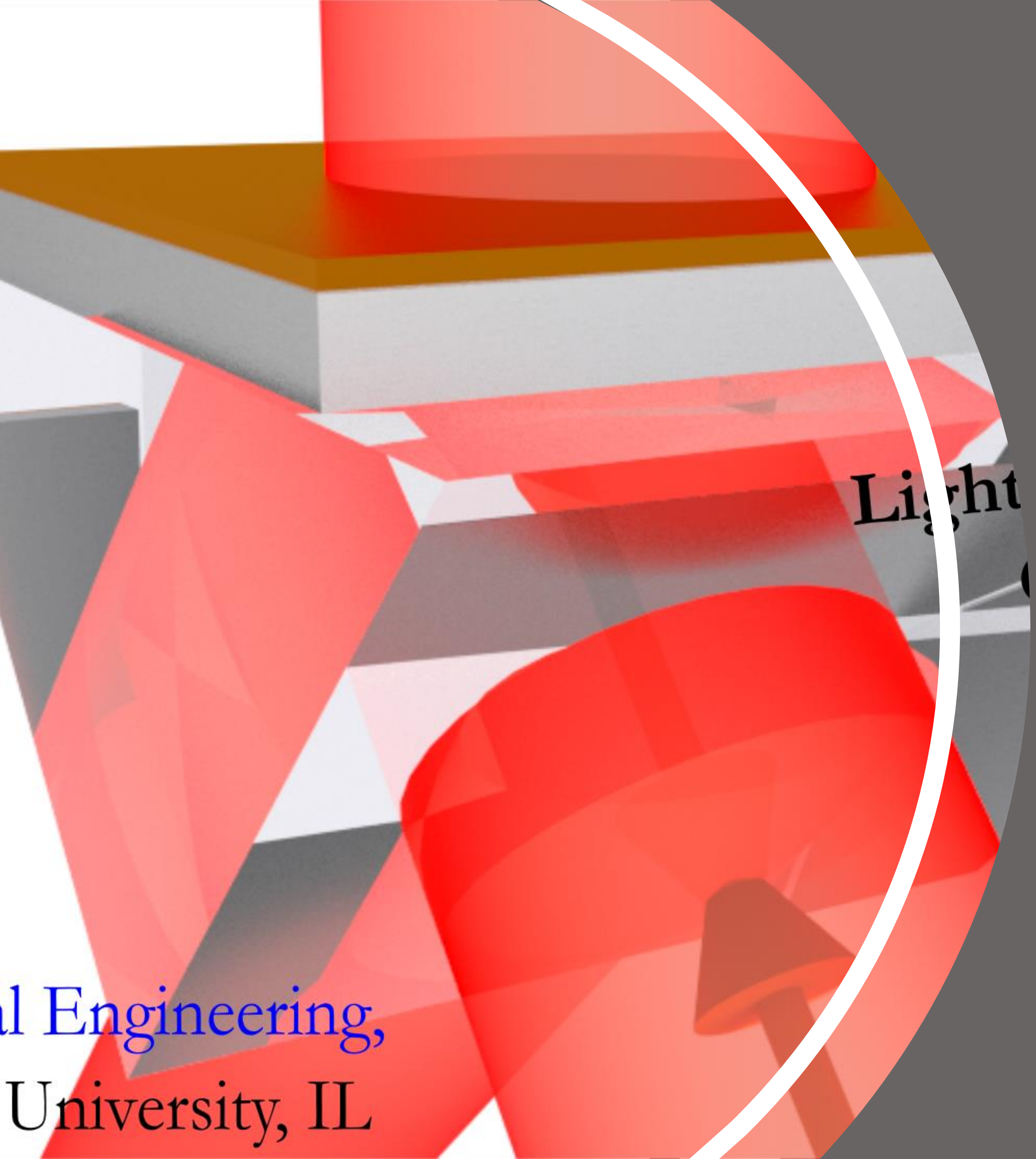


Figure 2: Plane wave reflection and refraction at an optical interface.



FRESNEL'S EQUATIONS

- As light hits the boundary of two materials, the power is split and a fraction of the power is refracted while the rest is reflected.
- In 1825, Fresnel derived a set of equations that defines the relation between the reflectance or the transmittance to the incident angle and the indices of the material.

FRESNEL'S FIELD REFLECTIVITY

The reflectivity for optical field components parallel to the incident plane as:

$$\rho_{\parallel} = \frac{{E^r}_{\parallel}}{{E^i}_{\parallel}} = \frac{n_1 \cos \theta_2 - n_2 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \qquad (1)$$

In order to eliminate θ_2, we can use Snell's Law:

$$\rho_{\parallel} = \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin(\theta_1)\right)^2} - n_2 \cos \theta_1}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin(\theta_1)\right)^2} + n_2 \cos \theta_1} \qquad (2)$$

Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as:

FIELD REFLECTIVITY

Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as:

$$\rho_{\perp} = \frac{E^{r}{}_{\perp}}{E^{i}{}_{\perp}} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \tag{3}$$

$$\rho_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_1\right)^2}}{n_1 \cos\theta_1 + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_1\right)^2}} \tag{4}$$

FIELD REFLECTIVITY

Power reflectivities for parallel and perpendicular field components are therefore:

$$R_{\parallel} = |\rho_{\parallel}|^2 = \left|\frac{E^r{}_{\parallel}}{E^i{}_{\parallel}}\right|^2 \tag{5}$$

and

$$R_{\perp} = |\rho_{\perp}|^2 = \left|\frac{E^r{}_{\perp}}{E^i{}_{\perp}}\right|^2 \tag{6}$$

FRESNEL'S POWER TRANSMISSION COEFFICIENTS

According to energy conservation, the power transmission coefficients can be found as:

$$T_{\parallel} = \left|\frac{{E^t}_{\parallel}}{{E^i}_{\parallel}}\right|^2 = 1 - |\rho_{\parallel}|^2 \tag{7}$$

and

$$T_{\perp} = \left|\frac{{E^t}_{\perp}}{{E^i}_{\perp}}\right|^2 = 1 - |\rho_{\perp}|^2 \tag{8}$$

In practice, for an arbitrary incidence polarization state, the input field can always be decomposed into $E_{\parallel}$ and $E_{\perp}$ components. Each can be treated independently.

FRESNEL'S SPECIAL CASE: NORMAL INCIDENCE

Normal incidence is when a light is launched perpendicular to the material interface. In this case, $\theta_1 = \theta_2 = 0$ and $\cos(\theta_1) = \cos(\theta_2) = 1$. The field reflectivity can be simplified as:

$$\rho_{\parallel} = \rho_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} \tag{9}$$

- If $n_1 > n_2$ there is no phase shift between incident and reflected field (the phase of both $\rho_{\parallel}$ and $\rho_{\perp}$ is zero).
- If $n_1 < n_2$ there is a π phase shift for both $\rho_{\parallel}$ and $\rho_{\perp}$ because they both become negative.

With normal incidence, the power reflectivity is:

$$R_{\parallel} = R_{\perp} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 \tag{10}$$

CRITICAL ANGLE - θ_c

- However, when light hits the boundary between high to low refractive index material, above a specific angle, called the critical angle, the light will be fully reflected.
- This phenomenon is called total internal reflection (TIR). According to Fresnel equations (2) and (4), total reflection ($|\rho_{\parallel}| = |\rho_{\perp}| = 1$) occurs when $\frac{n1}{n2}\sin(\theta_1) = 1$ and the critical angle is defined as:

$$\theta_c = \theta_1 = sin^{-1}\left(\frac{n_2}{n_1}\right) \qquad (11)$$

CRITICAL ANGLE

- Obviously, the necessary condition to have a critical angle depends on the interface condition.
- **if $n_1 < n_2$: there is no real solution** for $\theta_c = \sin^{-1}(n_2/n_1)$.
- It means that when a light beam goes from a low index material to a high index material, total reflection is not possible.
- **if $n_1 > n_2$: there is a real solution** for $\theta_c = \sin^{-1}(n_2/n_1)$.
- Therefore, total reflection can only happen when a light beam launches from a high index material to a low index material.

CRITICAL ANGLE

- It is important to note that at a larger incidence angle $\theta_1 > \theta_c$, $1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2 < 0$ and $\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$ becomes imaginary.
- Equations (2) and (4) show that if $\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$ is imaginary, both $\left|\rho_{\parallel}\right|^2$ and $|\rho_{\perp}|^2$ are equal to 1. The important conclusion is that for all incidence angles satisfying $\theta_1 = \theta_c$ total internal reflection will happen with $R = 1$.

CRITICAL ANGLE

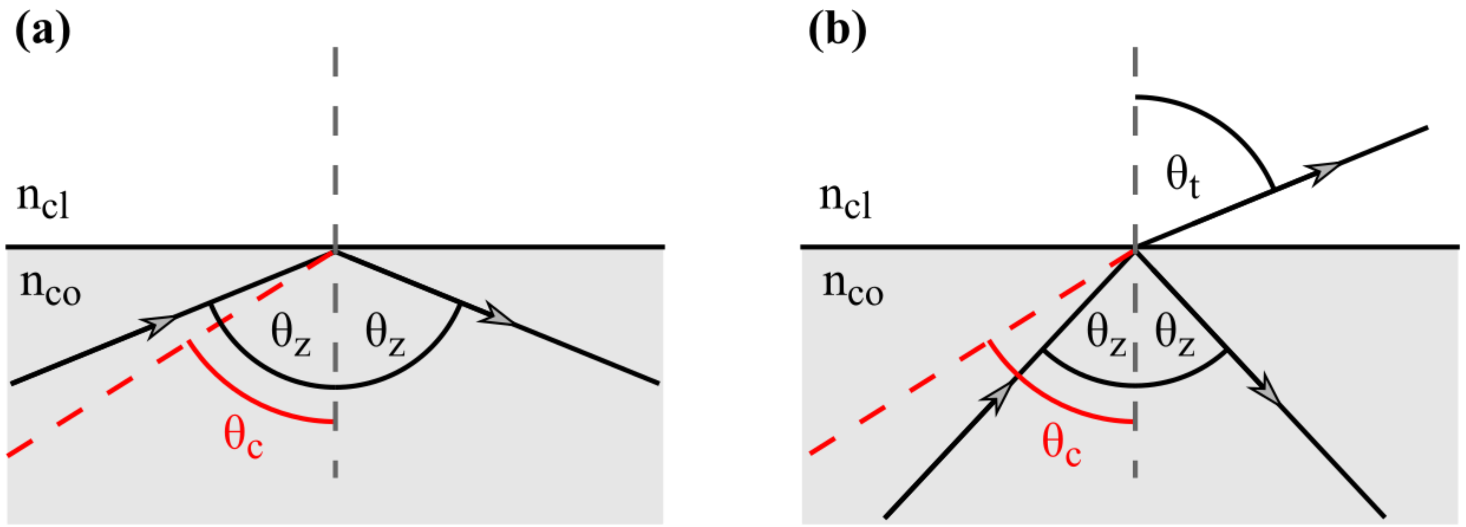
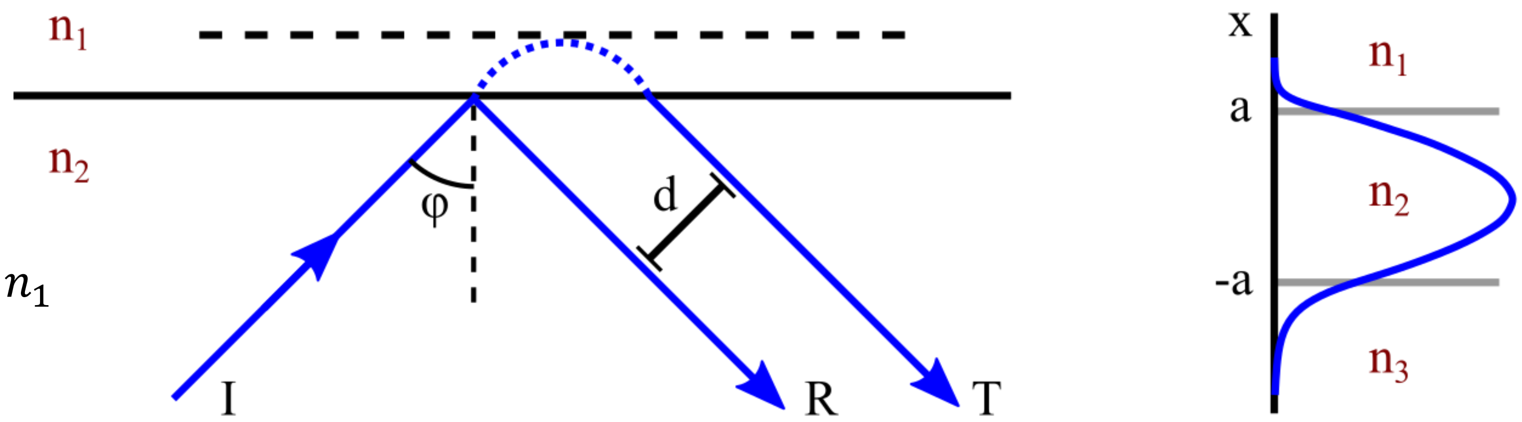


Figure 3: Reflection at a planar interface between unbounded regions of refractive indices n_{co} and n_{cl} showing (a) total internal reflection and (b) partial reflection and refraction.

EVANESCENT WAVE

- An evanescent wave is a side effect of TIR and appears beyond the boundary surface. Specifically, even though the entire incident wave is reflected back into the originating medium (TIR), there is some penetration into the medium with a lower n at the boundary. The evanescent wave is leading to the Goos-Hanchen shift.

$n_2 > n_1$



$n_2 > n_3 > n_1$

Figure 4: Total internal reflection and Goos-Hänchen shift. R is the behavior of the partially reflected beam, T is the behavior of the total internal reflection beam and d is the Goos-Hänchen shift.

EVANESCENT WAVE

- The transmitted wavevector is: $k_t = k_t \sin \theta_t \, \hat{x} + k_t \cos \theta_t \, \hat{z}$
- If $n_1 > n_2$, then $\sin \theta_t > 1$.
- Since $\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i$ (Snell's law), $\frac{n_1}{n_2} \sin \theta_i > 1$ for $\theta_i > \theta_c$ therefore $\cos \theta_t$ becomes complex:

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = j\sqrt{\sin^2 \theta_t - 1} \quad (12)$$

EVANESCENT WAVE

The electric field of the transmitted plane wave is given by $E_t = E_0 e^{j(\overline{k_t} \cdot \bar{r} - \omega t)}$

$$E_t = E_0 e^{j(\overline{k_t} \cdot \bar{r} - \omega t)} = E_0 e^{j[k_t \sin(\theta_t) x + k_t \cos(\theta_t) z - \omega t]} \quad (13)$$

$$E_t = E_0 e^{j\left[x k_t \sin(\theta_t) + z j k_t \sqrt{\sin^2(\theta_t) - 1} - \omega t\right]} \quad (14)$$

By substituting $k_t = \frac{\omega n_2}{c}$ we obtain:

$$E_t = E_0 e^{-\kappa z} e^{j(kx - \omega t)} \quad (15)$$

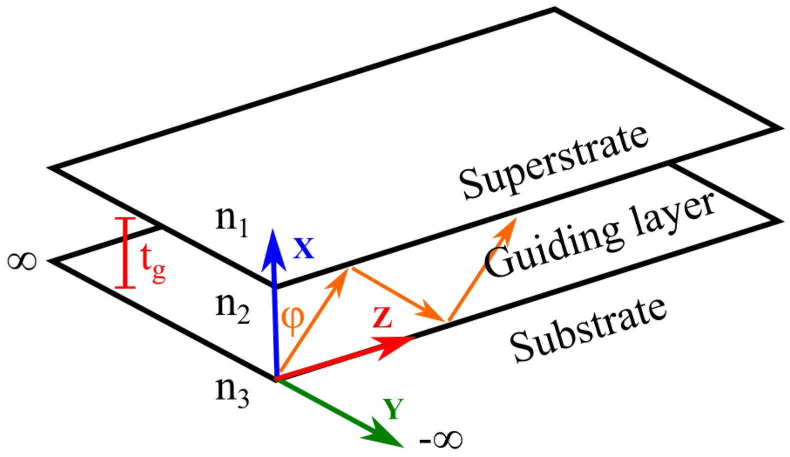
where $k = \frac{\omega n_1}{c}$ and $\kappa = \frac{\omega}{c} \sqrt{{n_1}^2 \sin^2 \theta_i - {n_2}^2}$

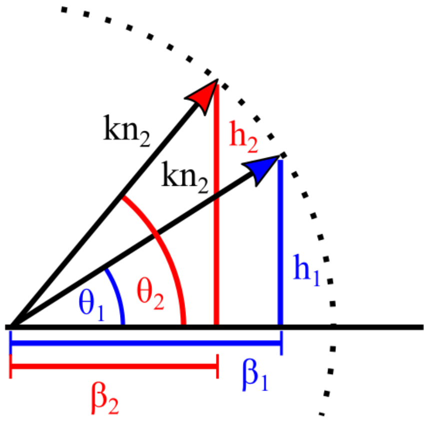
EVANESCENT FIELD - $E_t = E_0 e^{-\kappa \hat{z}} e^{j(k\hat{x} - \omega t)}$

1) Appears in the optically less dense medium.
2) Characterized by its propagation in the x direction.
3) Characterized by its exponential attenuation in the z direction.
4) No energy flows across the boundary.
5) The component of Poynting vector in the direction normal to the boundary is finite, but its time average vanishes (what is Poynting vector? what is time average Poynting vector?).
6) The Goos-Hanchen effect only occurs for linearly polarized light.
7) If the light is circularly or elliptically polarized, it will undergo the analogous Imbert–Fedorov effect.

BACKGROUND

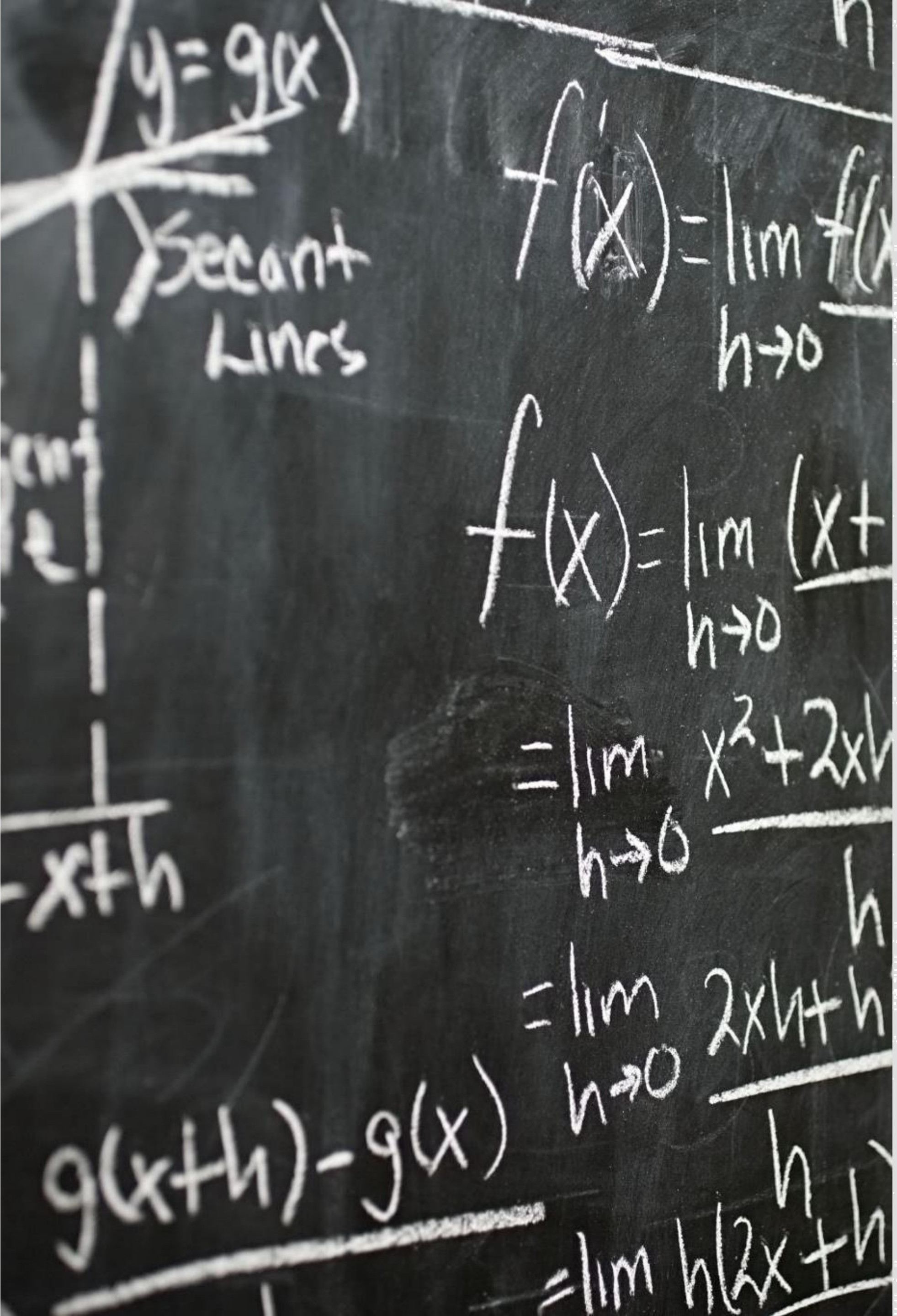
- The optical wave propagates in the waveguide as a mode.
- Mode is the spatial distribution of optical energy propagating inside the waveguide and constant in time.
- Each mode has a different reflection angle. As the order of the mode increases, the reflection angle and the propagation constant





AMP'ERE'S CIRCUITAL LAW

- These days, light is defined as an electromagnetic phenomenon, however, until 1821, electrostatics and magnetostatics were considered separate phenomena.
- In 1821, Danish physicist Hans-Christian Ørsted showed experimentally that flowing electric current creates a magnetic field around it that was observed as a shift in the needle of a compass that was next to the wire.
- This discovery led to the development of '*Amp'ere's circuital law*' which describes the relation between the magnetic flux density B and the flowing electric current I.



MAXWELL'S EQUATIONS

Maxwell's equations, or **Maxwell–Heaviside** equations, are a set of coupled partial differential equations that, together with the **Lorentz force law**, form the foundation of classical electromagnetism, classical optics, and electric circuits. The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc.

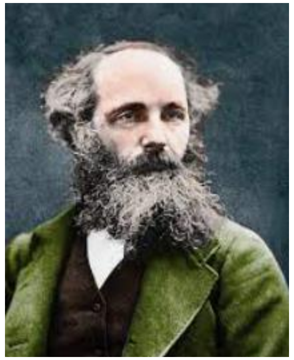
MAXWELL'S EQUATIONS

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho_{\text{ext}}$$

$$\nabla \cdot \vec{B} = 0$$



- The link between electricity and magnetism was completed by the work of James Clerk Maxwell.
- He took the four equations made by Gauss (also Coulomb), Faraday, and Amp'ere and by making some corrections he developed mathematically the connection between those equations.
- In 1861, Maxwell presented a set of coupled equations (around 20 equations) that describe electromagnetic phenomena varying in time which are called Maxwell's equations.
- The four equations known today were obtained by Oliver Heaviside, using vector notation to simplify 12 of the 20 equations into the 4 known Maxwell's equations.

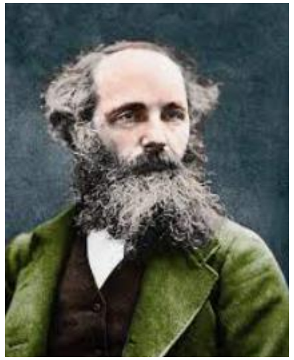
MAXWELL'S EQUATIONS

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

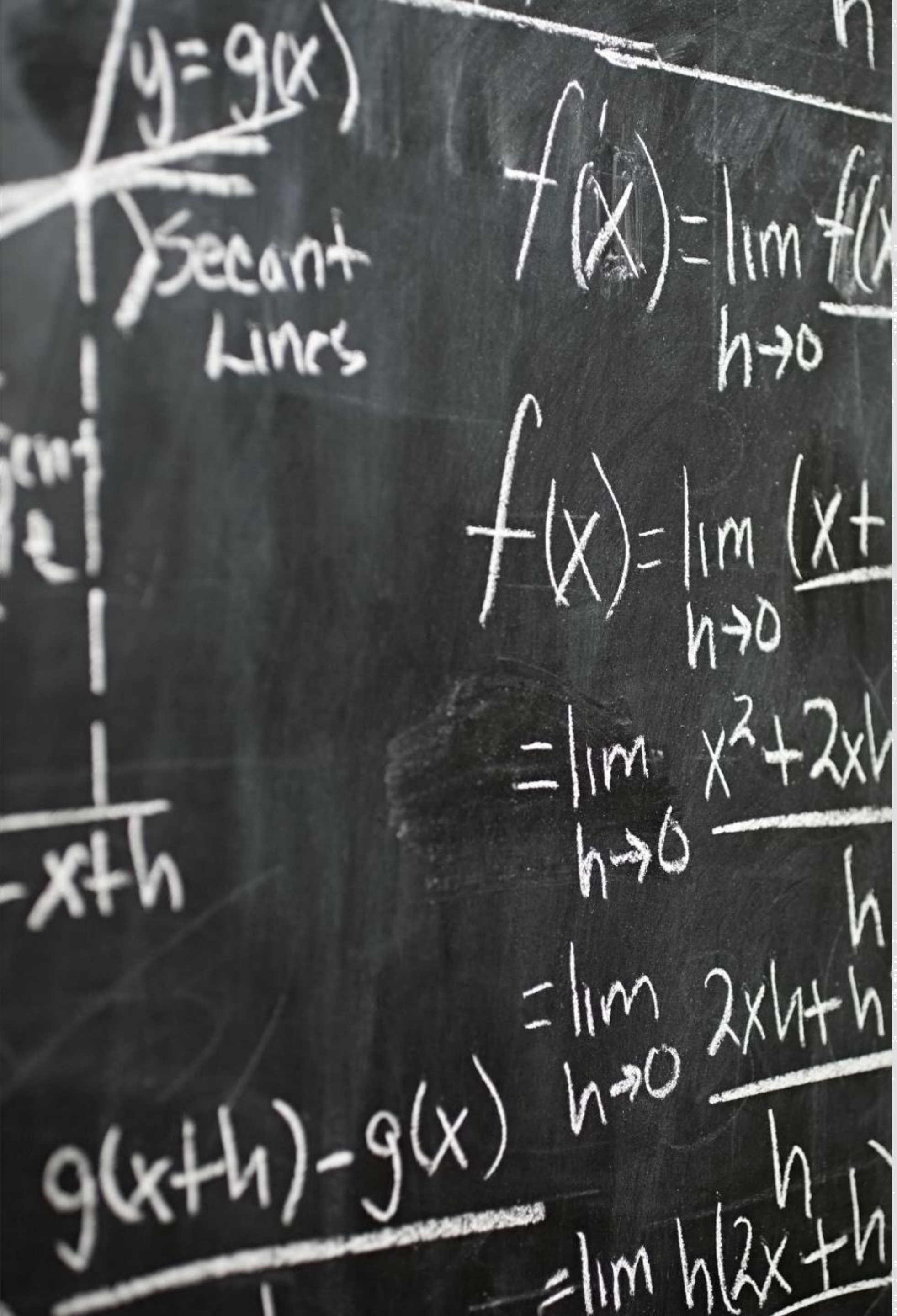
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho_{\text{ext}}$$

$$\nabla \cdot \vec{B} = 0$$



- These equations can be used as a mathematical model for phenomena in nature and for electrical and optical problems.
- In a paper published in 1865, Maxwell has derived a wave equation from his equations thus discovering electromagnetic waves.
- He suggested that light is an electromagnetic wave and showed this hypothesis to be consistent with experimental results. Therefore, he concluded that light is an electromagnetic wave.
- In 1886-1889, German physicist Heinrich Rudolf Hertz performed a series of experiments that proved that light is an electromagnetic wave as was analytically calculated by Maxwell.



MAXWELL'S EQUATIONS

Assumptions:

1. The parameters of the medium in a linear system don't dependent on the electric field E and the magnetic field H: $\varepsilon = \varepsilon_r \varepsilon_0 \quad \mu = \mu_0$.
2. The medium parameters μ and ε are constant and time independent.
3. The medium is isotropic $\Rightarrow$ μ and ε are direction independent.
4. The medium is dielectric $\Rightarrow J = 0$ and $\rho_{\text{ext}} = 0$

MAXWELL'S EQUATIONS

- Assuming linear, homogeneous and isotropic medium, Maxwell's equations are defined as

Faraday's law
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (16)$$

Ampere-Maxwell law
$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \qquad (17)$$

Gauss law
$$\nabla \cdot \vec{D} = \rho_{\text{ext}} \qquad (18)$$

Gauss's law for magnetism
$$\nabla \cdot \vec{B} = 0 \qquad (19)$$

where E is the electric field vector, D is the electric displacement field vector, H is the magnetic field vector and B is the magnetic flux density vector. ρ_{ext} and J are the charge and current densities, respectively.

MAXWELL'S EQUATIONS

- The current density is defined as $J = \sigma E$, where σ is the electric conductivity, and only exists in ohmic material, such as metals and semiconductors.
- In dielectric medium, $J = 0$ and $\rho_{\text{ext}} = 0$.
- D and B are related to the field vectors and are defined as

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$
$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

where ε_0 and μ_0 are the electric permittivity and magnetic permeability of vacuum, respectively, P is the polarization and M is the magnetization.

POLARIZATION AND MAGNETIZATION

In the case of isotropic material, the polarization and the magnetization are given by

$$\vec{P} = \varepsilon_0 \chi \vec{E} \qquad \vec{M} = \mu_0 \chi \vec{H}$$

and

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E} \tag{20}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M} = \mu_0 \mu_r \vec{H} = \mu \vec{H} \tag{21}$$

where μ is the permeability, ε is the permittivity, c is the speed of light in vacuum, χ is the electric susceptibility and ε_r is called the relative permittivity and μ_r the relative permeability, which in case of non-magnetic material is $\mu_r = 1$.

MAXWELL'S EQUATIONS

- Maxwell's equations for dielectric waveguide are given as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{16}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \tag{17}$$

$$\nabla \cdot \vec{D} = 0 \tag{18}$$

$$\nabla \cdot \vec{B} = 0 \tag{19}$$

where

$$\vec{B} = \mu \vec{H} \qquad \vec{D} = \varepsilon \vec{E}$$

MAXWELL'S EQUATIONS

Reminder:

The divergence of a vector function:

$$\vec{\nabla}\cdot\vec{f}=\frac{\partial f_x}{\partial x}+\frac{\partial f_y}{\partial y}+\frac{\partial f_z}{\partial z} \qquad (22)$$

The Laplacian of a vector function:

$$\nabla^2\cdot\vec{f}=\left(\frac{\partial^2 f_x}{\partial x^2}+\frac{\partial^2 f_x}{\partial y^2}+\frac{\partial^2 f_x}{\partial z^2},\frac{\partial^2 f_y}{\partial x^2}+\frac{\partial^2 f_y}{\partial y^2}+\frac{\partial^2 f_y}{\partial z^2},\frac{\partial^2 f_z}{\partial x^2}+\frac{\partial^2 f_z}{\partial y^2}+\frac{\partial^2 f_z}{\partial z^2}\right) \qquad (23)$$

The rotor/curl of a vector function:

$$\vec{\nabla}\times\vec{f}\equiv\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ f_x & f_y & f_z\end{vmatrix}\equiv\left(\frac{\partial f_z}{\partial y}-\frac{\partial f_y}{\partial z}\right)\hat{x}-\left(\frac{\partial f_z}{\partial x}-\frac{\partial f_x}{\partial z}\right)\hat{y}+\left(\frac{\partial f_y}{\partial x}-\frac{\partial f_x}{\partial y}\right)\hat{z} \qquad (24)$$

The BAC CAB law:

$$\mathrm{A}\times(\mathrm{B}\times\mathrm{C})=\mathrm{B}(\mathrm{A}\cdot\mathrm{C})-\mathrm{C}(\mathrm{A}\cdot\mathrm{B})$$

$$\vec{\nabla}\times\left(\vec{\nabla}\times\vec{f}\right)=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{f}\right)-\left(\vec{\nabla}\cdot\vec{\nabla}\right)\vec{f}=\vec{\nabla}\left(\vec{\nabla}\cdot\vec{f}\right)-\nabla^2\vec{f}$$

THE WAVE EQUATION

We operate with rotor/curl (24) on Eq. (16) and use Eq. (20):

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\mu \frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{H}\right) = -\frac{\mu\varepsilon\partial^2 \vec{E}}{\partial t^2} \qquad (25)$$

Using BAC CAB law:

$$\vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = \vec{\nabla}\left(\vec{\nabla} \cdot \vec{E}\right) - \nabla^2 \vec{E}$$

$$\vec{D} = \varepsilon\vec{E} \;\Rightarrow\; \vec{\nabla} \cdot \vec{D} = 0 \;\Rightarrow\; \vec{\nabla} \times \left(\vec{\nabla} \times \vec{E}\right) = -\nabla^2 \vec{E} \qquad (26)$$

The wave equation is:

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (27)$$

THE WAVE EQUATION

The wave equation:

$$\nabla^2 \vec{E} = \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} \qquad (28)$$

where ε is the permittivity and μ is the permeability.

$\varepsilon = \varepsilon_0 \varepsilon_r \quad \varepsilon_0 = 8.854 \times 10^{-12}\ \text{F/m}$

$\mu = \mu_0 \mu_r \quad \mu_0 = 4\pi \times 10^{-7}\ \text{H/m}$

c is the light speed in non-magnetic medium: $c = 1/\sqrt{\varepsilon_0 \mu_0}$.

n is the refractive index: $n = \sqrt{\varepsilon_r}$

Another form of the wave equation:

$$\nabla^2 \vec{E}(r,t) = \frac{n^2(r)}{c^2} \frac{\partial^2 \vec{E}(r,t)}{\partial t^2} \qquad (29)$$

THE WAVE EQUATION SOLUTION

The solution of the wave equation for monochromatic harmonic wave (sinusoidal wave) in time:

$$\vec{E}(r,t) = \vec{E}(r)e^{j\omega t} \tag{30}$$

where ν is the frequency, $c = \lambda_0 \cdot \nu$ and $\omega = 2\pi\nu$.

k_0 is the propagation constant in air (in the course we will use k instead of k_0).

$$k_0 = \frac{2\pi}{\lambda_0} = \frac{\omega}{c} \tag{31}$$

where ω is the angular frequency.

MAXWELL'S EQUATIONS

Considering light as a transverse electromagnetic wave that oscillates perpendicular to the direction of light propagation (z) as

$$\tilde{E} = E(x, y)e^{j(\omega t - \beta z)}$$
$$\tilde{H} = H(x, y)e^{j(\omega t - \beta z)}$$

where $\omega = 2\pi f$ is the angular frequency and β is the propagation constant.

MAXWELL'S EQUATIONS

Substituting the wave equations in Maxwell's Equations (16) and (17), we obtain a set of equations

$$\frac{\partial E_z}{\partial y} + j\beta E_y = -j\omega\mu_0 H_x$$

$$-j\beta E_x - \frac{\partial E_z}{\partial x} = -j\omega\mu_0 H_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu_0 H_z$$

and

$$\frac{\partial H_z}{\partial y} + j\beta H_y = j\omega\varepsilon_0 n^2 E_x$$

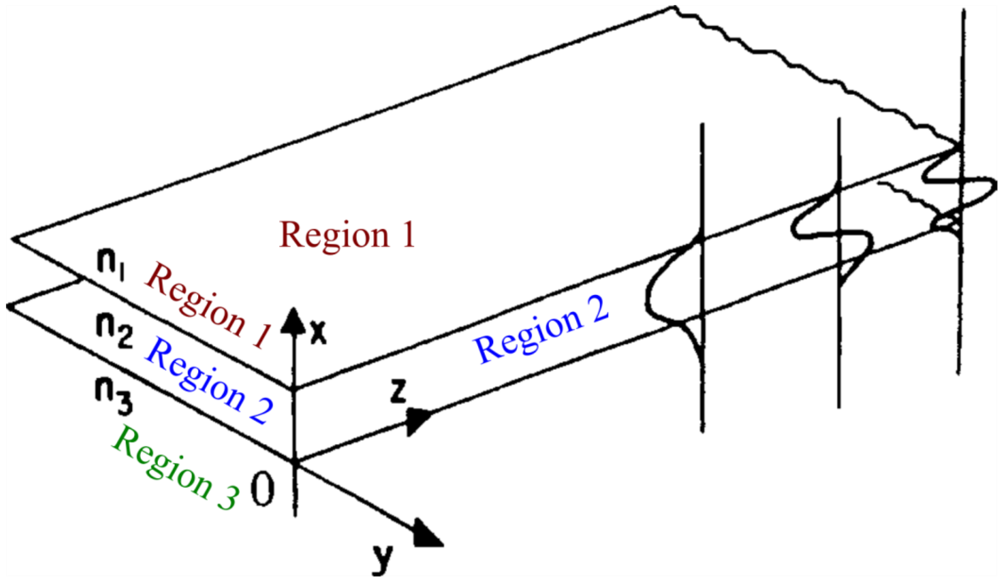
$$j\beta H_x - \frac{\partial H_z}{\partial x} = -j\omega\varepsilon_0 n^2 E_y$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\varepsilon_0 n^2 E_z$$

- These six equations define each electromagnetic field component and can be used to analytically calculate the mode distribution for slab waveguide. Other waveguide configurations, such as rib waveguide, are too complicated and the mode can be calculated only numerically.

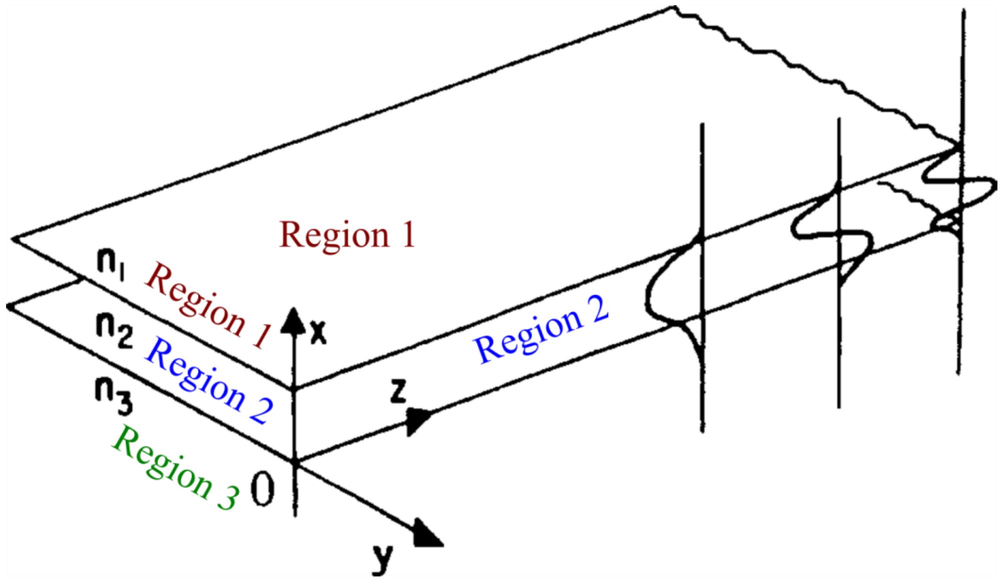
THEORETICAL DESCRIPTION OF THE MODES OF A THREE-LAYER SLAB WAVEGUIDE

- A slab waveguide is characterized by parallel planar boundaries with respect to one (x) direction, but is infinite in extent in the lateral directions (z and y).
- Note: since it is infinite in two dimensions, it is non-practical structure for OIC, but it forms the basis for the analysis of practical waveguides of rectangular cross section.



ASSUMPTIONS

- The layers are all assumed to be infinite in extent in the y and z directions.
- The layers 1 and 3 are also assumed to be semi-infinite in the x direction.
- Light waves are assumed to be propagating in the z direction with β_z.



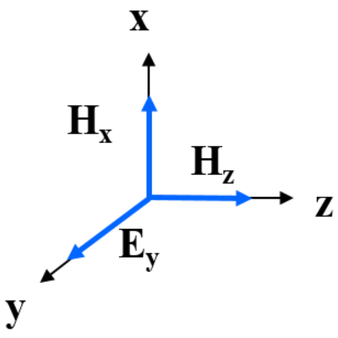
MAXWELL'S EQUATIONS

- Assuming the dimensions of the slab waveguide with infinite width on the y-axis, the electric and magnetic fields do not vary on the y-axis and we obtain $\partial E/\partial y = 0$ and $\partial H/\partial y = 0$.
- Substituting each relations separately in the field equations, we get the following separated solutions.

TE mode

$(H_x, 0, H_z)$

$(0, E_y, 0)$



$$\frac{d^2E_y}{dx} + (k^2n^2 - \beta^2)E_y = 0$$

$$H_x = -\frac{\beta}{\omega\mu_0}E_y$$

$$H_z = \frac{j}{\omega\mu_0}\frac{dE_y}{dx}$$

$$E_x = E_z = H_y = 0$$

and

$$\frac{d}{dx}\left(\frac{1}{n^2}\frac{dH_y}{dx}\right) + \left(k^2 - \frac{\beta^2}{n^2}\right)H_y = 0$$

$$E_x = \frac{\beta}{\omega\varepsilon_0 n^2}H_y$$

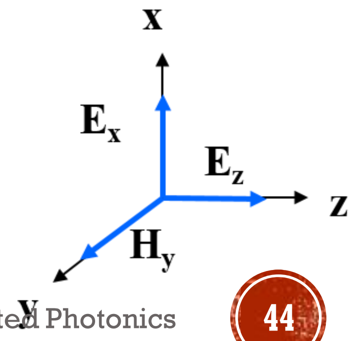
$$E_z = \frac{j}{\omega\varepsilon_0 n^2}\frac{dH_y}{dx}$$

$$E_y = H_x = H_z = 0$$

TM mode

$(0, H_y, 0)$

$(E_x, 0, E_z)$



MAXWELL'S EQUATIONS

- Each set of equations defines the field component of two types of modes in a waveguide. The first set corresponds to the transverse electric (TE - S polarization) mode and the second set to the transverse magnetic (TM - P polarization) mode.
- The 'transverse' means that the field vector is orthogonal to the propagation direction, therefore, having zero longitudinal component. In optical waveguide it is described as 'quasi' because the transverse field is very small.
- It is worth noting that there is another type of mode called transverse electric and magnetic (TEM) mode where $E_z = H_z = 0$. However, dielectric waveguides don't support them.

MAXWELL'S EQUATIONS

Figure below shows the fundamental quasi-TE and quasi-TM modes for a rectangle buried waveguide with dimensions of 1×1.5 μm.

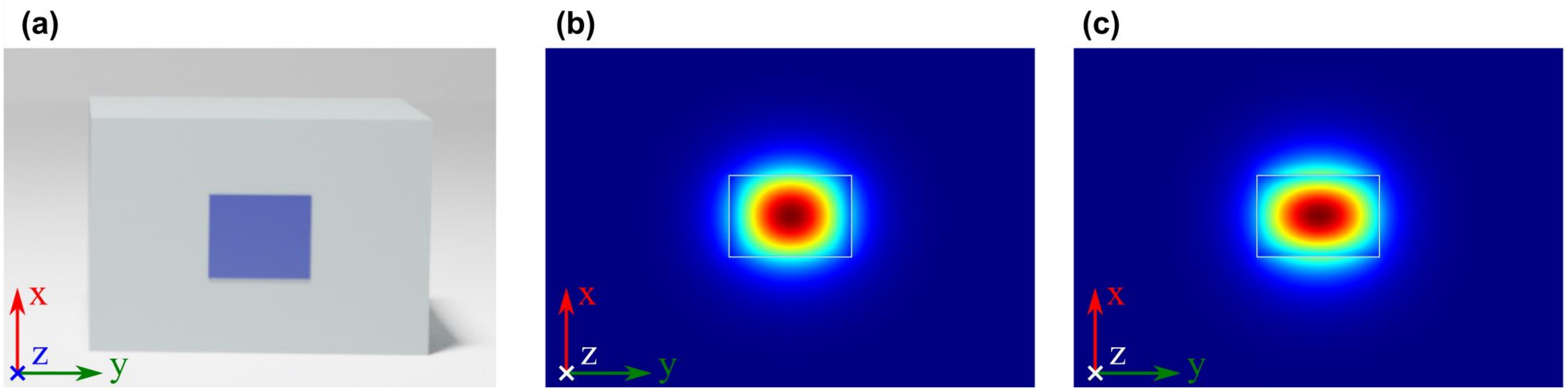


Figure 5: (a) Illustration of a buried waveguide. The field distribution of (b) transverse electric (TE) fundamental mode and (c) transverse magnetic (TM) fundamental mode of buried waveguide with dimensions of 1×1.5 μm.

MAXWELL'S EQUATIONS

It shows that for each mode type, quasi-TE mode or quasi-TM mode, the electric field distribution of the mode is different. The difference in the field distribution between each mode can be utilized for different applications.

- Due to the evanescent field in the x-axis, TM mode can be used for applications that involve overlayer interaction such as plasmons.
- In TE mode the evanescent tail in the y-axis and can be used for side interaction-based applications.

OPTICAL WAVEGUIDE MODES 2

Prof. Alina Karabchevsky, www.alinakarabchevsky.com

Integrated Photonics Course 377-2-5599

School of ECE

Ben-Gurion University of the Negev, Israel

OPTICAL MODES

Mathematical definition of a mode is that it is an electromagnetic field which is a solution of Maxwell's wave equation:

$$\nabla^2 E(r,t) = [n^2/c^2]\partial^2 E(r,t)/\partial t^2 \quad (34)$$

where E is the electric field vector, r is the radius vector, $n(r)$ is the index of refraction, and c is the speed of light in a vacuum. For monochromatic waves, the solutions of Eq. (34) have the form:

$$E(r,t) = E(r)e^{j\omega t} \quad (35)$$

$$\nabla^2 E(r,t) + k^2 n^2(r)E(r) = 0 \quad (36)$$

where $k \equiv \omega/c = 2\pi/\lambda$. If we assume, for convenience, a uniform plane wave propagating in the z direction. $E(r)E(x,y)\exp(-j\beta z)$, β is the a propagation constant, then Eq. (36) becomes:

$$\partial^2 E_y(x)/\partial x^2 + \partial^2 E_y(x)/\partial y^2 + [k^2 n^2 - \beta^2]E_y(x) = 0 \quad (37)$$

REGIONS

Since the waveguide is assumed infinite in the y direction, by writing Eq. (37) separately for the three regions in x, we get:

Region 1: $0 \leq x \leq \infty$

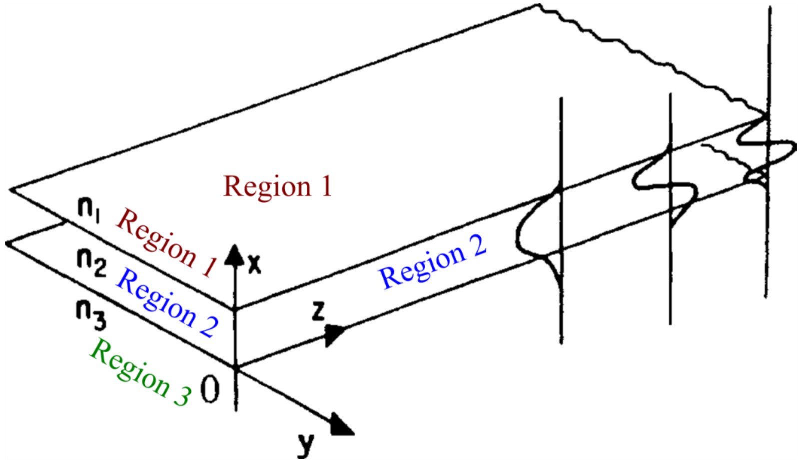
$$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_1}^2 - \beta^2) E_y(x) = 0$$

Region 2: $-t_g \leq x \leq 0$

$$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_2}^2 - \beta^2) E_y(x) = 0$$

Region 3: $-\infty \leq x \leq -t_g$

$$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_3}^2 - \beta^2) E_y(x) = 0$$

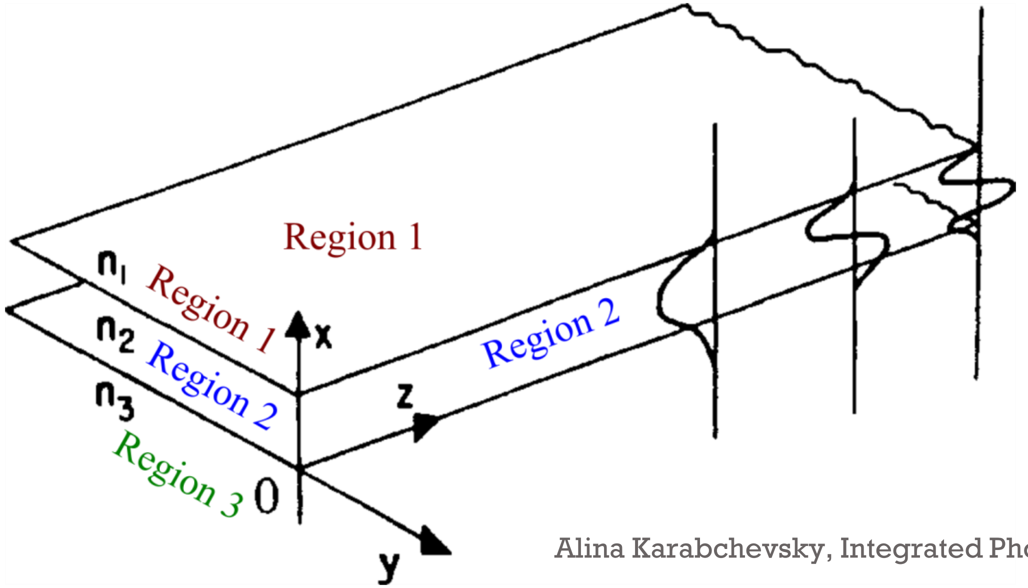


REGIONS

The solutions of Eq. (34) are either sinusoidal or exponential functions of x in each of the regions, depending on whether $(k^2 {n_i}^2 - \beta^2)$ for $i = 1, 2, 3$, is greater than or less than zero. Of course, $E(x, y)$ and $\partial E(x, y)/\partial x$ must be continuous at the interface between layers.

Waveguiding condition:

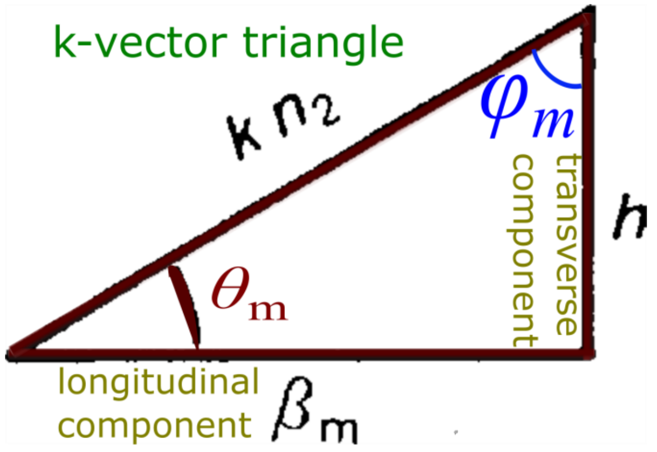
$n_2 > n_3 \geq n_1$ or $n_2 > n_1 \geq n_3$



REGION 1

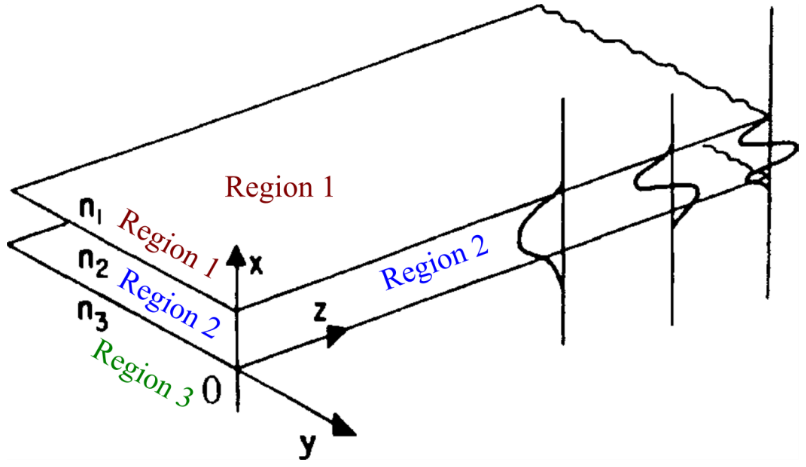
Domain 1: $0 \leq x \leq \infty$

$$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_1}^2 - \beta^2) E_y(x) = 0$$



- From the waveguiding condition in layer 2

$$k n_2 \sin\phi_2 > k n_1 \;\Rightarrow\; \beta > k n_1 \;\Rightarrow\; k^2 {n_1}^2 - \beta^2 < 0$$

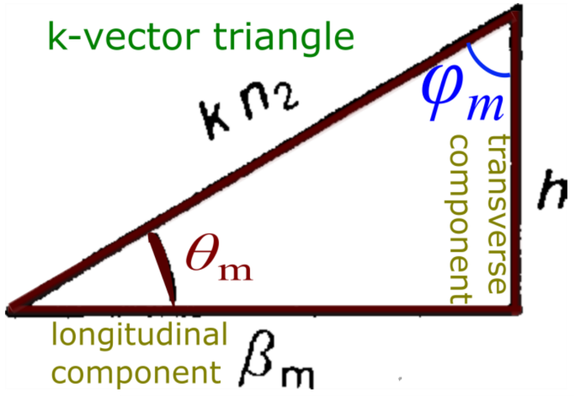


- Exponential solution of the wave equation

$$E_y(x) = A e^{-qx}$$

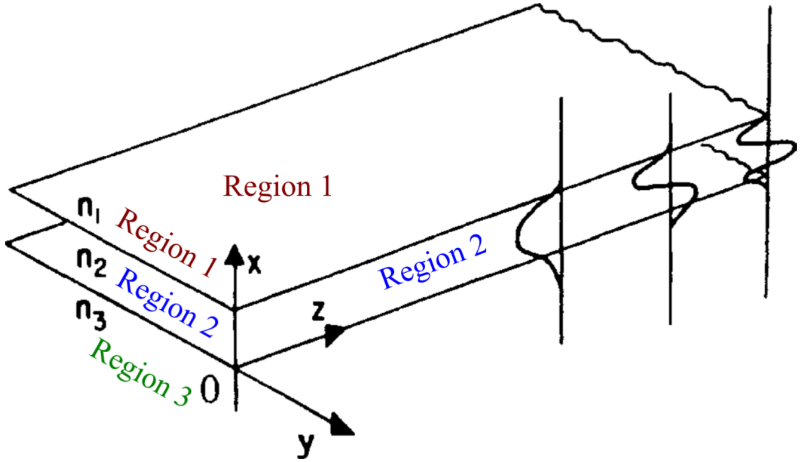
While $q = \sqrt{\beta^2 - k^2 {n_1}^2}$

REGION 2



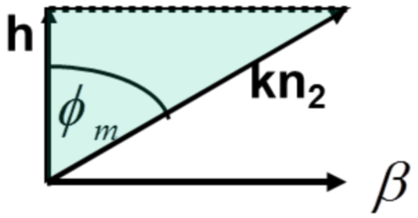
Domain 2: $-t_g \leq x \leq 0$

$$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_2}^2 - \beta^2) E_y(x) = 0$$



- From the waveguiding condition in layer 2

$$kn_2 > kn_2 \sin\phi\,, \quad |\sin| < 1 \Rightarrow \beta < kn_2 \;\Rightarrow\; k^2 {n_2}^2 - \beta^2 > 0$$

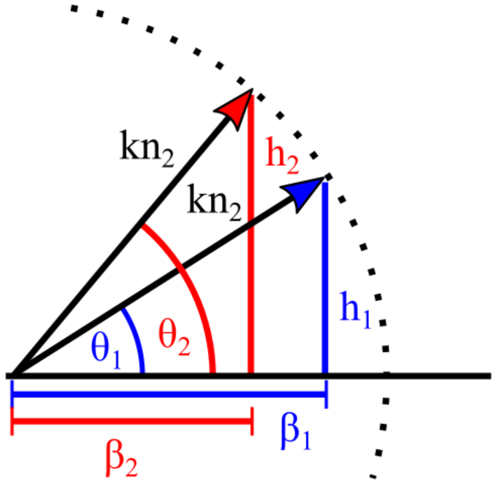


- Sinusoidal solution of the wave equation

$$E_y(x) = B\cos(hx) + C\sin(hx)$$

While $h = \sqrt{k^2 {n_2}^2 - \beta^2}$ is the propagation constant in x direction.

WAVE OPTICS VS. RAY OPTICS ANALYSIS IN REGION 2



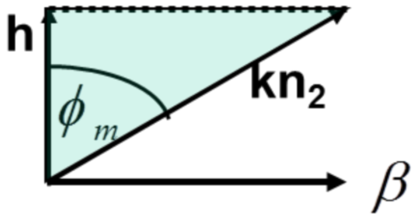
Domain 2: $-t_g \leq x \leq 0$

$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_2}^2 - \beta^2) E_y(x) = 0$

- Using Pythagorean relation:

$\beta^2 + h^2 = k^2 {n_2}^2$

$k_2 = \omega n_2/c$ with ω constant. kn_2, β and h are the propagation constants with units of 1/length.



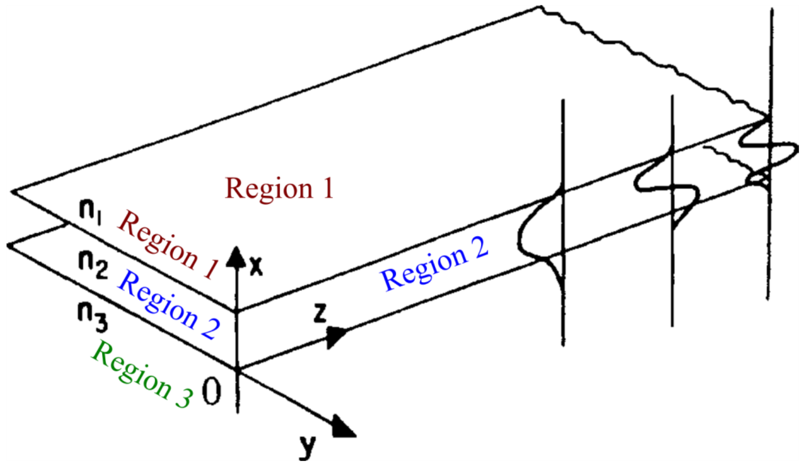
- Plane wave propagates in z direction with angle of:

$\phi_m = \tan^{-1}(\beta_m/h_m)$

While $q = \sqrt{k^2 {n_2}^2 - \beta^2}$

ϕ_m, h_m and β_m belong to the discrete mode m.

REGION 3



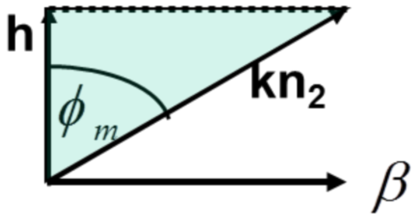
Domain 3: $-\infty \leq x \leq -t_g$

$$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_3}^2 - \beta^2) E_y(x) = 0$$

- From the waveguiding condition in layer 2

$$k n_2 \sin \phi > k n_3 \;\Rightarrow\; \beta > k n_3 \;\Rightarrow\; k^2 {n_3}^2 - \beta^2 > 0$$

- Exponential solution of the wave equation



$$E_y(x) = D e^{p(x+t_g)}$$

While $p = \sqrt{\beta^2 - k^2 {n_3}^2}$ is the propagation constant in x direction and $x + t_g < 0$.

TE SOLUTION IN REGIONS: SUMMARY

- Wave equation

$\partial^2 E_y(x)/\partial x^2 + (k^2 {n_3}^2 - \beta^2) E_y(x) = 0$

with $i = 1, 2, 3$

Domain 1: $0 \leq x \leq \infty$

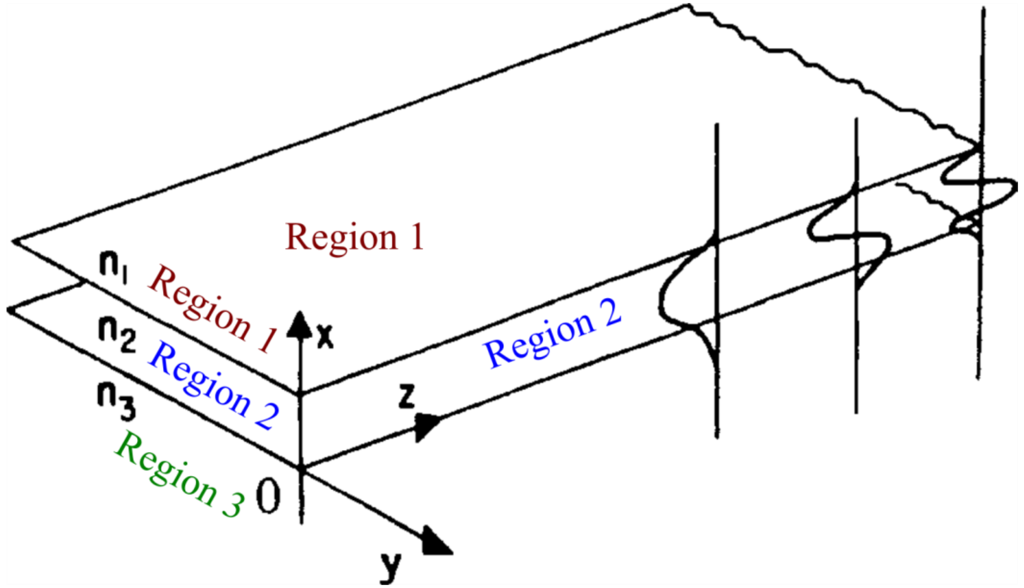
$E_y(x) = Ae^{-qx}$ with $q = \sqrt{\beta^2 - k^2 {n_1}^2}$

Domain 2: $-t_g \leq x \leq 0$

$E_y(x) = B\cos(hx) + C\sin(hx)$ with $h = \sqrt{k^2 {n_2}^2 - \beta^2}$

Domain 3: $-\infty \leq x \leq -t_g$

$E_y(x) = De^{p(x+t_g)}$ with $p = \sqrt{\beta^2 - k^2 {n_3}^2}$ and $x + t_g < 0$



CONSTANTS

Solution: $E_y(x)$	Domain
Ae^{-qx}	$0 \le x \le \infty$
$B\cos(hx) + C\sin(hx)$	$-t_g \le x \le 0$
$De^{p(x+t_g)}$	$-\infty \le x \le -t_g$

I $E_{t1} = E_{t2}$

$$E_y(x = 0^+) = E_y(x = 0^-) \Rightarrow A = B$$

II $E_{t2} = E_{t3}$

$$E_y\left(x = -t_g{}^+\right) = E_y\left(x = -t_g{}^-\right) \Rightarrow B\cos(-t_g h) + C\sin(-t_g h) = D$$

III $H_{t1} = H_{t2}$

$$H_z(0^+) = H_z(0^-) \Rightarrow A = -\frac{hC}{q}$$

$$H_z = j\frac{1}{\omega\mu}\frac{\partial E_y}{\partial x}$$

SURVEY: BOUNDARY CONDITIONS

- EasyPolls:

Boundary conditions

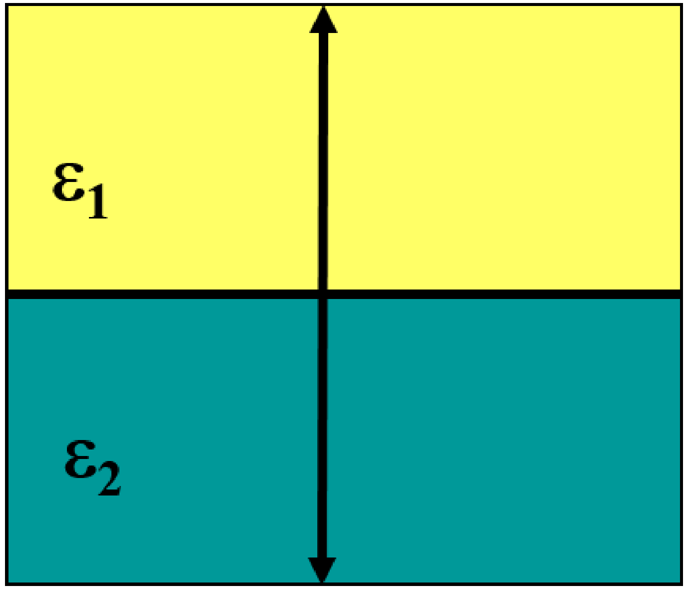
- ○ The tangential components of the electric field are continuous on the boundary
- ○ The tangential components of the electric displacement field are continuous on the boundary
- ○ The vertical components of the electric displacement field are continuous between the two media
- ○ The vertical components of the electric field are continuous between the two media

results vote

BOUNDARY CONDITIONS

- The boundary conditions define the behavior of the electric and the magnetic fields on the boundary.
- Assume two different media with permittivity of ε_1 and ε_2, as shown in the figure.
- The electric field - E and the magnetic field H can be decomposed to the tangential (t) and vertical (n) components.

$$E = E_n + E_t \qquad H = H_n + H_t$$



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE ELECTRIC FIELD - E_t

- Assume electric field in medium 1 (ε_1). From Faraday's law:

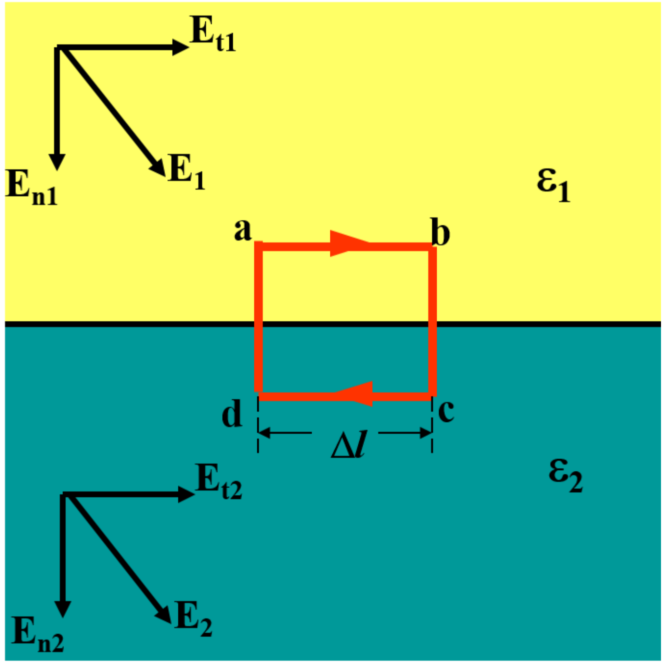
$$\oint E \cdot dl = -\iint \frac{\partial B}{\partial t} dA = 0$$

- Faraday's law for closed loop $a \Rightarrow b \Rightarrow c \Rightarrow d$ is:

$$\oint E \cdot dl = \int_a^b \ldots + \int_b^c \ldots + \int_c^d \ldots + \int_d^a \ldots = 0$$

- Assuming $a - d$ and $b - c$ equal 0 then:

$$\oint E \cdot dl = \int_a^b \ldots + \int_c^d \ldots$$



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE ELECTRIC FIELD - E_t

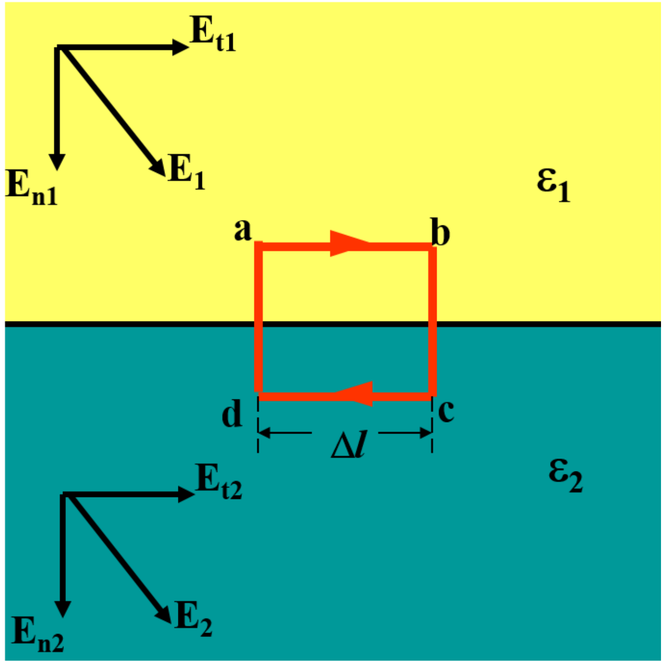
- $\int_a^b \ldots \approx E_{t1}\Delta l$ and $\int_c^d \ldots \approx -E_{t2}\Delta l$ therefore $E_{t1}\Delta l - E_{t2}\Delta l = 0$ and:

$$\boxed{E_{t1} = E_{t2}}$$

- The tangential components of the electric field are continuous on the boundary.
- In addition, $D_i = \varepsilon_{ij}E_j$ therefore:

$$\boxed{\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}}$$

The tangential components of the electric displacement field are not continuous on the boundary.



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE ELECTRIC FIELD - E_n

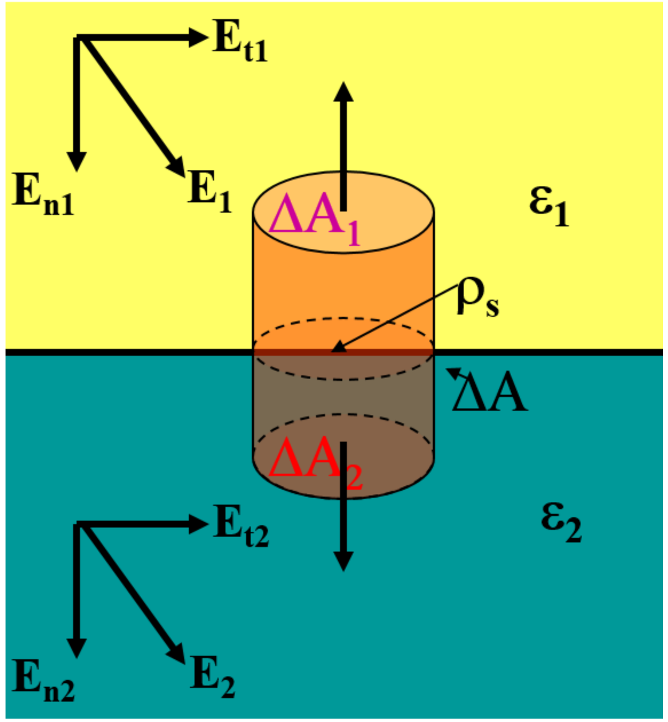
- From Gauss's law:

$$\oint D \cdot dA = \iiint \rho \, dv = Q_{\text{enclosed}}$$

- The figure shows that the surrounded charge is a surface.
- We write Gauss's law as:

$$\oint D \cdot dA = \iint \rho_s \, dA$$

where ρ_s (units of [C/m^2] is the charge on the boundary.



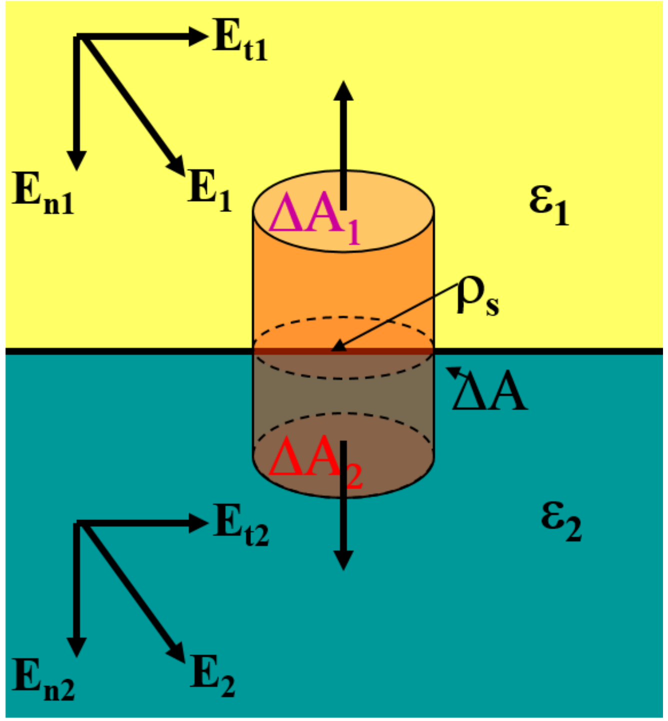
BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE ELECTRIC FIELD - E_n

- We write the equation as:

$$-D_{n1}\Delta A_1 + D_{n2}\Delta A_2 = \rho_s \Delta A$$

- Vertical vectors are defined far from the boundary and the electric field is in medium 1.
- Since $\Delta A_1 = \Delta A_2 = \Delta A$ therefore:

$$-D_{n1} + D_{n2} = \rho_s$$



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE ELECTRIC FIELD - E_n

- In addition:

$$-\varepsilon_1\varepsilon_0 E_{n1} + -\varepsilon_2\varepsilon_0 E_{n2} = \rho_s$$

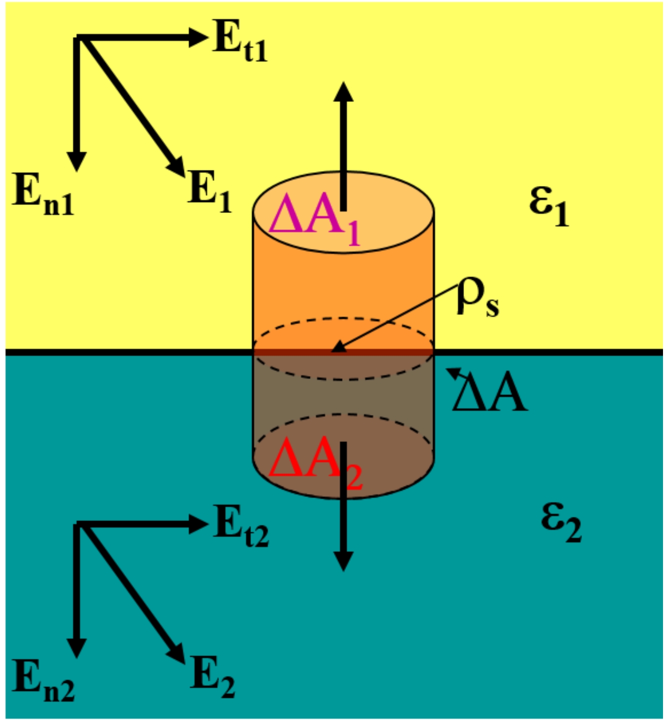
- Without charge on the boundary, we get:

$$\boxed{D_{n1} = D_{n2}}$$

and

$$\boxed{\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}}$$

The vertical components of the electric displacement field are continuous between the two media but the vertical components of the electric field are not.



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE MAGNETIC FIELD - H_t

- Assume boundary between two media with different permeability of μ_1 and μ_2.
- Assume Ampere's law without currents on the boundary:

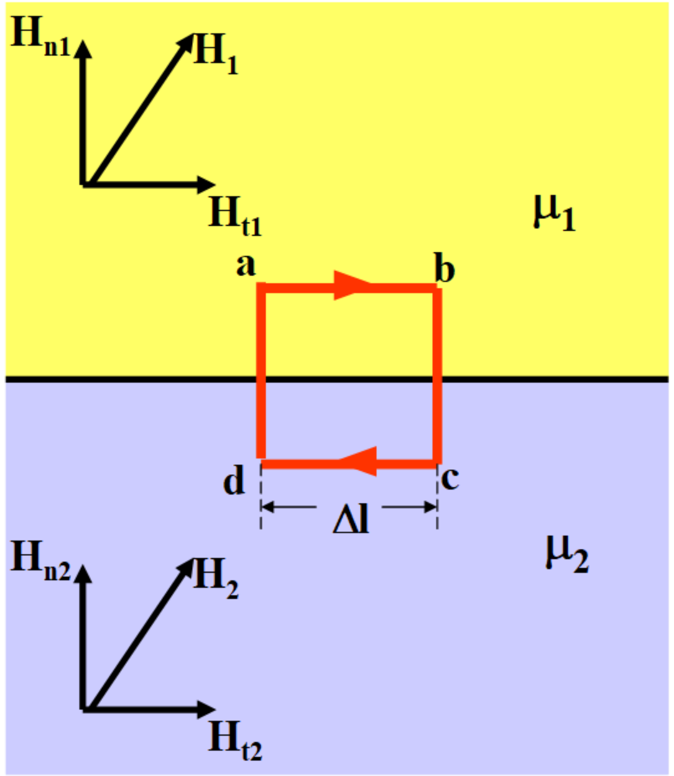
$$\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) dA = 0$$

For closed loop is:

$$\oint \ldots = \int_a^b \ldots + \int_b^c \ldots + \int_c^d \ldots + \int_d^a \ldots = 0$$

- Assuming $a - d$ and $b - c$ equal 0 then:

$$\oint H \cdot dl = \int_a^b \ldots + \int_c^d \ldots = 0$$



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE MAGNETIC FIELD - H_t

- $\int_a^b \ldots \approx H_{t1}\Delta l$ and $\int_c^d \ldots \approx -H_{t2}\Delta l$ therefore $H_{t1}\Delta l - H_{t2}\Delta l = 0$ and so:

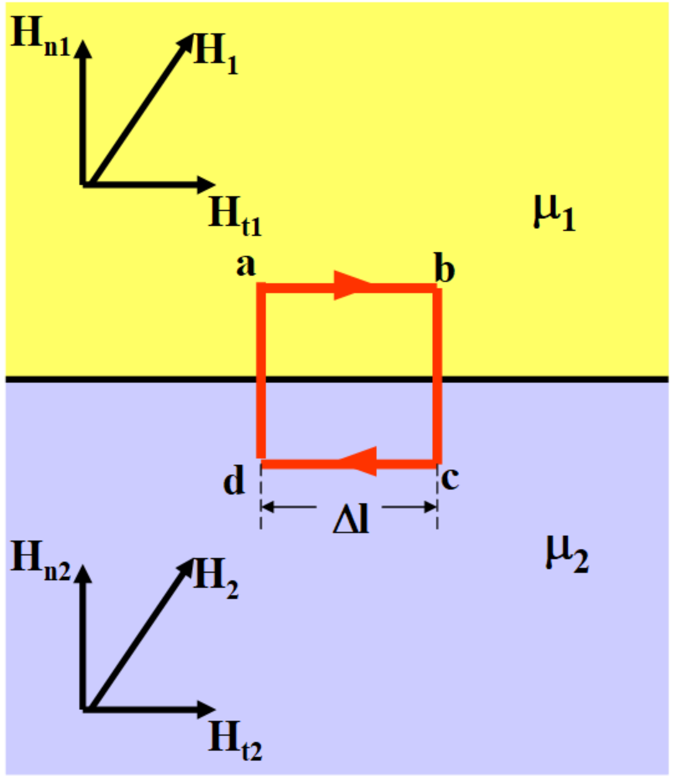
$$\boxed{H_{t1}\Delta l = H_{t2}\Delta l}$$

The tangential components of the magnetic field are continuous on the boundary between the two media.

- From $B = \mu H$ we get:

$$\boxed{\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}}$$

The tangential component of the magnetic flux density are not continuous on the boundary between the two media.



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE MAGNETIC FIELD - H_n

- From magnetic Gauss's law:

$$\oint B \cdot dA = 0$$

- From the figure:

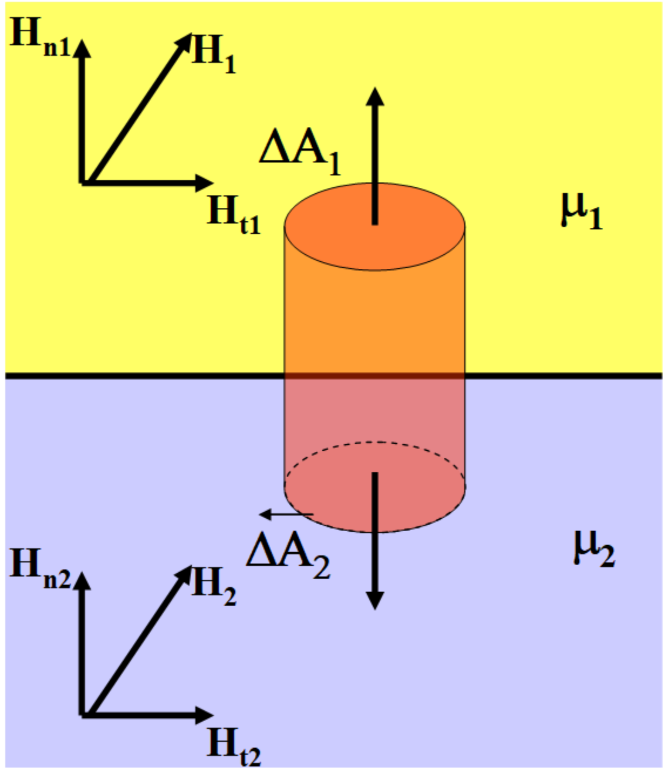
$$\oint \ldots = \oint_{\text{top}} \ldots + \oint_{\text{side}} \ldots + \oint_{\text{bottom}} \ldots = 0$$

- We shrink the cylinder - $\oint_{\text{side}} \ldots \rightarrow 0$ therefore:

$$\oint_{\text{top}} \ldots \approx B_{n1}\Delta A \quad \text{and} \quad \oint_{\text{bottom}} \ldots \approx B_{n2}\Delta A$$

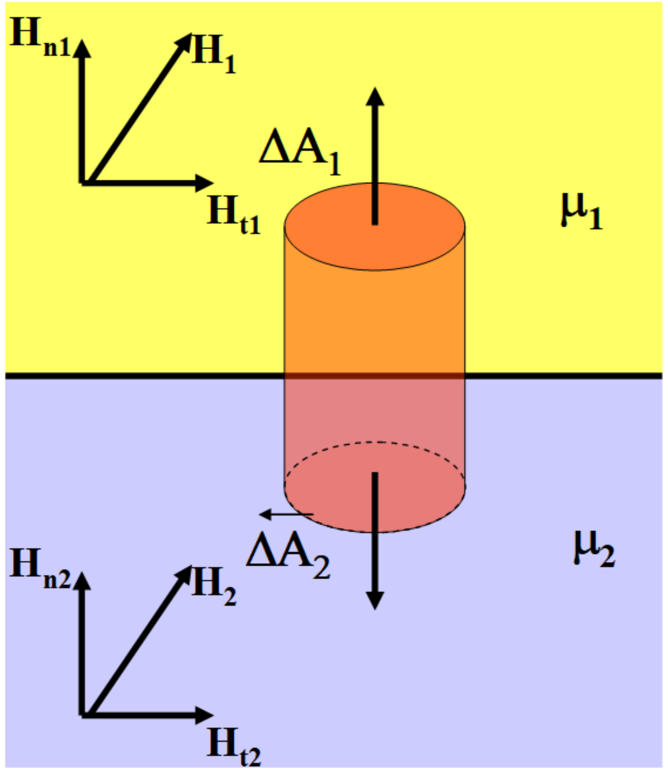
- We get:

$$\boxed{B_{n1} = B_{n2}} \qquad \boxed{\mu_1 H_{n1} = \mu_2 H_{n2}}$$



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE MAGNETIC FIELD - H_n

The vertical components of the magnetic flux density are continuous on the boundary, but the vertical components of the magnetic field are not.



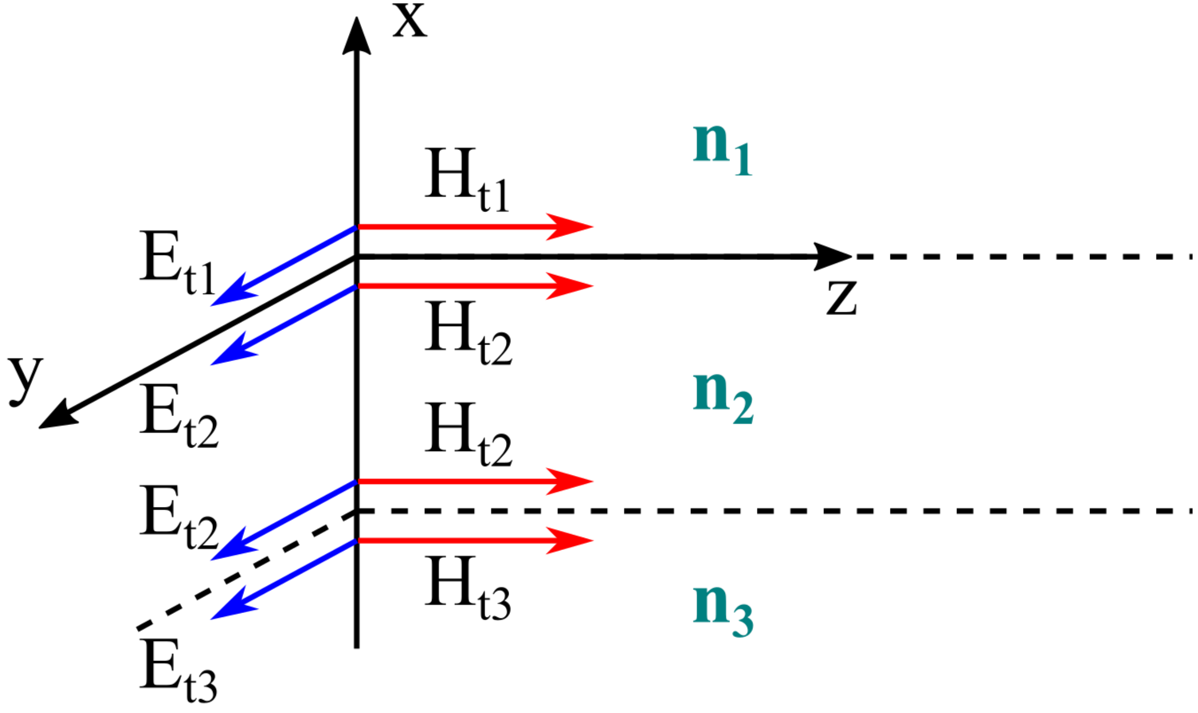
BOUNDARY CONDITIONS

I. $E_{t1} = E_{t2}$

II. $E_{t2} = E_{t3}$

III. $H_{t1} = H_{t2}$

IV. $H_{t2} = H_{t3}$



CONSTANTS

Solution: $E_y(x)$	**Domain**
Ae^{-qx}	$0 \le x \le \infty$
$B\cos(hx) + C\sin(hx)$	$-t_g \le x \le 0$
$De^{p(x+t_g)}$	$-\infty \le x \le -t_g$

I $E_{t1} = E_{t2}$

$$E_y(x = 0^+) = E_y(x = 0^-) \Rightarrow A = B$$

II $E_{t2} = E_{t3}$

$$E_y\left(x = -t_g{}^+\right) = E_y\left(x = -t_g{}^-\right) \Rightarrow B\cos(-t_g h) + C\sin(-t_g h) = D$$

III $H_{t1} = H_{t2}$

$$H_z(0^+) = H_z(0^-) \Rightarrow A = -\frac{hC}{q}$$

OPTICAL WAVEGUIDE MODES

Prof. Alina Karabchevsky, www.alinakarabchevsky.com

Integrated Photonics Course 377-2-5599

School of ECE

Ben-Gurion University of the Negev, Israel

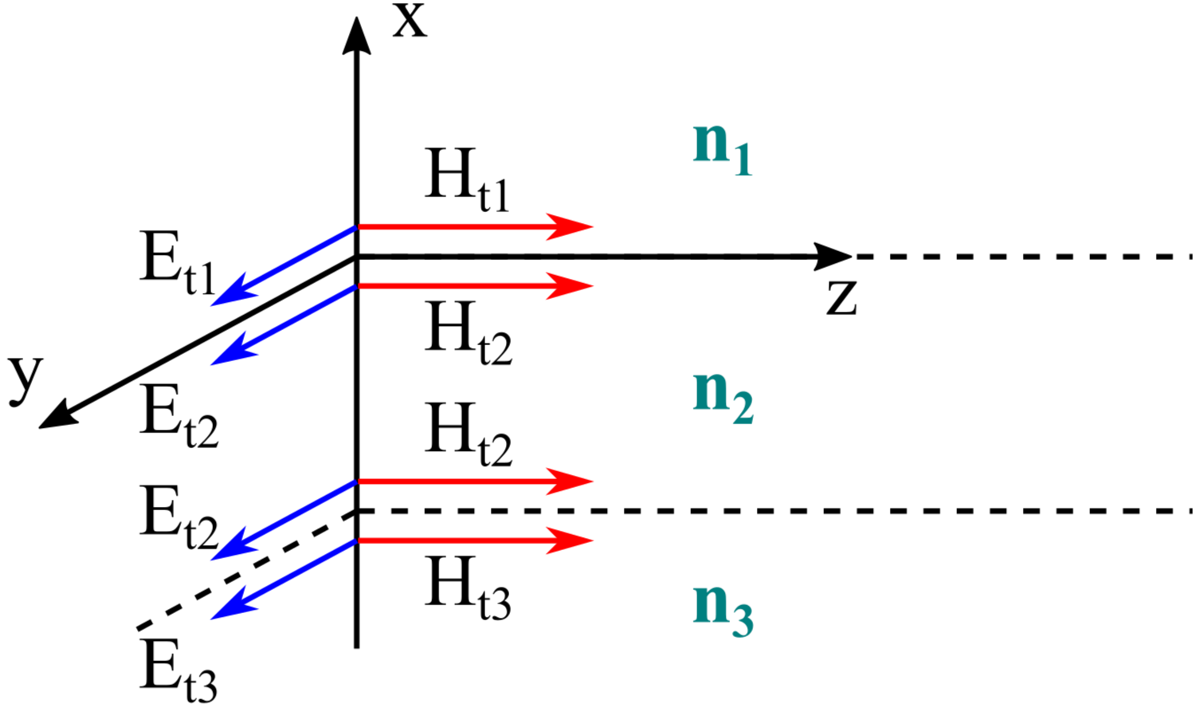
BOUNDARY CONDITIONS

I. $E_{t1} = E_{t2}$

II. $E_{t2} = E_{t3}$

III. $H_{t1} = H_{t2}$

IV. $H_{t2} = H_{t3}$



CONSTANTS

Solution: $E_y(x)$	**Domain**
Ae^{-qx}	$0 \le x \le \infty$
$B\cos(hx) + C\sin(hx)$	$-t_g \le x \le 0$
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I $E_{t1} = E_{t2}$

$$E_y(x = 0^+) = E_y(x = 0^-) \Rightarrow A = B$$

II $E_{t2} = E_{t3}$

$$E_y\left(x = -t_g{}^+\right) = E_y\left(x = -t_g{}^-\right) \Rightarrow B\cos(-t_g h) + C\sin(-t_g h) = D$$

III $H_{t1} = H_{t2}$

$$H_z(0^+) = H_z(0^-) \Rightarrow A = -\frac{hC}{q}$$

CONSTANTS

Let $K = A$, we formulate B, C, D via K

$$K = B, \qquad K\left[\cos(t_g h) - \frac{q}{h}\sin(t_g h)\right] = D, \qquad C = -\frac{Kq}{h}$$

Solution: $E_y(x)$	**Domain**
Ke^{-qx}	$0 \le x \le \infty$
$K\left[\cos(hx) - \frac{q}{h}\sin(hx)\right]$	$-t_g \le x \le 0$
$K\left[\cos(ht_g) - \sin(ht_g)\right]e^{p(x+t_g)}$	$-\infty \le x \le -t_g$

$$K_m = 2h_m\sqrt{\frac{\omega\mu}{|\beta_m|\left(t_g + \frac{1}{q_m} + \frac{1}{p_m}\right)\left({h_m}^2 + {q_m}^2\right)}}$$

CONSTANTS

To find one remaining constant K, we normalized the field so that it has a power flow of 1 W/m in the y direction per unit width using the Poynting vector:

$$-\frac{1}{2}\int_{-\infty}^{\infty} E_y H_x^* \, dx = \frac{\beta_m}{2\omega\mu}\int_{-\infty}^{\infty} \left|\mathcal{E}_y(x)\right|^2 dx = 1 \qquad (38)$$

we have:

$$\begin{aligned}
&\int_{-\infty}^{\infty} \left|\mathcal{E}_y(x)\right|^2 dx = \\
&\int_{-\infty}^{0} \left|K_m\left[\cos\left(h_m t_g\right) - \sin\left(h_m t_g\right)\right] \exp\left[p_m\left(x + t_g\right)\right]\right|^2 dx \\
&+ \int_{-tg}^{0} |K_m[\cos(h_m x) - (q_m/h_m)\sin(h_m x)]|^2 \, dx \\
&+ \int_{0}^{\infty} |K_m \exp(-q_m x)|^2 dx = \frac{2\omega\mu}{\beta_m}
\end{aligned} \qquad (39)$$

Solving (39) yields:

$$K_m = 2h_m \sqrt{\frac{\omega\mu}{|\beta_m|\left(t_g + \frac{1}{q_m} + \frac{1}{p_m}\right)\left({h_m}^2 + {q_m}^2\right)}} \qquad (40)$$

POSSIBLE OPTICAL MODES IN A SLAB WAVEGUIDE

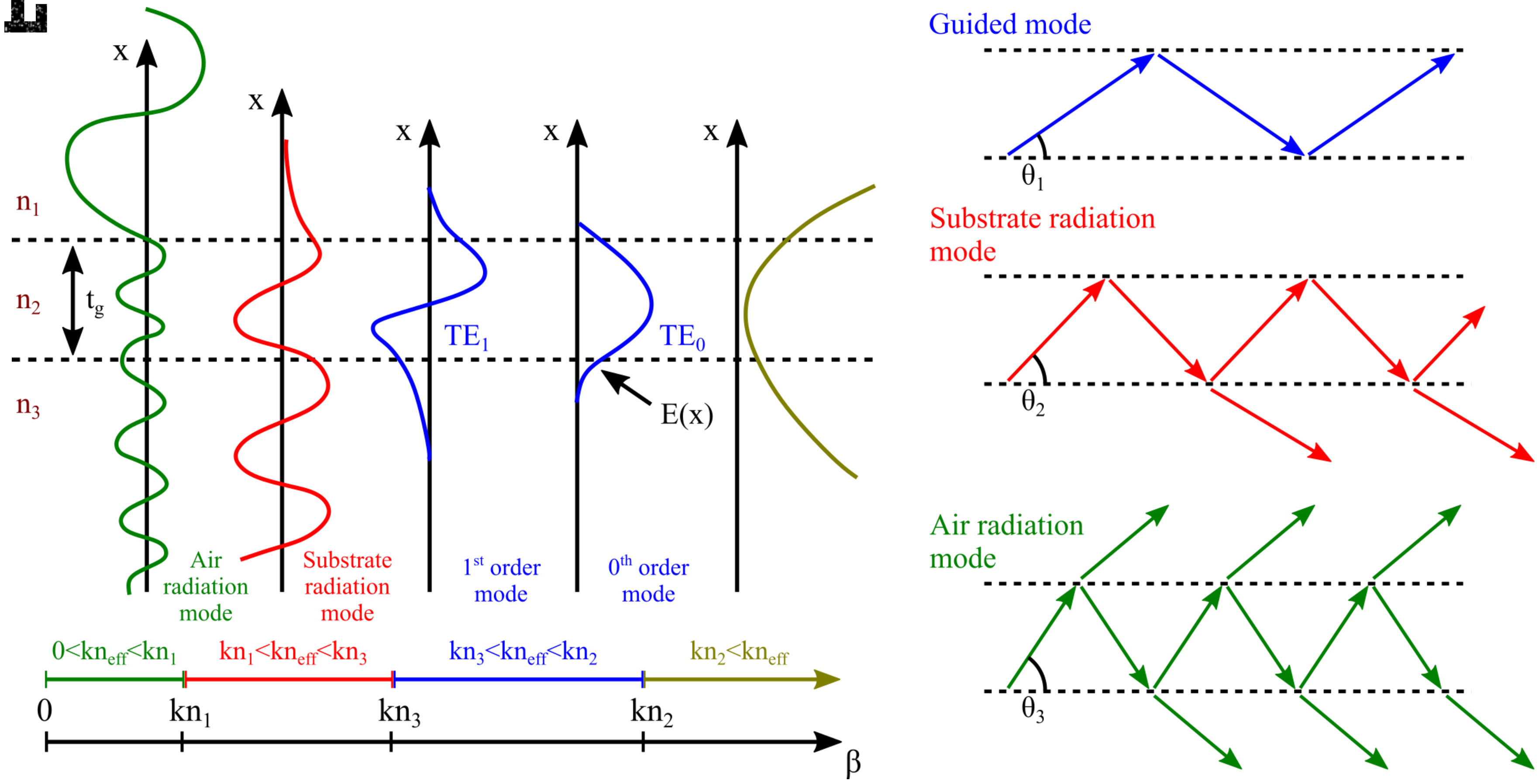


Figure 6: Diagram of the possible modes in a slab waveguide

H.W.

Submission due is next week

[1] Derive the expressions for the constants of the TE solution of the wave equation.

[2] Derive the solution of the TM wave equation.

Submit the detailed derivations to: alinak@bgu.ac.il.

BOUNDARY CONDITION IV

$$H_{t2} = H_{t3} \tag{41}$$

$$H_z\left(x = -{t_g}^{+}\right) = H_z\left(x = -{t_g}^{-}\right) \tag{42}$$

$$\frac{\partial E_y\left(x = -{t_g}^{+}\right)}{\partial x} = \frac{\partial E_y\left(x = -{t_g}^{-}\right)}{\partial x} \tag{43}$$

- **Transcendental equation** for solving the allowed β graphically or numerically.

$$\tan(ht_g) = \frac{p+q}{h\left(1 - \frac{pq}{h^2}\right)}$$

EXAMPLE: FINDING β FOR TE MODES

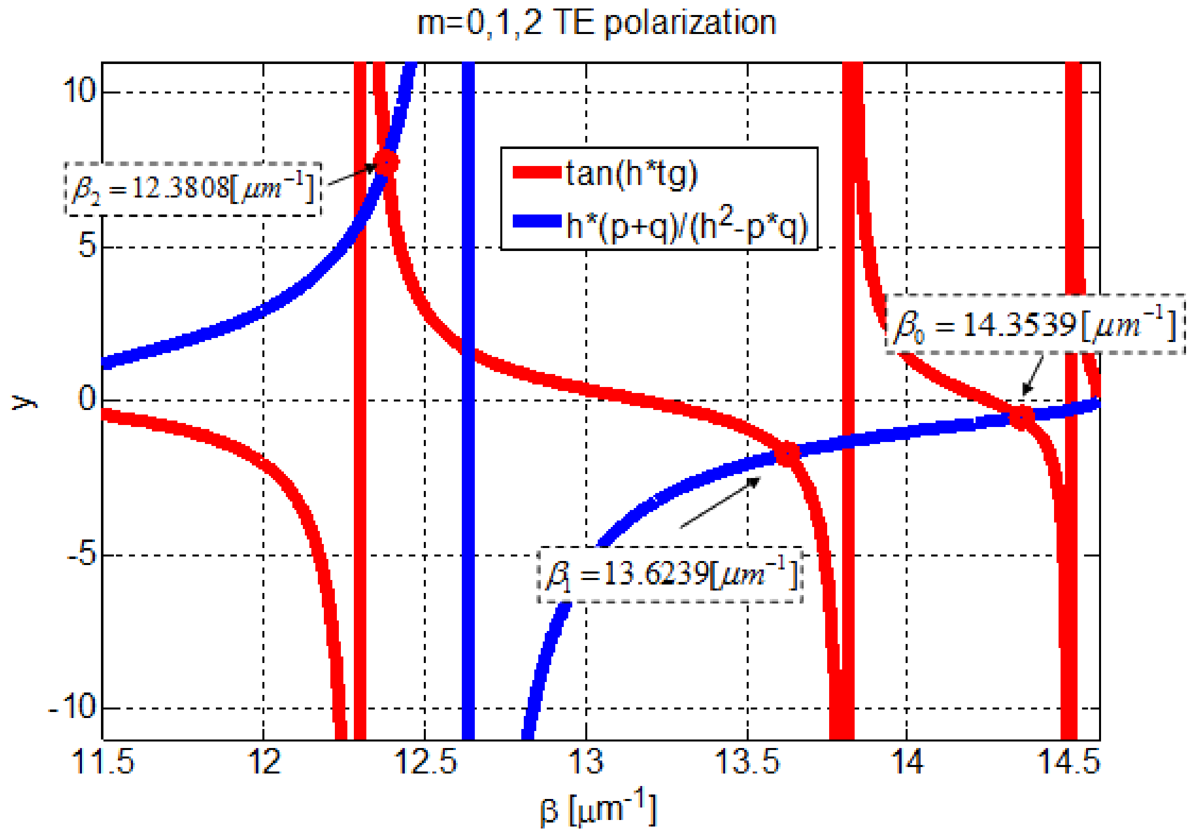


Figure 7: Finding the propagation constant using a numerical simulation.

THE SYMMETRIC WAVEGUIDE

- **The guiding condition**

$$n_2 > n_3 = n_1$$

- At cutoff, the point at which the field becomes oscillatory in Regions 1 and 3, the magnitude of β is given by:

$$\beta = kn_1 = kn_3$$

- By substituting β in equations of h, p and q

When $\boldsymbol{\beta = kn_3 = kn_1}$:

$$q = \sqrt{\beta^2 - k^2{n_1}^2}_{\beta=kn_1} = 0$$

$$p = \sqrt{\beta^2 - k^2{n_3}^2}_{\beta=kn_3} = 0$$

$$h = \sqrt{k^2{n_2}^2 - \beta^2} = k\sqrt{{n_2}^2 - {n_1}^2}$$

THE SYMMETRIC WAVEGUIDE

- By substituting $q = 0, p = 0$ in transcendental equation - $\tan(ht_g) = \frac{p+q}{h\left(1-\frac{pq}{h^2}\right)}$

$$\tan(ht_g)_{p=0,q=0} = 0$$

$m_s = 0, 1, 2, \ldots$ is number of a symmetric mode

$ht_g = m_s\pi$ with $m_s = 0, 1, 2, \ldots \Rightarrow$ points of $\tan = 0$, therefore:

$$kt_g\sqrt{{n_2}^2 - {n_1}^2} = m_s\pi \tag{44}$$

$${n_2}^2 - {n_1}^2 = \underbrace{(n_2 - n_1)}_{\Delta n}(n_2 + n_1) = \Delta n(n_2 + n_1); \quad k = \frac{2\pi}{\lambda_0}$$

$$\Delta n = \frac{{m_s}^2\pi^2{\lambda_0}^2}{{t_g}^2(n_2 + n_1)(2\pi)^2} = \frac{{m_s}^2{\lambda_0}^2}{4{t_g}^2(n_2 + n_1)} \tag{45}$$

THE SYMMETRIC WAVEGUIDE

- The **cut-off condition**

$$\Delta n = n_2 - n_1 > \frac{{m_s}^2{\lambda_0}^2}{4{t_g}^2(n_2+n_1)} \underbrace{\approx}_{n_2 \cong n_1} \frac{{m_s}^2{\lambda_0}^2}{8{t_g}^2 n_2}$$

- The lowest-order mode $m_s = 0$ of the symmetric waveguide does not exhibit a cutoff.
- All other modes do exhibit a cutoff.
- In principle, any wavelength could be guided in this mode even with an incrementally small Δn.
- For small Δn and/or large λ_0/t_g:
  1) Poor confinement.
  2) Relatively large evanescent tails of the mode extending into the substrate.

THE ASYMMETRIC WAVEGUIDE

- **The guiding condition** $n_2 > n_3 \gg n_1$
- Since $n_2 > n_3$

$$\Delta n = n_2 - n_3 > \frac{{m_s}^2{\lambda_0}^2}{4{t_g}^2(n_2 + n_3)}$$

To approximate the asymmetric waveguide, we substitute

$t_g = 2t_g$ and $m_s = 2m_a + 1 \Rightarrow \Delta n = n_2 - n_3 > \frac{(2m_a+1)^2{\lambda_0}^2}{4(2t_g)^2(n_2+n_3)}$

- Cutoff condition

$$\Delta n = n_2 - n_3 > \frac{(2m_a + 1)^2{\lambda_0}^2}{16{t_g}^2(n_2 + n_3)} \underbrace{\approx}_{n_2 \cong n_3} \frac{(2m_a + 1)^2{\lambda_0}^2}{32n_2{t_g}^2}$$

where the m_a is the asymmetric mode order.

- Note: This is an estimated model. For a more accurate solution, the transcendental equation, shown in slide 78, needs to be solved

SYMMETRIC VS. ASYMMETRIC WAVEGUIDE

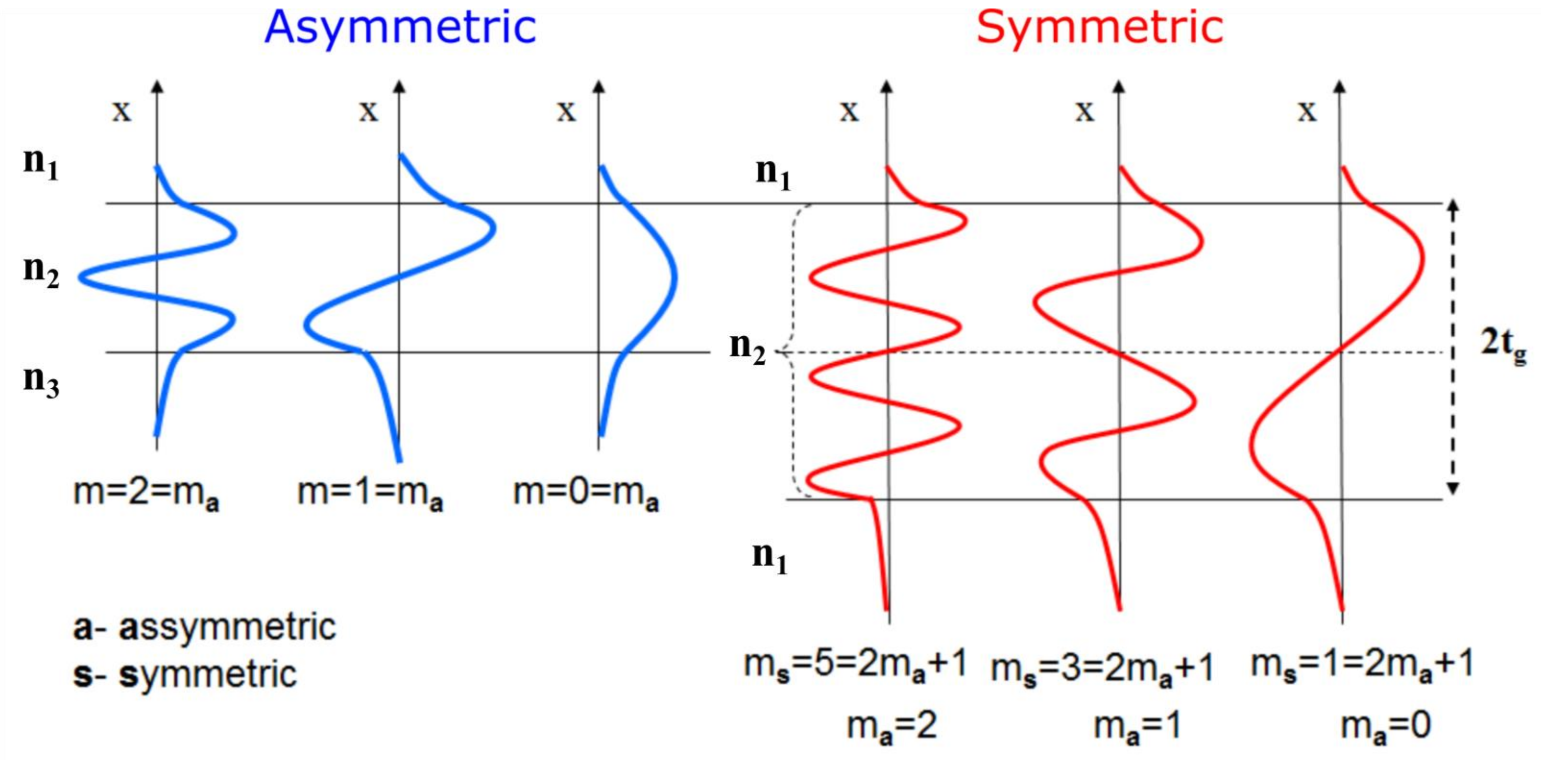


Figure 8: Cross-section of the modes for symmetric and asymmetric waveguide.

EXPERIMENTAL OBSERVATION OF WAVEGUIDE MODES

- The lowest order mode $(m = 0)$ appears as a single band of light, while higher order modes have a correspondingly increased number of bands.

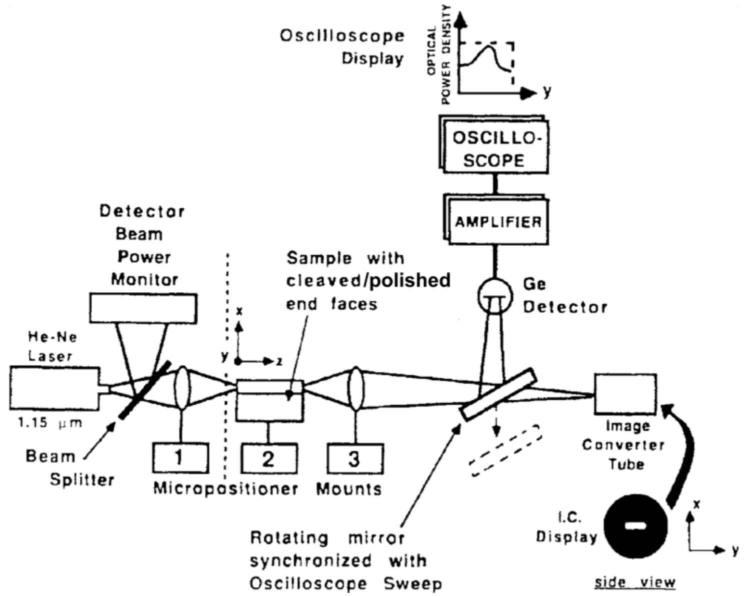
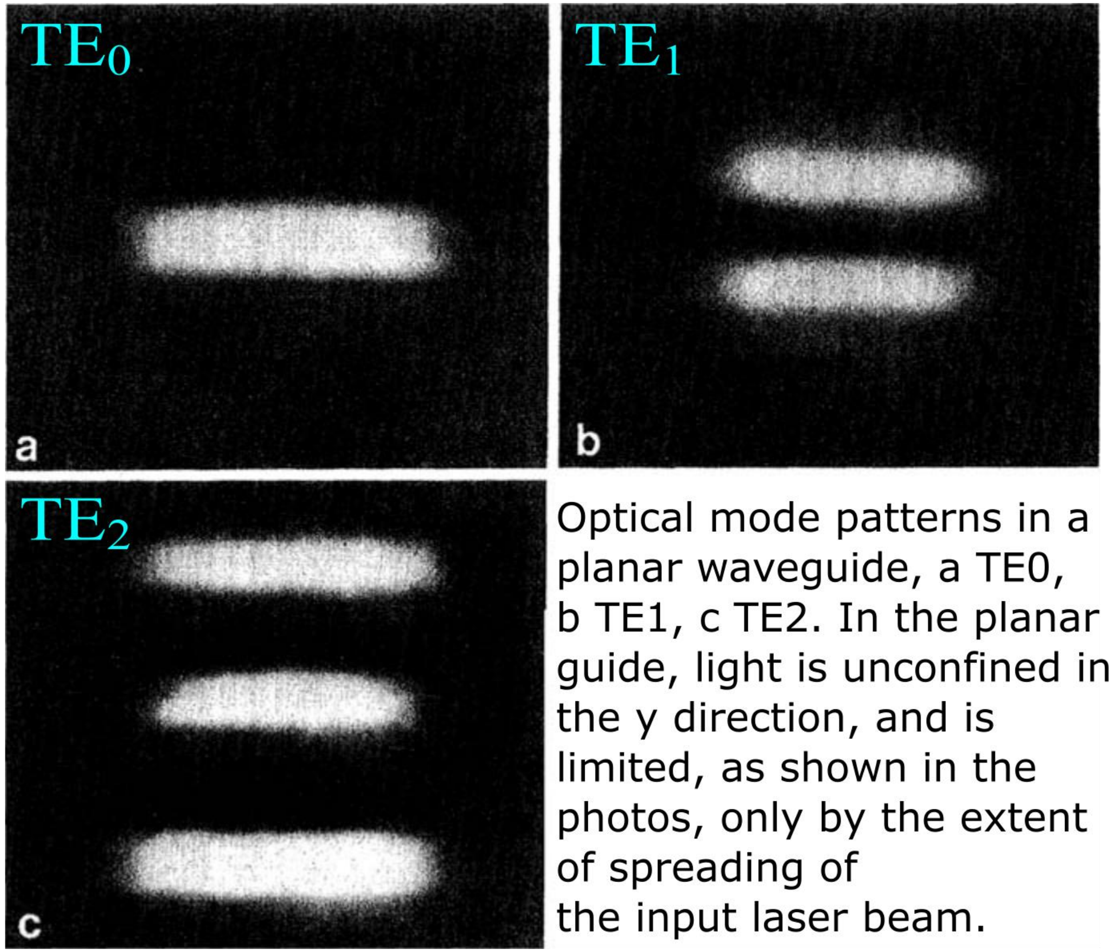


Figure 9: Diagram of an experimental setup that can be used to measure optical mode shapes.

OPTICAL MODES IN SLAB WAVEGUIDE



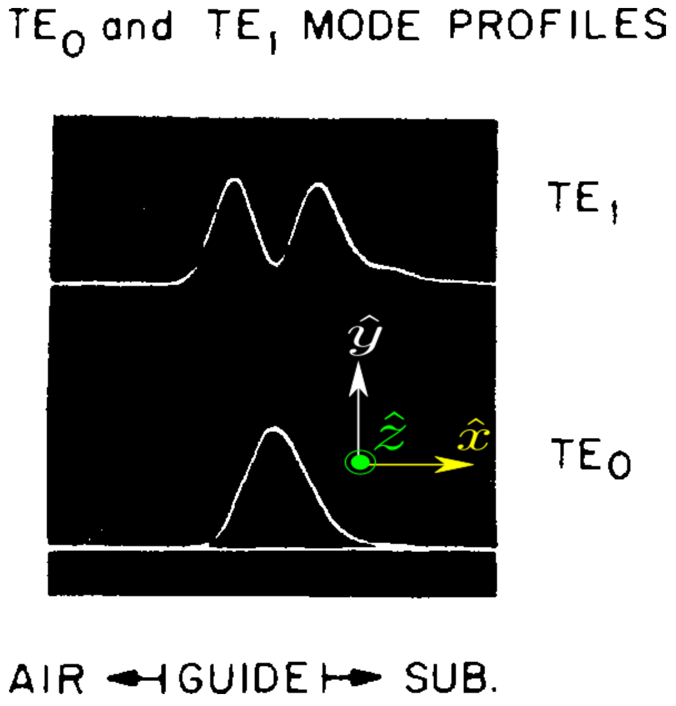
Optical mode patterns in a planar waveguide, a TE0, b TE1, c TE2. In the planar guide, light is unconfined in the y direction, and is limited, as shown in the photos, only by the extent of spreading of the input laser beam.

OPTICAL MODES IN PLANAR WAVEGUIDE

- The light image appears as a band rather than a spot because it is confined by the waveguide only in the x direction. Since the waveguide is much wider than its thickness, the laser beam is essentially free to diverge in the y direction.
- To obtain a quantitative display of the mode profile, i.e., optical power density vs. distance across the facet of the waveguide, a rotating mirror is used to scan the image of the waveguide facet across a photodetector that is masked to a narrow slit input. The electrical signal from the detector is then fed to the vertical scale of an oscilloscope on which the horizontal sweep has been synchronized with the mirror scan rate.
- The result is in the form of graphic displays of the mode shape, like those shown in the next frame:

EXPERIMENTAL MODE ANALYSIS

- Note that the modes have the theoretically predicted sinusoidal shape in waveguide guiding layer and exponential shape beyond it. Optical power density, or intensity, which is proportional to E^2.
- Details of the mode shape, like the rate of exponential decay (or extinction) of the evanescent ”tail” extending across the waveguide-substrate and waveguide-air interfaces, depend strongly on the values of δ at the interface.



EXPERIMENTAL MODE ANALYSIS

- As can be seen in Fig. above, the extinction is much sharper at the waveguide-air interface where $\Delta n \simeq 2.5$ than at the waveguide-substrate plane where $\Delta n \simeq 0.01 – 0.1$.
- A system like that shown in Fig. 8 is particularly useful for analysis of mode shapes in semiconductor waveguides, which generally support only one or two modes because of the relatively small Δn at the waveguide-substrate interface. Generally, the position of the focused input laser beam can be moved toward the center of the waveguide to selectively pump the zeroth order mode, or toward either the air or substrate interface to select the first order mode.
- It becomes very difficult to visually resolve the light bands in the case of higher-order, multimode waveguides because of spatial overlapping, even though the modes may be electromagnetically distinct and non-coupled one to another.
- Waveguides produced by depositing thin films of oxides, nitrides or glasses onto glass or semiconductor substrates usually are multi-mode, supporting 3 or more modes, because of the larger waveguide substrate Δn.

EXPERIMENTAL OBSERVATION OF WAVEGUIDE MODES

A. Karabchevsky A.V. Kavokin, Sci.Rep. 2016

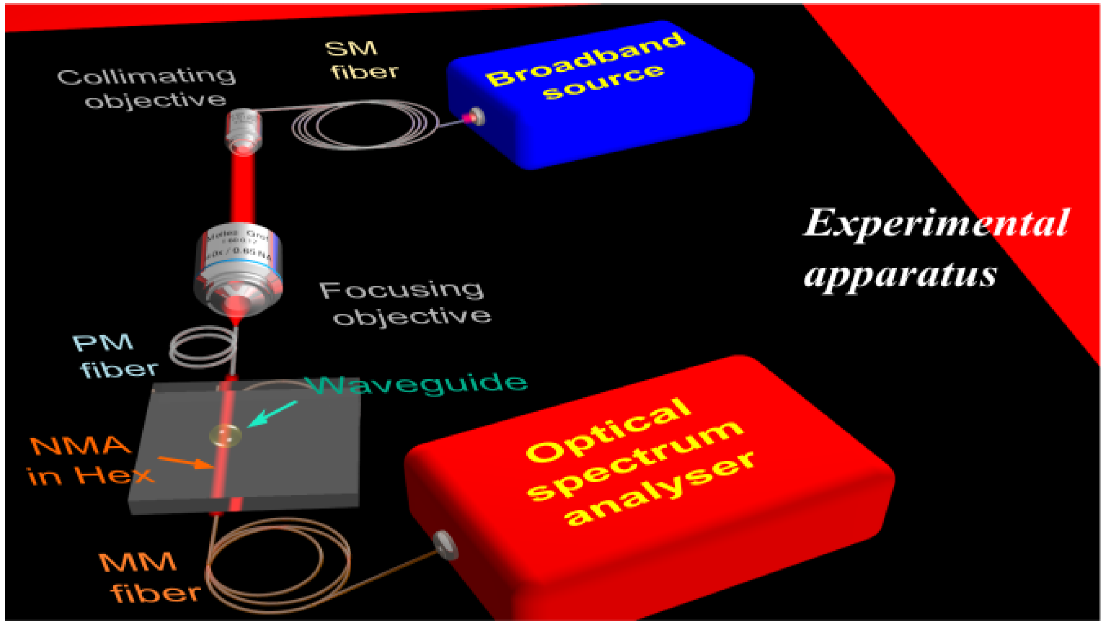


Figure 10: Butt-coupling to a channel waveguide.

ION EXCHANGE CHANNEL OPTICAL WAVEGUIDES

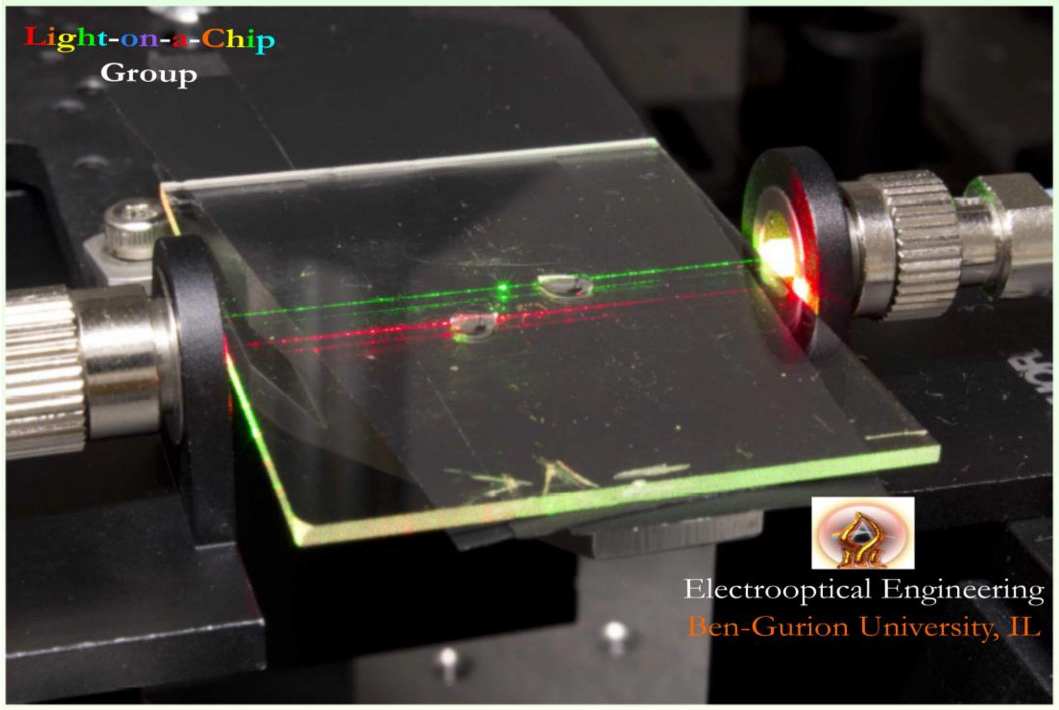


Figure 11: Butt-coupling to a channel waveguide.

COMMON LATERALLY CONFINED PASSIVE WAVEGUIDE STRUCTURES

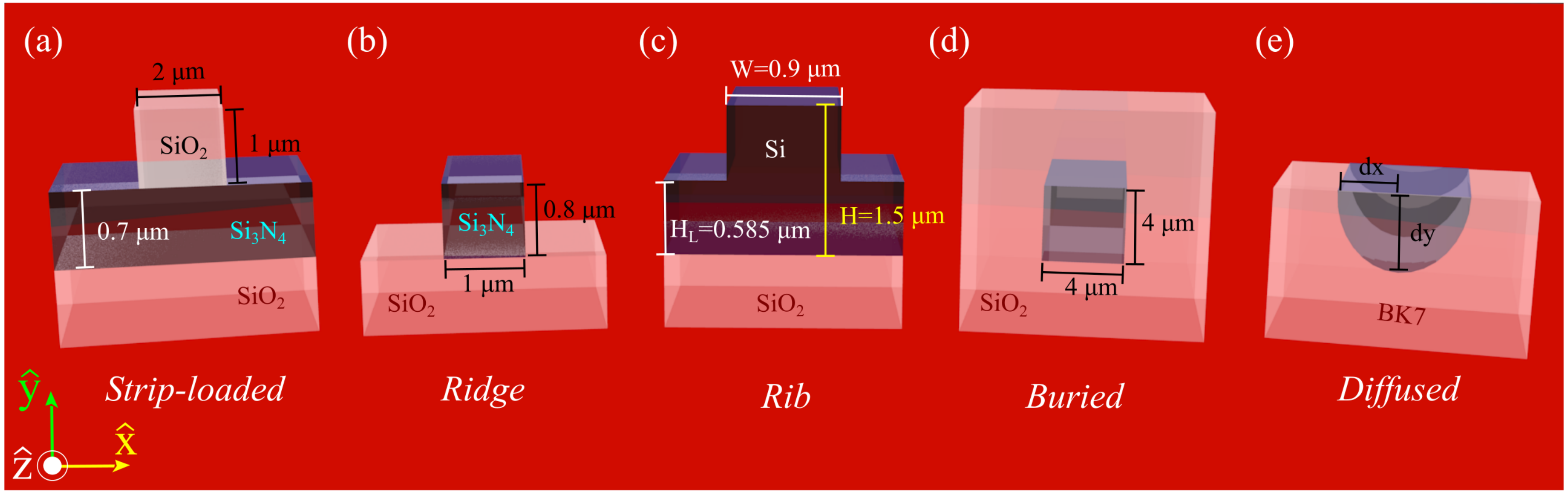


Figure 12: Common laterally confined passive waveguide structures: (a) a strip-loaded waveguide made of nitride on a silica substrate covered by silica, (b) a ridge waveguide made of nitride on a silica substrate, (c) a rib waveguide made of silicon on a silica substrate, (d) a waveguide buried in silica glass, and (e) a diffused waveguide in borosilicate glass from A. Katiyi and A. Karabchevsky *Lightwave Technology* 35:14, 2902 - 2908, (2017)

E_x MODES OF AN OPTICAL WAVEGUIDE

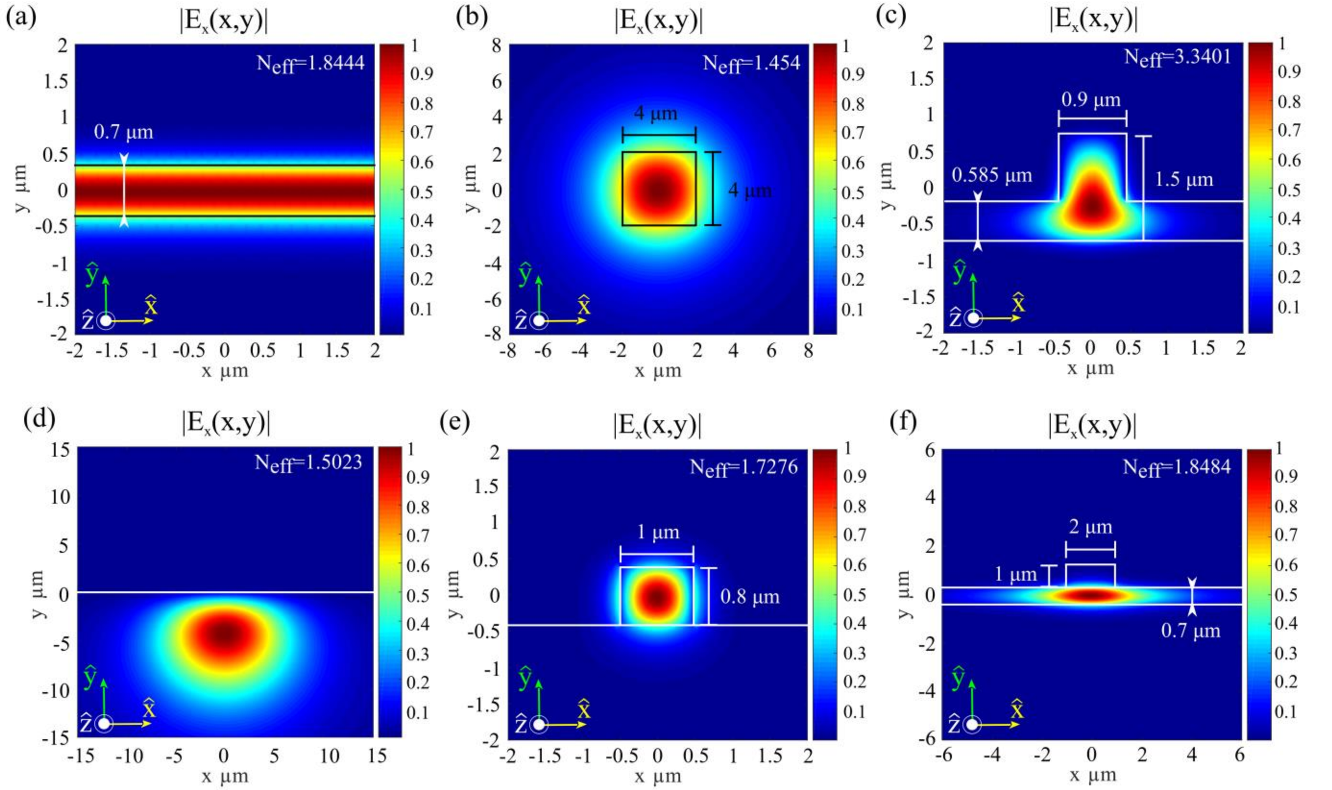


Figure 13: Quasi-TE polarization. Colormaps of $|E_x(x,y)|$, normalized to the maximum amplitude in single-mode waveguides: (a) slab, (b) buried, (c) rib, (d) diffused, (e) ridge, and (f) strip-loaded from A. Katiyi and A. Karabchevsky 2017.

E_y MODES OF AN OPTICAL WAVEGUIDE

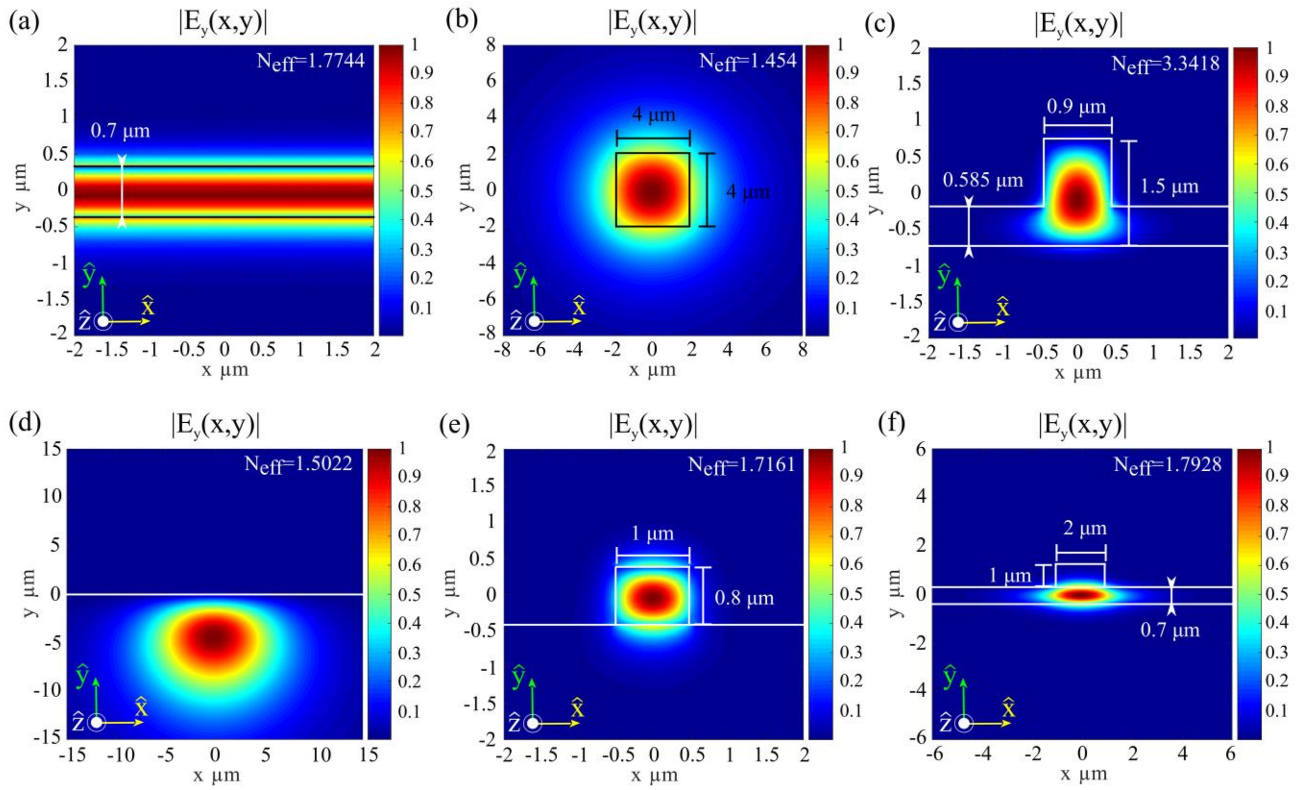


Figure 14: Quasi-TM polarization. Colormaps of $\left|E_y(x,y)\right|$, normalized to the maximum amplitude in single-mode waveguides: (a) slab, (b) buried, (c) rib, (d) diffused, (e) ridge, and (f) strip-loaded from A. Katiyi and A. Karabchevsky 2017.

STRIP LOADED WAVEGUIDE

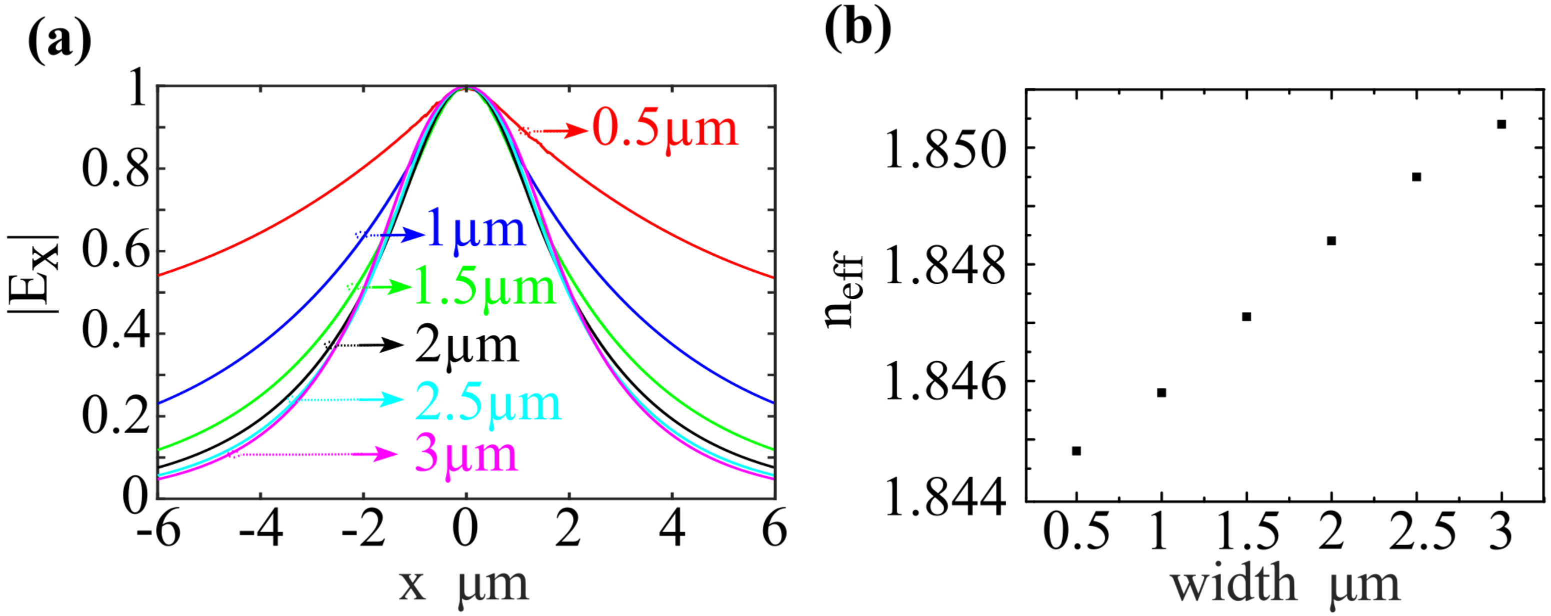
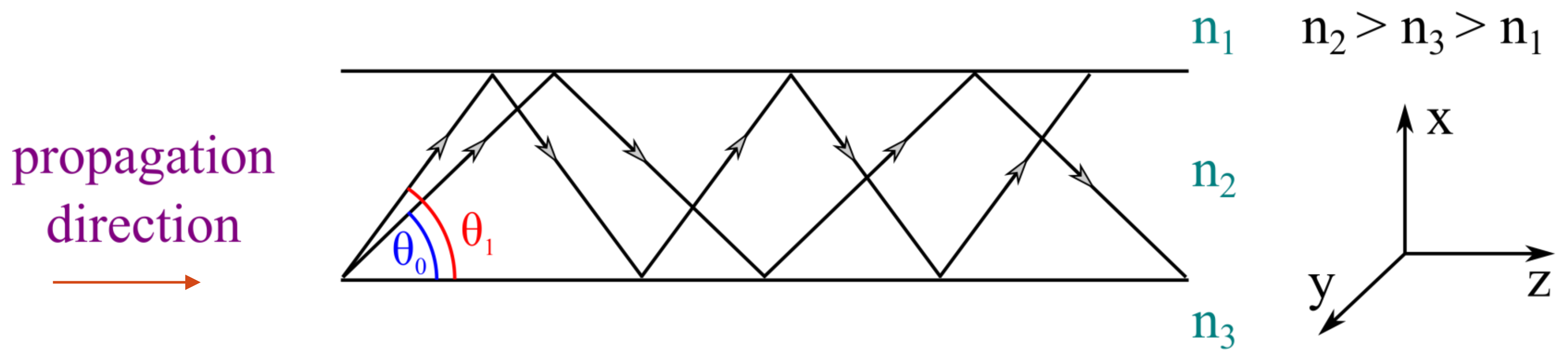


Figure 15: Quasi-TE polarization. $|E_x(x, y)|$ modes are normalized to the maximum amplitude while varying the width of the strip in the strip-loaded waveguides. The indicated values represent the cross-section profiles (a) and the change in the effective mode index (b) of the waveguide width with the strip width from A. Katiyi and A. Karabchevsky 2017.

THE PRISM COUPLER USED AS A DEVICE FOR MODAL ANALYSIS

- The prism coupler has the property that it selectively couples light into (or out of) a particular mode, depending on the angle of incidence (or emergence). The mode-selective property of the prism coupler, results from the fact that light in each mode within a waveguide propagates at a different velocity, and continuous phase-matching is required for coupling. The particular angle of incidence required to couple light into a given mode or the angle of emergence of light coupled out of a given mode can both be accurately calculated.

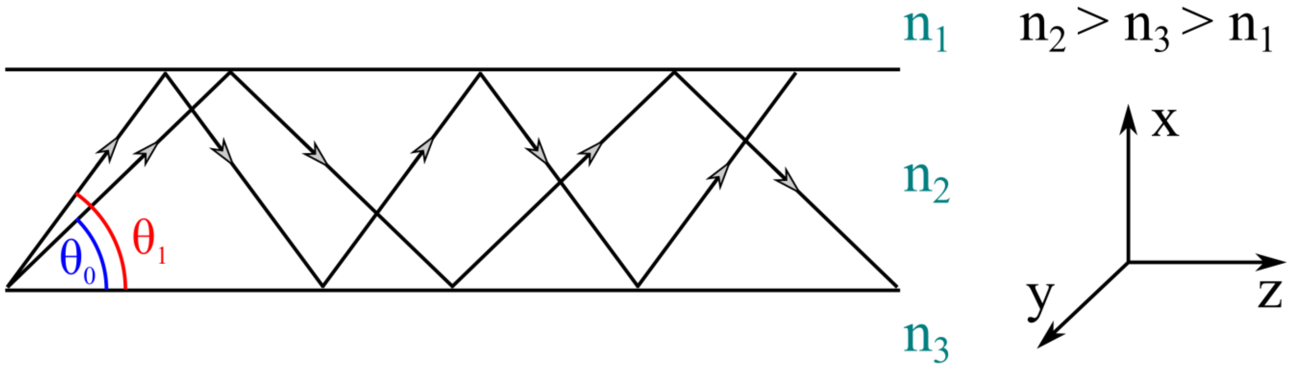


THE RAY-OPTIC APPROACH TO OPTICAL MODE THEORY

- Plane waves propagating along the z direction, support one or more optical modes. The light propagating in each mode travels in the z direction with a different phase velocity, which is characteristic of that mode. This description of wave propagation is generally called the physical-optic approach. An alternative method, the so-called ray-optic approach.
- The light propagating in the z direction is considered to be composed of plane waves moving in zig-zag paths in the x-z plane undergoing total internal reflection at the interfaces bounding the waveguide. The plane waves comprising each mode travel with the same phase velocity. However, the angle of reflection in the zigzag path is different for each mode, making the z component of the phase velocity different. The plane waves are generally represented by rays drawn normal to the planes of constant phase which explains the name ray-optic.

RAY PATTERNS IN THE THREE-LAYER PLANAR WAVEGUIDE

- The ray patterns shown here, correspond to two modes, say the TE_0 and TE_1, propagating in three layers waveguide with $n_2 > n_3 > n_1$. The electric (***E***) and magnetic (***H***) fields of these plane waves traveling along zig-zag paths would add vectorially to give the ***E*** and ***H*** distributions of the waves comprising the same two modes, propagating in the z direction. Both the ray-optic and physical-optic formulations can be used to represent either TE waves, with components E_y, H_z, and H_x, or TM waves, with components H_y, E_z and E_x.



THE DISCRETE NATURE OF THE PROPAGATION CONSTANT β

- The solution of Maxwell's equation subject to the appropriate boundary conditions requires that only certain discrete values of β are allowed. Thus, there are only a limited number of guided modes that can exist when β is in the range (see Fig. 5 slide 6)

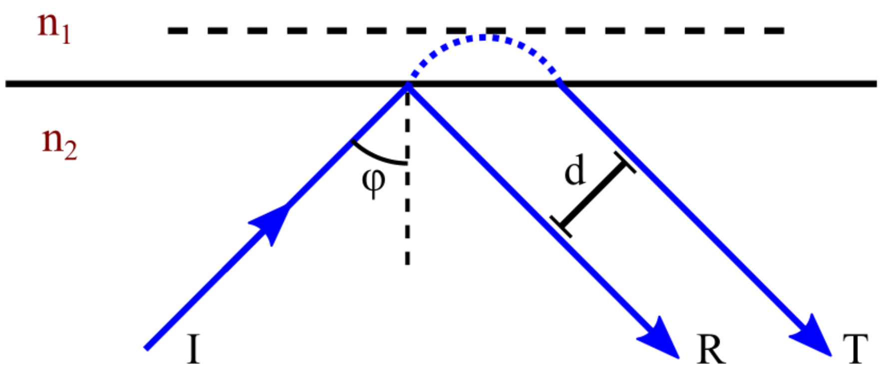
$$kn_3 \leq \beta \leq kn_2 \qquad (46)$$

- The plane wavefronts that are normal to the zig-zag rays are assumed to be infinite, or at least larger than the cross section of the waveguide that is intercepted; otherwise, they would not fit the definition of a plane wave, which requires a constant phase over the plane.
- Thus, there is much overlapping of the waves as they travel in the zig-zag path. To avoid decay of optical energy due to destructive interference as the waves travel through the guide, the total phase change for a point on a wavefront that travels from the $n_2 - n_3$ interface to the $n_2 - n_1$ interface and back again must be a multiple of 2π.

GOOS-HANCHEN SHIFT

$$2kn_2t_g \sin\theta_m - 2\phi_{23} - 2\phi_{21} = 2m\pi \qquad (47)$$

t_g is the thickness of the waveguiding Region 2, θ_m is the angle of reflection with respect to the z direction, m is the mode number and ϕ_{23} and ϕ_{21} are the phase changes suffered upon total internal reflection at the interfaces. The phases $-2\phi_{23}$ and $-2\phi_{21}$, represent the Goos-Hanchen shifts. These phase shifts can be interpreted as penetration of the zig-zag ray (for a certain depth δ) into the confining layers 1 and 3 before it is reflected.



GOOS-HANCHEN SHIFT

The phase shift of ϕ_{23} and ϕ_{21} for TM waves can be calculated from:

$$\tan\phi_{23} = \frac{\sqrt{{n_2}^2 \sin^2\phi_2 - {n_3}^2}}{n_2\cos\phi_2} \tag{48}$$

$$\tan\phi_{21} = \frac{\sqrt{{n_2}^2 \sin^2\phi_2 - {n_1}^2}}{n_2\cos\phi_2} \tag{49}$$

The phase shift of ϕ_{23} and ϕ_{21} for TM waves can be calculated from:

$$\tan\phi_{23} = \frac{{n_2}^2\sqrt{{n_2}^2 \sin^2\phi_2 - {n_3}^2}}{{n_3}^2 n_2\cos\phi_2} \tag{50}$$

$$\tan\phi_{21} = \frac{{n_2}^2\sqrt{{n_2}^2 \sin^2\phi_2 - {n_1}^2}}{{n_1}^2 n_2\cos\phi_2} \tag{51}$$

GOOS-HANCHEN SHIFT

The substitution of either (49) or (51) into (47) results in a transcendental equation in only one variable, θ_m or ϕ_m, where:

$$\phi_m = \frac{\pi}{2} - \theta_m \tag{52}$$

For a given m, the parameters n_1, n_2, n_3, t, ϕ_m (or θ_m) can be calculated. Thus, a discrete set of reflection angles ϕ_m are obtained corresponding to the various modes.

VALID SOLUTIONS

- Valid solutions do not exist for all values of m. There is a cutoff condition on allowed values of m for each set of n_1, n_2, n_3 and t, corresponding to the point at which ϕ_m becomes less than the critical angle for total internal reflection at either the $n_2 - n_3$ or the $n_2 - n_1$ interface.
- For each allowed mode, there is a corresponding propagation constant β_m given by:

$$\beta_m = kn_2 \sin\phi_m = kn_2 \cos\theta_m \qquad (53)$$

- The velocity of the light parallel to the waveguide is then given by:

$$v = c\left(\frac{k}{\beta}\right) \qquad (54)$$

- The effective index of refraction for the guide is:

$$n_{\text{eff}} = \frac{c}{v} = \frac{\beta}{k} \qquad (55)$$

THE FORMATION OF MODES (STANDING WAVES)

- The Figure below schematically shows the formation of modes (standing waves) for (a) the fundamental mode and (b) a higher-order mode, respectively, through the interference of light waves.
- The solid line represents a positive phase front and a dotted line represents a negative phase front, respectively. The electric field amplitude becomes the maximum (minimum) at the point where two positive (negative) phase fronts interfere.