LECTURE 2 - FROM RAY OPTICS TO MODE ANALYSIS

Prof. Alina Karabchevsky, www.alinakarabchevsky.com

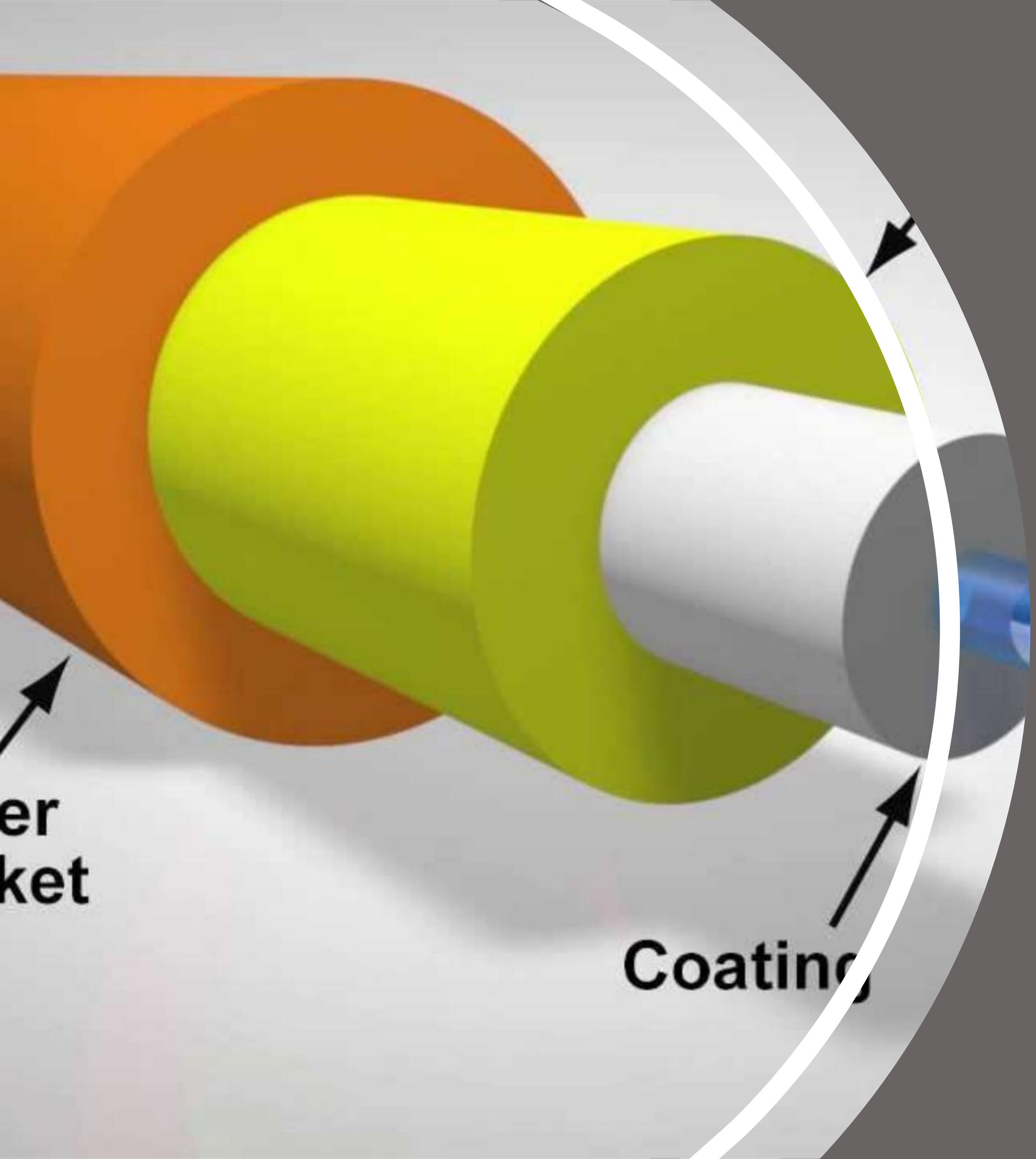
Fundamentals of Fiber Optics Communication 377-2-5060

School of ECE

Ben-Gurion University of the Negev, Israel

OUTLINE

- Introduction
- Reflection and refraction
- Optical field
- Ray optics
- Fiber configurations
- Bibliography



OPTICAL FIBER

In its simplest form, a step-index fiber consists of a cylindrical core surrounded by a cladding layer whose index is slightly lower than the core.

Both core and cladding use silica as the base material; the difference in the refractive indices is realized by doping the core, or the cladding, or both.

- Dopants such as GeO_2 and P_2O_5 increase the refractive index of silica and are suitable for the core.
- Dopants such as B_2O_3 and fluorine decrease the refractive index of silica and are suitable for the cladding.

REFLECTION AND REFRACTION

<u>Assumptions:</u>

- Plane wave propagation.
- Linear medium.
- Isotropic medium.
- Smooth planar optical interface.

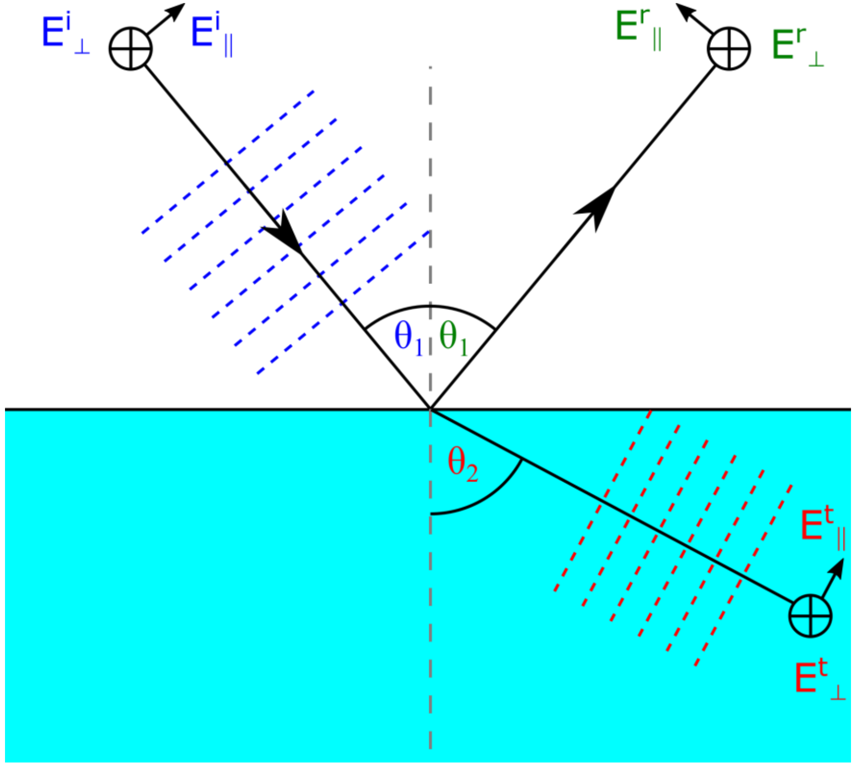


Figure 1: Plane wave reflection and refraction at an optical interface [1].

FRESNEL'S FIELD REFLECTIVITY

The reflectivity for optical field components parallel to the incident plane as:

$$\rho_{\parallel} = \frac{E^{r}{}_{\parallel}}{E^{i}{}_{\parallel}} = \frac{n_1 \cos \theta_2 - n_2 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \tag{1}$$

In order to eliminate θ_2, we can use Snell's Law:

$$\rho_{\parallel} = \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_1\right)^2} - n_2 \cos \theta_1}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_1\right)^2} + n_2 \cos \theta_1} \tag{2}$$

Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as:

FIELD REFLECTIVITY

Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as:

$$\rho_{\perp} = \frac{E^{r}{}_{\perp}}{E^{i}{}_{\perp}} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \tag{3}$$

$$\rho_{\perp} = \frac{n_1 \cos\theta_1 - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_1\right)^2}}{n_1 \cos\theta_1 + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta_1\right)^2}} \tag{4}$$

FIELD REFLECTIVITY

Power reflectivities for parallel and perpendicular field components are therefore:

$$R_{\parallel} = \left|\rho_{\parallel}\right|^2 = \left|\frac{E^r{}_{\parallel}}{E^i{}_{\parallel}}\right|^2 \tag{5}$$

and

$$R_{\perp} = |\rho_{\perp}|^2 = \left|\frac{E^r{}_{\perp}}{E^i{}_{\perp}}\right|^2 \tag{6}$$

FRESNEL'S POWER TRANSMISSION COEFFICIENTS

According to energy conservation, the power transmission coefficients can be found as:

$$T_{\parallel} = \left|\frac{E^{t}{}_{\parallel}}{E^{i}{}_{\parallel}}\right|^2 = 1 - \left|\rho_{\parallel}\right|^2 \tag{7}$$

and

$$T_{\perp} = \left|\frac{E^{t}{}_{\perp}}{E^{i}{}_{\perp}}\right|^2 = 1 - |\rho_{\perp}|^2 \tag{8}$$

In practice, for an arbitrary incidence polarization state, the input field can always be decomposed into $E_{\parallel}$ and $E_{\perp}$ components. Each can be treated independently.

SPECIAL CASE: NORMAL INCIDENCE

Normal incidence is when a light is launched perpendicular to the material interface. In this case, $\theta_1 = \theta_2 = 0$ and $\cos\theta_1 = \cos\theta_2 = 1$. The field reflectivity can be simplified as:

$$\rho_{\parallel} = \rho_{\perp} = \frac{n_1 - n_2}{n_1 + n_2} \qquad (9)$$

- $\boldsymbol{n_1 > n_2}$**:** there is no phase shift between incident and reflected field (the phase of both $\rho_{\parallel}$ and $\rho_{\perp}$ is zero)
- $\boldsymbol{n_1 < n_2}$**:** there is a phase shift for both $\rho_{\parallel}$ and $\rho_{\perp}$ because they both become negative

With normal incidence, the power reflectivity is:

$$\rho_{\parallel} = \rho_{\perp} = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 \qquad (10)$$

CRITICAL ANGLE

Critical angle is defined as an incident angle θ_1 at which total reflection happens at the interface. According to Fresnel Equations (2) and (4), total reflection ($\rho_{\parallel} = \rho_{\perp} = 1$) occurs when $\frac{n_1}{n_2}\sin\theta_1 = 1$ or

$$\theta_1 = \theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{11}$$

where θ_c is the critical angle.

- Obviously, the necessary condition to have a critical angle depends on the interface condition. First, if $n_1 < n_2$, there is not real solution for $\sin^{-1}(n_2/n_1)$.
- It means that when a light beam goes from a low index material to a high index material, total reflection is not possible.

CRITICAL ANGLE

- Second, if $n_1 > n_2$, a real solution can be found for $\theta_c = \sin^{-1}(n_2/n_1)$. Therefore, total reflection can only happen when a light beam launches from a high index material to a low index material.
- It is important to note that at a larger incidence angle $\theta_1 > \theta_c$, $1 - (n_1/n_2 \cdot \sin\theta_1)^2 < 0$ and $\sqrt{1 - (n_1/n_2 \cdot \sin\theta_1)^2}$ becomes imaginary.

Equations (2) and (4) show that if $\sqrt{1 - (n_1/n_2 \cdot \sin\theta_1)^2}$ is imaginary, $|\rho_\parallel|^2 = |\rho_\perp|^2 = 1$. The important conclusion is that for all incidence angles satisfying $\theta_1 = \theta_c$, total internal reflection will happen with $R = 1$.

(see the side effect of TIR which is Evanescent field in Lecture 1)

CRITICAL ANGLE

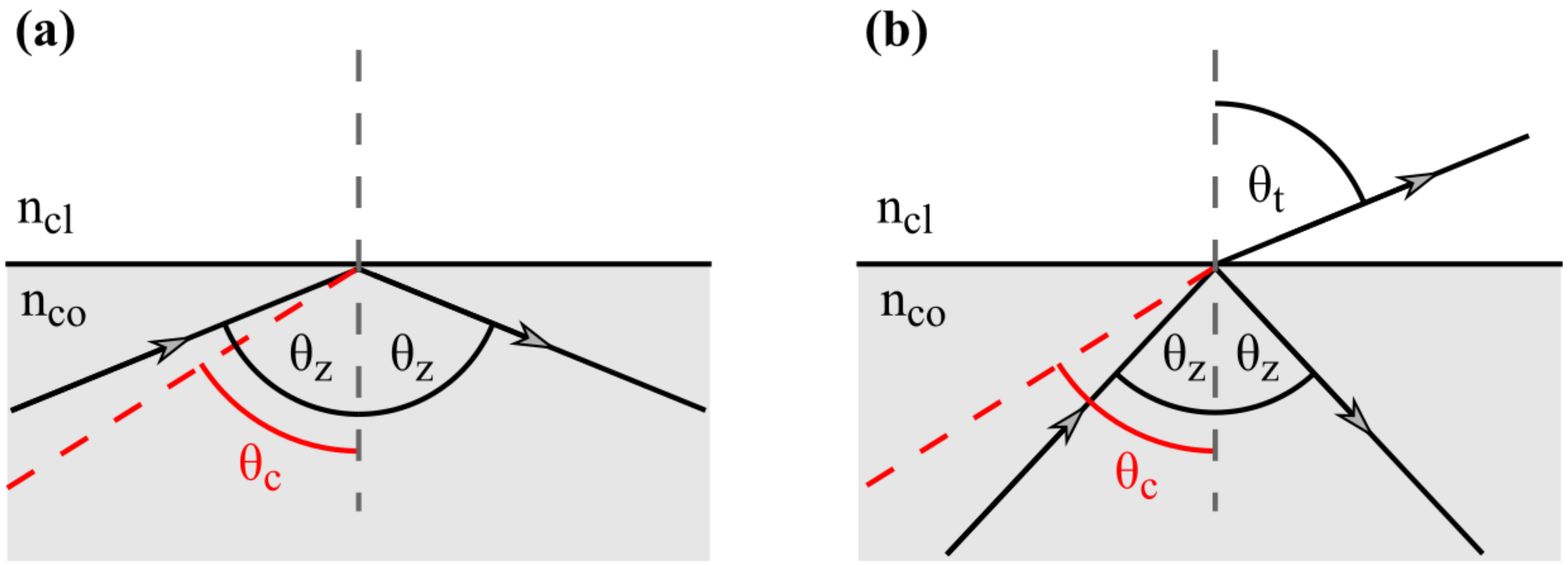


Figure 2: Reflection at a planar interface between unbounded regions of refractive indices n_{co} and n_{cl} showing (a) total internal reflection and (b) partial reflection and refraction.

OPTICAL FIELD PHASE SHIFT BETWEEN THE INCIDENT AND THE REFLECTED BEAMS

- When $\theta_1 = \theta_c$ (partial reflection and partial transmission), both $\rho_{\parallel}$ and $\rho_{\perp}$ are real and therefore there is no phase shift for the reflected wave at the interface.

- When total internal reflection happens ($\theta_1 > \theta_c$), $\sqrt{1-(n_1/n_2 \cdot \sin\theta_1)^2}$ is imaginary. Fresnel Equations (2) and (4) can be written as:

$$\rho_{\parallel} = \frac{jn_1\sqrt{1-(n_1/n_2 \cdot \sin\theta_1)^2} - n_2\cos\theta_1}{jn_1\sqrt{1-(n_1/n_2 \cdot \sin\theta_1)^2} + n_2\cos\theta_1} \tag{12}$$

$$\rho_{\perp} = \frac{n_1\cos\theta_1 - jn_2\sqrt{1-(n_1/n_2 \cdot \sin\theta_1)^2}}{n_1\cos\theta_1 + jn_2\sqrt{1-(n_1/n_2 \cdot \sin\theta_1)^2}} \tag{13}$$

OPTICAL FIELD PHASE SHIFT BETWEEN THE INCIDENT AND THE REFLECTED BEAMS

Therefore, the phase shift for the parallel and the perpendicular electrical field components is:

$$\Delta\Phi_{\parallel} = \arg\left(\frac{{E^r}_{\parallel}}{{E^i}_{\parallel}}\right) = -2\tan^{-1}\left(\frac{\sqrt{{n_1}^2\sin^2\theta_1 - {n_2}^2}}{n_1\cos\theta_1}\right) \quad (14)$$

$$\Delta\Phi_{\perp} = \arg\left(\frac{{E^r}_{\perp}}{{E^i}_{\perp}}\right) = -2\tan^{-1}\left(\frac{\sqrt{{n_1}^2/{n_2}^2\cdot\sin^2\theta_1 - 1}}{n_2\cos\theta_1}\right) \quad (15)$$

This optical phase shift happens at the optical interface, which has to be considered in optical waveguide design.

BREWSTER ANGLE

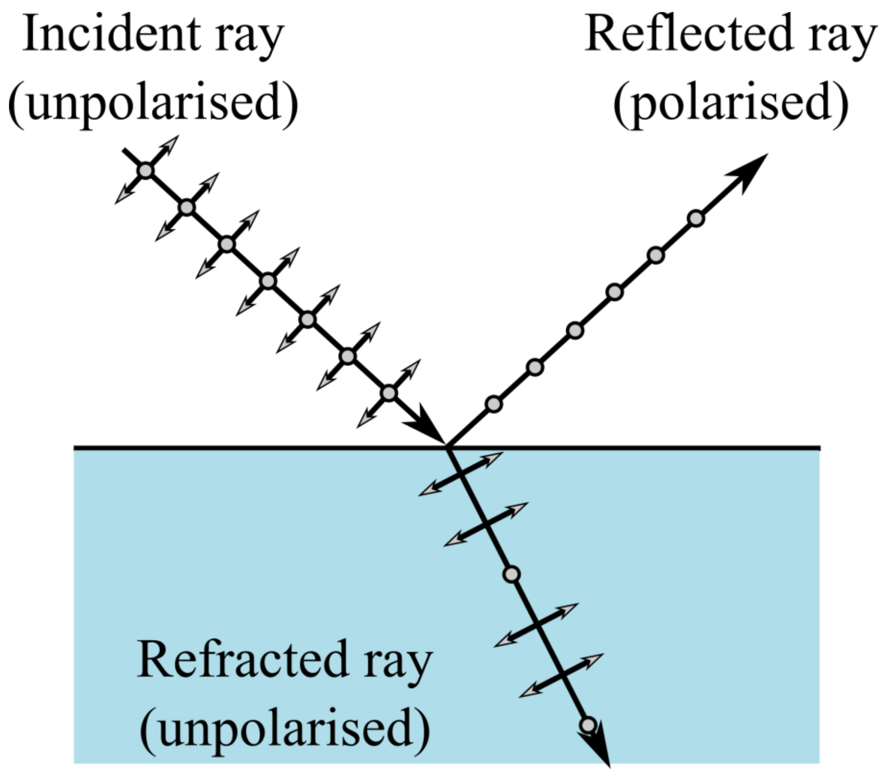


Figure 3: Brewster angle for unpolarized ray.

BREWSTER ANGLE - θ_B

Consider a light beam launching onto an optical interface.

If the input electrical field is parallel to the incidence plane, there is a specific incidence angle θ_B at which the reflection is equal to zero.

θ_B is defined as the *Brewster angle*.

Consider Fresnel Equation (2) for parallel field components. For $\rho_{\parallel} = 0$, the only solution is tan $\tan \theta_1 = n_2 = n_1$ and Brewster angle is defined as:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right) \qquad \theta_B + \theta_R = 90 \qquad (16)$$

BREWSTER ANGLE - θ_B

Two important points we need to note:

- The Brewster angle is only valid for the polarization component which has the electrical field vector parallel to the incidence plane. For the perpendicular polarized component, no matter how we choose θ_1, total transmission will never happen.
- $\rho_{\parallel} = 0$ happens only at one angle $\theta_1 = \theta_B$. This is very different from the critical angle where total reflection happens for all incidence angles within the range of

$$\theta_c < \theta_1 < \pi.$$

Brewster angle is often used to minimize the optical reflection and it can also be used to select the polarization.

BREWSTER ANGLE - θ_B

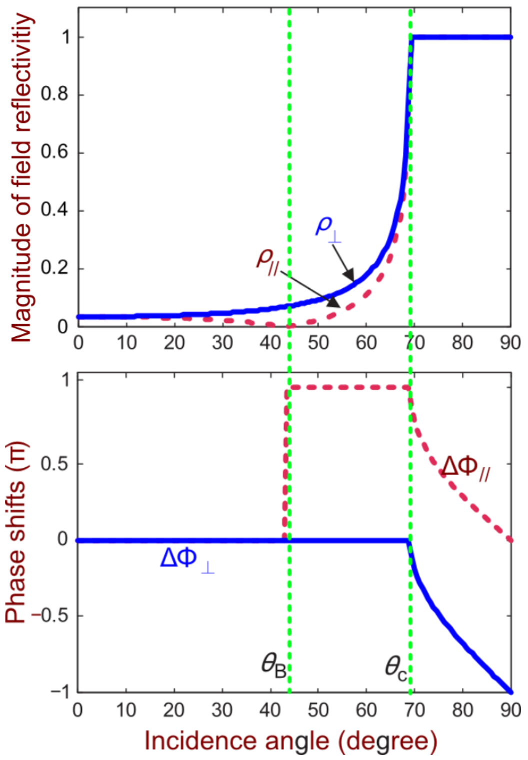


Figure 4: Field reflectivities and phase shifts vs. incidence angle. Optical interface is $n_1 = 1.5$ and $n_2 = 1.4$ [1].

RAY BOUNCING IN A FIBER

To confine and guide the lightwave signal within the fiber core, a total internal reflection is required at the core-cladding interface. This requires the refractive index of the core to be higher than the index of the cladding.

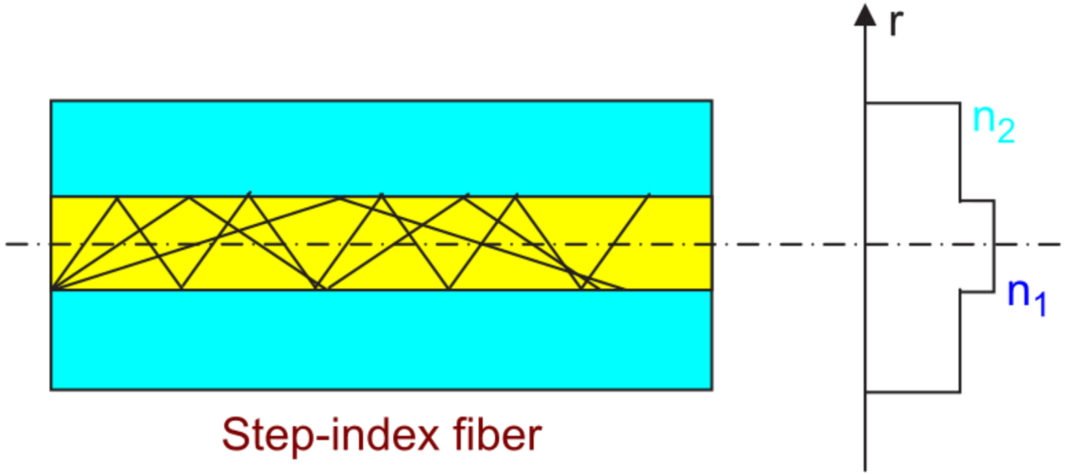


Figure 5: Index profiles of step-index [1].

GEOMETRIC OPTICS ANALYSIS

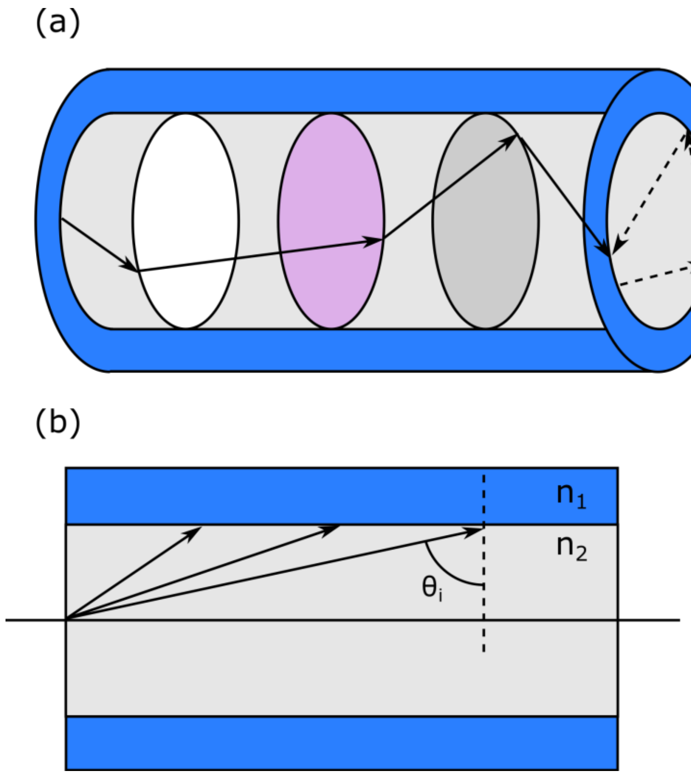


Figure 6: Illustration of fiber propagation modes in geometric optics: (a) skew ray trace and (b) meridional ray trace [1].

NOMENCLATURE

Nomenclature for describing circular fibers. Cartesian coordinates x, y, z and cylindrical polar coordinates ρ, ϕ, z are oriented so that the z-axis lies along the fiber axis. A representative graded profile varies over the core and is uniform over the cladding, assumed unbounded.

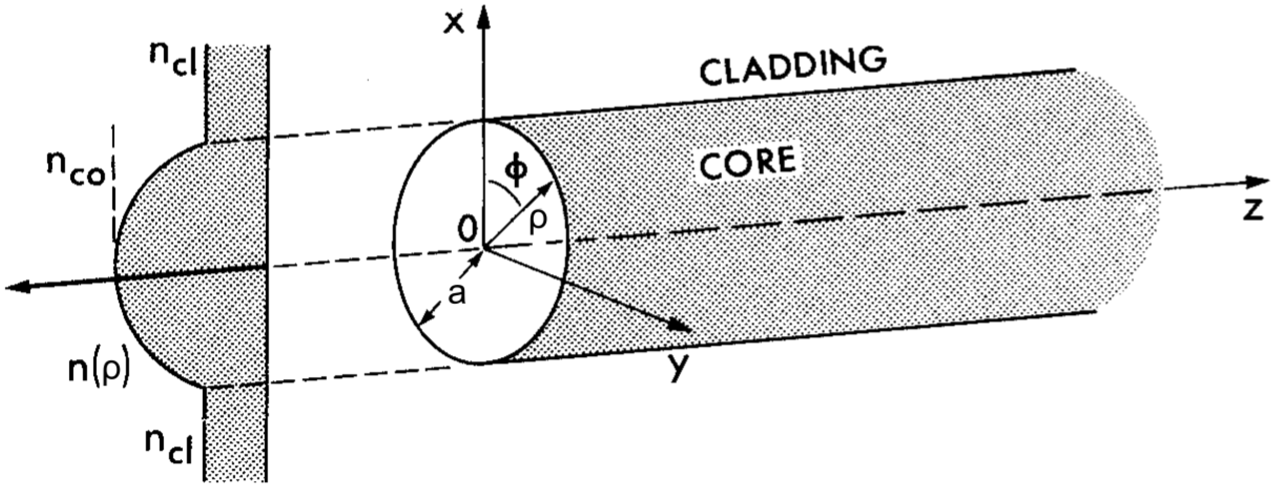


Figure 7: Schematics of fiber circular cross-section [2].

V-NUMBER

V-number, known as the waveguide parameter or waveguide frequency, is dimensionless parameter defined as

$$V = \frac{2\pi a}{\lambda}\sqrt{{n_{\mathrm{co}}}^2 - {n_{cl}}^2} \qquad (17)$$

Where n_{co} is the maximum value of $n(\rho)$, a the core radius, and λ is the free-space wavelength of light.

The quantity $\sqrt{{n_{\mathrm{co}}}^2 - {n_{cl}}^2}$ is often referred to as the numerical aperture (NA) of the fiber, while a related expression $\sqrt{{n_{\mathrm{co}}}^2 - {n_{cl}}^2}$ is sometimes called the local numerical aperture.

$$n(\rho) = n_{\mathrm{co}} \qquad 0 \le \rho < a$$

$$n(\rho) = n_{\mathrm{cl}} \qquad a \le \rho < \infty$$

$$n_{\mathrm{co}} > n_{\mathrm{cl}}$$

CONSTRUCTION OF RAY PATHS OF A STEP-INDEX FIBER

Between reflections the ray follows a straight line and, on reflection from the interface, its direction is determined by Snell's laws. Thus, the incident ray, reflected ray and normal, or radial direction, lie in the same plane, and the angles of incidence and reflection relative to the normal are equal.

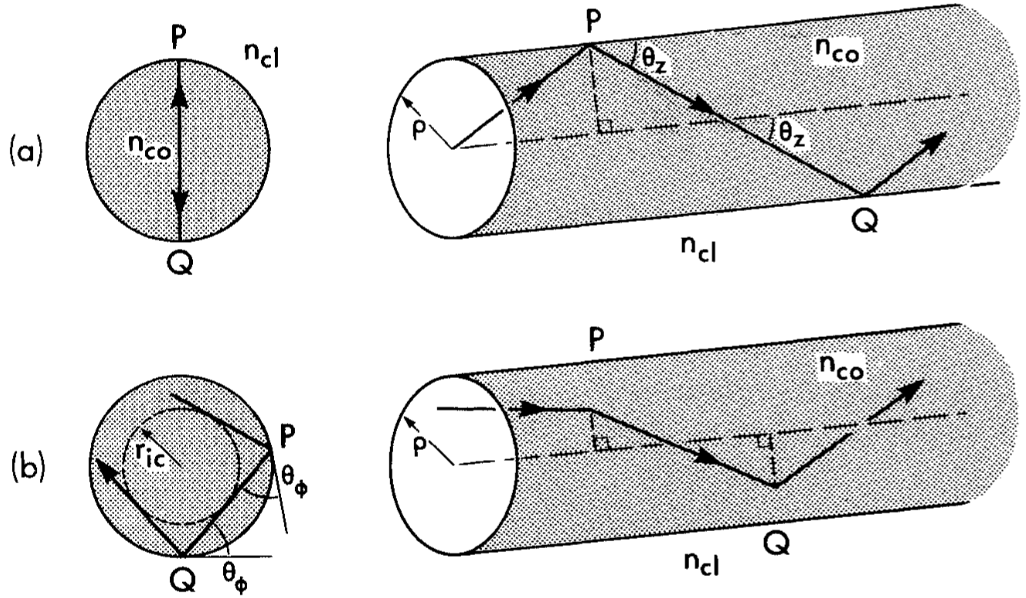


Figure 8: (a) the zig-zag path of a meridional ray and (b) the helical path of a skew ray, together with their projections onto the core cross-section [2].

MERIDIONAL RAYS

Meridional rays - rays which cross the fiber axis between reflections.

Note: Meridional rays lie in a plane of width 2ρ through the axis. Consequently, they have properties identical with rays of the corresponding planar waveguide.

We label meridional rays with the angle θ_z between the path and the z direction. The ranges of θ_z for bound and refracting meridional rays are given by **bound ray** (Eq. 18) and **refracting ray** (Eq. 19):

$$0 \leq \theta_z < \frac{\pi}{2} - \theta_c \tag{18}$$

$$\frac{\pi}{2} - \theta_c \leq \theta_z < \frac{\pi}{2} \tag{19}$$

BOUND (EQ. 18) AND REFRACTING (EQ. 19) RAYS

$$n_{\text{co}} \cos\theta_t = n_{\text{cl}} \cos\theta_t \tag{20}$$

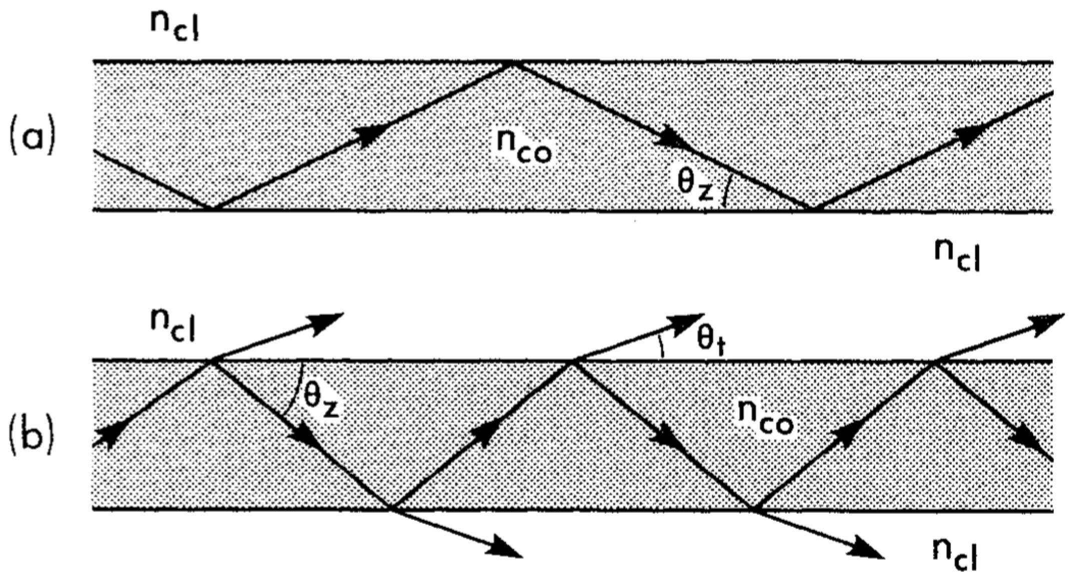


Figure 9: Zig-zag paths within the core of a step-profile planar waveguide for (a) bound rays and (b) refracting rays [2].

SKEW RAYS

Skew rays - rays which never cross the fiber axis.

- Skew rays, follow a helical path, whose projection onto the cross-section is a regular polygon-not necessarily closed.
- To specify the trajectory of a skew ray in addition to the inclination θ_z to the axial direction, we need a second angle to indicate the skewness.
- We define θ_ϕ to be the angle in the core cross-section between the tangent to the interface and the projection of the ray path. By geometry θ_ϕ has the same value at every reflection.
- The skew-ray directions which are not included in either bound or refracting rays belong to a third class of rays called **tunneling rays**.

MERIDIONAL AND SKEW RAYS

Angles for describing reflection of a ray incident at P on the interface of a step-profile fiber. Relative to the normal PN, the angle of incidence or reflection is α. Both incident and reflected rays make angles θ_z with the axial direction PQ, and θ_ϕ in the cross-section between the tangent PT and the path projection, i.e. PR for the reflected ray.

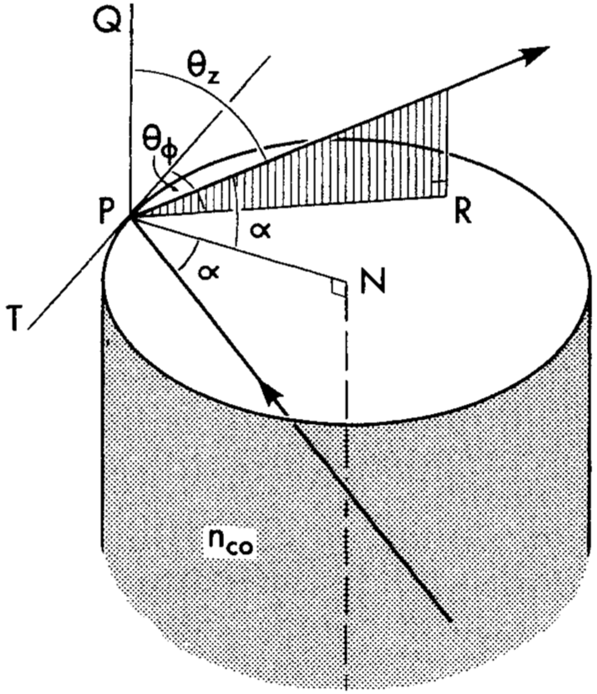


Figure 10: Ray reflected from a fiber [2].

CHARACTERIZATION OF RAYS ON STEP-PROFILE FIBERS

Table 1: Characterization of rays on step-profile fibers.

Rays classification	Condition
Bound rays	$0 \leq \theta_z < \theta_c$
Refracting rays	$0 \leq \alpha < \alpha_c$
Tunneling rays	$\theta_c \leq \theta_z \leq \frac{\pi}{2}$
Tunneling rays	$\alpha_c \leq \alpha < \frac{\pi}{2}$

NUMERICAL APERTURE

Numerical aperture (NA) is a parameter which defines the maximal acceptance angle of an optical system.

$$\mathrm{NA} = n_0 \sin(\theta_{\max}) \tag{21}$$

where n_0 is refractive index in the acceptance angle.

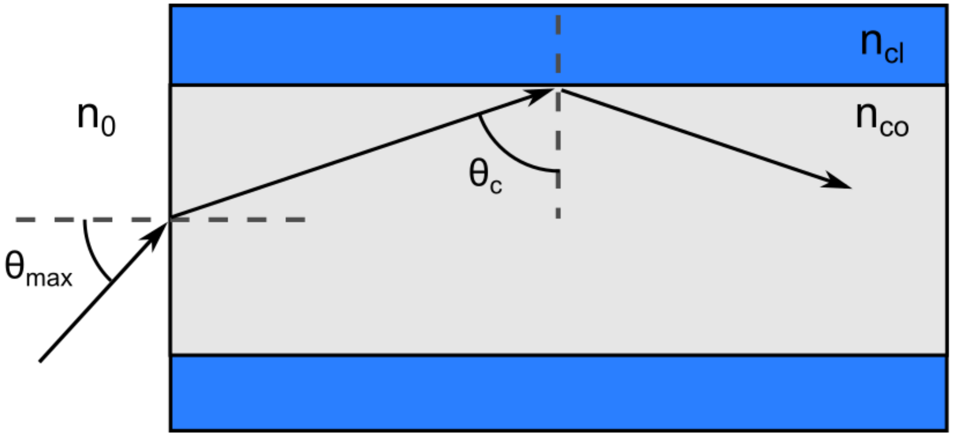


Figure 11: Illustration of numerical aperture in optical fiber.

NUMERICAL APERTURE

For coupling to guided mode of the optical fiber:

- TIR must occur inside the core and $\theta_i > \theta_c$.
- $\theta_c = \sin^{-1}(n_2/n_1)$ is the critical angle of the core-cladding interface.
- Since $\sin\theta_1 = \sqrt{1-\sin^2\theta_i}$ by Snell's Law: $n_0 \sin(\theta_{\max}) = n_1\sqrt{1-\sin^2\theta_i}$.
- For the maximal acceptance angle $\theta_i = \theta_c$. By using $\sin\theta_c = n_2/n_1$ we get the numerical aperture as

$$NA = n_0 \sin(\theta_{\max}) = n_1\sqrt{1-\sin^2\theta_c} = \sqrt{{n_1}^2 - {n_2}^2} \qquad (22)$$

NUMERICAL APERTURE

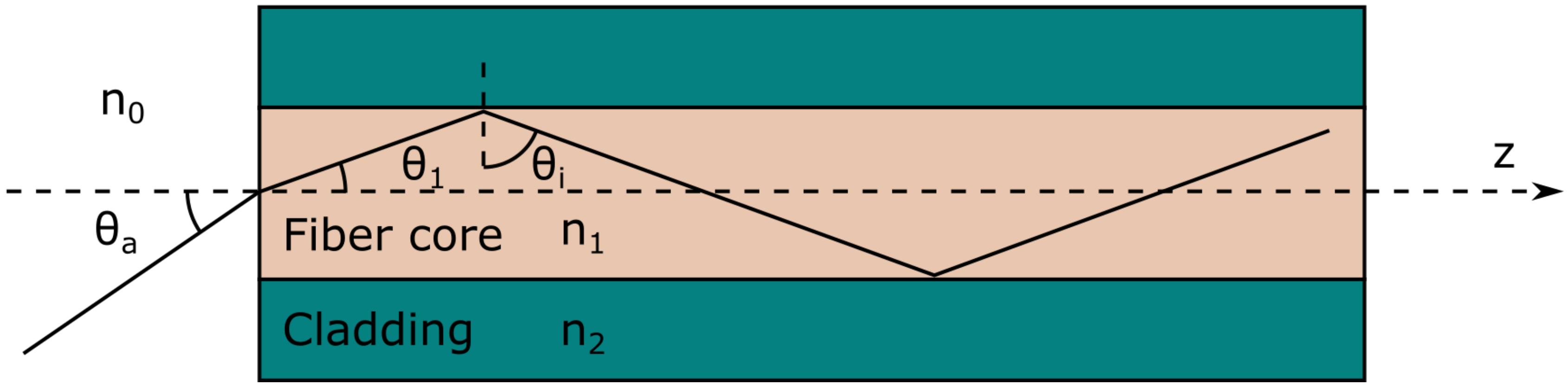


Figure 12: Illustration of light coupling into a step-index fiber.

NUMERICAL APERTURE

- Above this angle, the light propagates with partially internal reflection in the fiber and the power will be lost after few reflections.
- Below this angle, the light propagates in the fiber with total internal reflection without losses in the fiber.

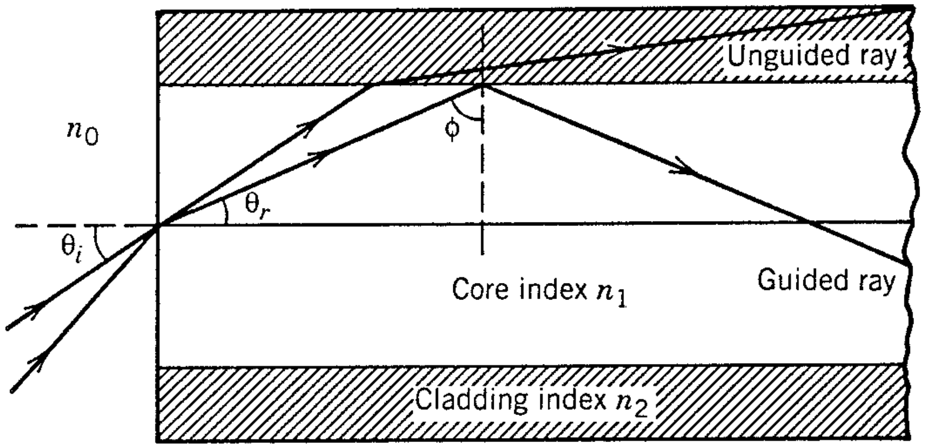


Figure 13: Light confinement through total internal reflection in step-index fibers [3].

NUMERICAL APERTURE

Numerical aperture: general case

$$\mathrm{NA} = \sqrt{{n_1}^2 - {n_2}^2} \geq \theta_a$$

In case of a weak optical waveguide, $\Delta = (n_1 - n_2)/n_1 \ll 1\%$. In this case, the expression of numerical aperture can be simplified as:

Numerical aperture: weak waveguide

$$\mathrm{NA} = n_1\sqrt{2\Delta}$$

Typically, parameters of a single-mode fiber are $\mathrm{NA} \approx 0.1 - 0.2$ and $\Delta \approx 0.2 - 1$. Therefore, $\theta_a \approx \sin^{-1}(\mathrm{NA}) \approx 5.7 - 11.5$.

NUMERICAL APERTURE AND V-NUMBER

With the definition of the numerical aperture, the V-number and the cutoff wavelength of a fiber can be expressed as a function of NA:

$$V = \frac{2\pi a}{\lambda}\text{NA}$$

$$\lambda_c = \frac{\pi d}{2.405}\text{NA}$$

where d is the core diameter of the step-index fiber.

<u>Example:</u>

For a typical standard single-mode fiber with $n_1 = 1.47$, $n_2 = 1.467$ and $d = 9\ \mu\text{m}$ - $NA = 0.0939$

The maximum incident angle at the fiber input is $\theta_a = \sin^{-1}(0.0939) = 5.38$ and the cutoff wavelength is $\lambda_c = 1.1\ \mu\text{m}$.

NUMERICAL APERTURE

- Large NA (and Δ) is good in terms of the coupling and collection of light efficiency but tends to become multimode.
- Large NA Unsuitable for high-speed communication because of a limitation such as modal dispersion.
- Small NA fibers are therefore used for high-speed optical communication systems since they improve the fiber's bandwidth.
- Plastic optical fibers (POFs) have high NA (0.4-0.5) and used for short-haul communications (e.g. within an automobile). They are usually short due to the high attenuation losses of plastic.

NUMERICAL APERTURE

- **Low numerical aperture** - small acceptance angle.
- **High numerical aperture** - big acceptance angle.

In commercial single mode fiber for telecommunication region (1.55 μm), the numerical aperture in 0.13 and the acceptance angle is ~7.45 degrees. Therefore, coupling to single-mode fiber is challenging (also small core of 9.8 μm).

Figure 14: θ_a - fiber acceptance angle. Light can be coupled to an optical fiber only when the incidence angle is smaller than the numerical aperture (if bigger, no TIR in the core → loss to radiation at core-cladding interface).

TYPES OF FIBERS

Optical fiber have two common configurations: step-index and graded-index.

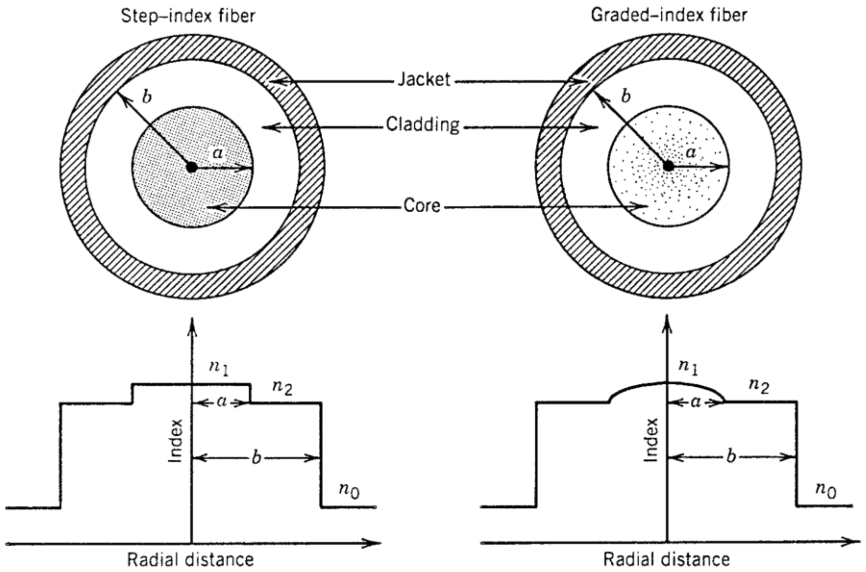


Figure 15: Cross section and refractive-index profile for step-index and graded-index fibers [3].

STEP-INDEX FIBER

- In step-index fiber, the core refractive index is constant.
- Longer ray trace will have bigger optical path distance and short ray trace will have smaller optical path distance.
- As a result, this index profile causes intermodal dispersion.

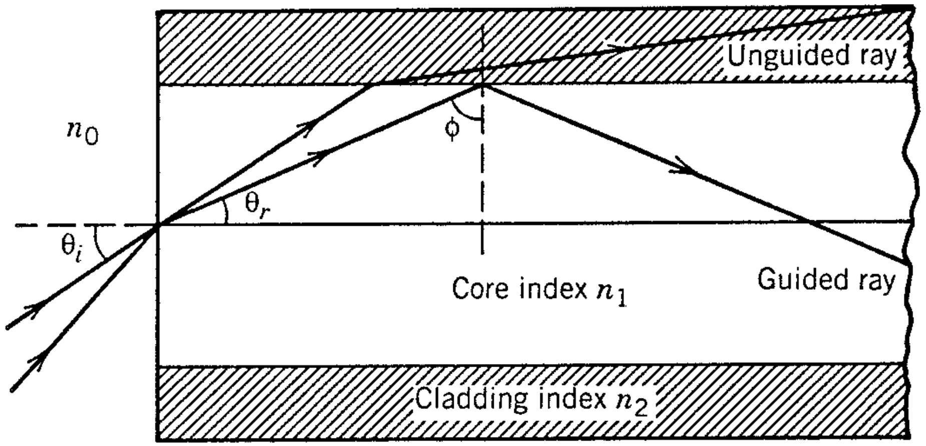


Figure 16: Light confinement through total internal reflection in step-index fibers [3].

STEP-INDEX FIBER - TIME DELAY

The shortest path for $\theta = 0$ the length is $D = L$. The longest path for $\theta = \phi_c$ the length is $D = \frac{L}{\sin \phi_c}$.

The time delay for the shortest and longest paths is defined as:

$$\Delta T = \frac{n_1}{c}\left(\frac{L}{\sin \phi_c} - L\right) = \frac{L}{c}\frac{{n_1}^2}{n_2}\Delta \qquad (23)$$

The estimate bit rate is obtained by the condition $B\Delta T < 1$. Therefore:

$$BL < \frac{n_2}{{n_1}^2}\frac{c}{\Delta} \qquad (24)$$

Common communication fibers ($\Delta = 2 \cdot 10^{-3}$) can communicate data at a bit rate of 10 Mb/s over distances up to 10 km and may be suitable for some local-area networks.

GRADED-INDEX FIBER

In graded-index fiber, the core refractive index decreases gradually from the maximum value n_1.

$$f(x) = \begin{cases} n_1[1 - \Delta(\rho/a)^{\alpha}], & \rho < a \\ n_1[1 - \Delta] = n_2 \quad , & \rho \geq a \end{cases} \tag{25}$$

where a is the core radius and α determines the index profile ($\alpha = 2$ - a parabolic-index fiber).

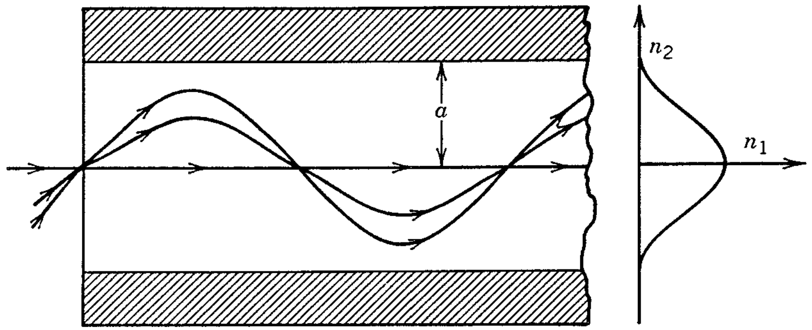
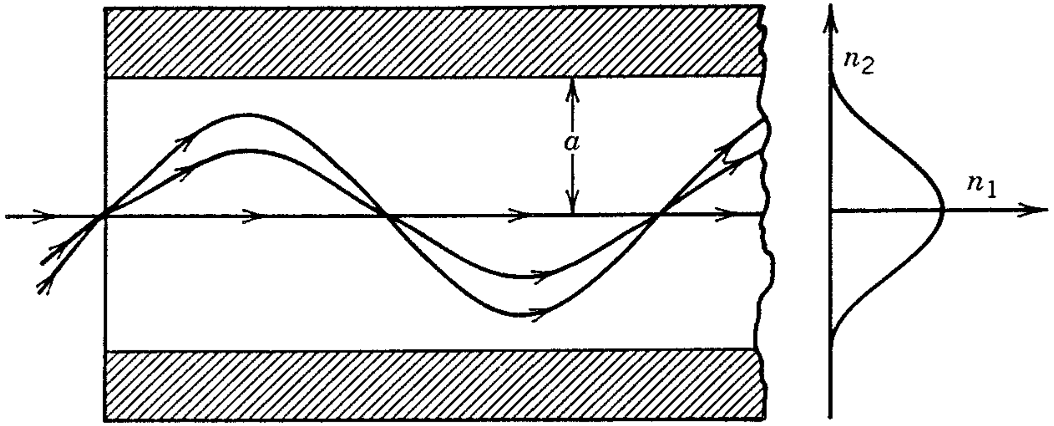


Figure 17: Ray trajectories in a graded-index fiber [3].

GRADED-INDEX FIBER



- Longer ray trace will pass in lower index and short ray trace in higher index.
- Therefore, longer ray trace will have in shorter optical path distance and short ray trace will have in longer optical path distance.
- It is therefore possible for all rays to arrive together at the fiber output.
- As a result, this index profile reduces the intermodal dispersion.

GRADED-INDEX FIBER

The trajectory of a paraxial ray is obtained by:

$$\frac{d^2\rho}{dz^2} = \frac{1}{n}\frac{dn}{d\rho}$$

where ρ the radial distance of the ray from the axis.

For $\rho < a$ and $\alpha = 2$, the solution is an equation of harmonic oscillator

$$\rho = \rho_0 \cos(pz) + \frac{\rho'_0}{p}\sin(pz) \qquad (26)$$

where $p = \sqrt{2\Delta/a^2}$ and ρ_0 and ρ'_0 are the position and the direction of the input ray, respectively.

GRADED-INDEX FIBER

$$\Delta T/L = n_1\Delta^2/8c \qquad (27)$$

The limiting bit rate-distance product is:

$$BL < 8c/n_1\Delta^2 \qquad (28)$$

The BL of this fiber can be improved in 3 orders of magnitude compared to step-index fiber and can communicate data at a bit rate of 100 Mb/s over distances up to 100 km.

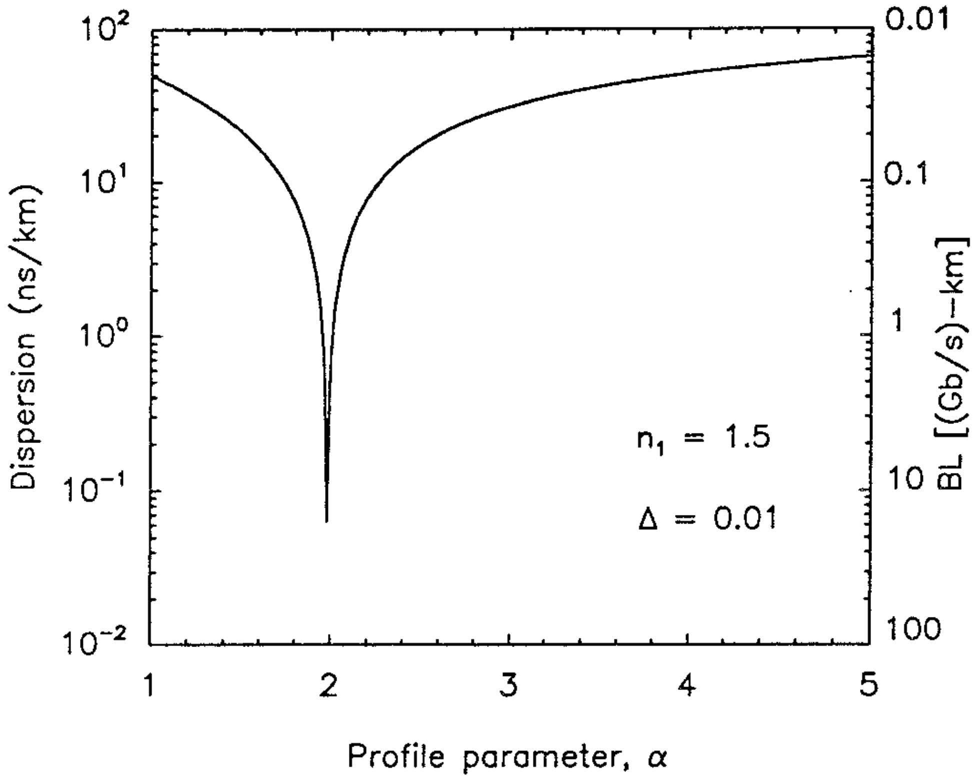


Figure 18: Variation of intermodal dispersion $\Delta T/L$ with the profile parameter α for a graded-index fiber [3].

H.W. : CHARACTERIZATION OF RAYS ON GRADED-PROFILE FIBERS

Graded-profile fibers:

Explain graphically and analytically.

[1] Construction of ray paths

[2] Classify the meridional, skew and tunneling rays

BIBLIOGRAPHY

[1] Rongqing Hui and Maurice O'Sullivan. Fiber optic measurement techniques. Academic Press, 2009.

[2] Allan W Snyder and John Love. Optical waveguide theory. Springer Science & Business Media, 2012.

[3] Govind P Agrawal. Fiber-optic communication systems, volume 222. John Wiley & Sons, 2012.