

Metamaterial Engineered Transparency due to nullifying of multipole moments

ANAR K. OSPANOVA^{1,2,3}, ALINA KARABCHEVSKY², ALEXEY A. BASHARIN^{1,3,4,*}

¹ National University of Science and Technology (MISIS), Department of Theoretical Physics and Quantum Technologies, 119049 Moscow, Russia

² Electrooptical Engineering Unit and Ilse Katz Institute for Nanoscale Science & Technology, Ben-Gurion University, Beer-Sheva 84105, Israel

³ Politecnico di Torino, Department of Electronic and Telecommunications, Torino 10129, Italy

⁴ National University of Science and Technology (MISIS), The Laboratory of Superconducting metamaterials, 119049 Moscow, Russia

*Corresponding author: alexey.basharin@gmail.com

Received XX Month XXXX; revised XX Month, XXXX; accepted XX Month XXXX; posted XX Month XXXX (Doc. ID XXXXX); published XX Month XXXX

Here, we propose novel transparency effect in cylindrical all-dielectric metamaterials. We show that cancellation of multipole moments of the same kind lead to almost zero radiation losses in all-dielectric metamaterials due to the counter-directed multipolar moments in metamolecule. Nullifying of multipoles, mainly dipoles and suppression of higher multipoles results in ideal transmission of incident wave through the designed metamaterial. The observed effect could pave the road to new generation of light-manipulating transparent metadevices such as filters, waveguides, cloaks and more.

OCIS codes: (160.3918) Metamaterials; (040.2235) Far infrared or terahertz.

<http://dx.doi.org/10.1364/OL.99.099999>

Narrow transmission peak called “transparency window” in the optical spectral range [1], is one of the most promising effects in nanophotonics. This effect provides new field in electronic and optical applications. Namely, slowlight propagation and long pulse delays for the storage of optical data in matter, frequency selectivity for narrow-band filters, enhanced nonlinear effects, strong light-matter interaction in photonics [2]. For the first time, observed in quantum systems, soon this phenomenon imitated in classical objects. Experimentally classical Electromagnetically Induced Transparency (EIT) obtained in metamaterials in microwave frequency range [3]. At the same time, metamaterials are manmade materials exhibiting unnatural properties like negative refraction, cloaking, strong field localization and others [4-8]. Since metamaterials are free for geometrical modifications, they may be tuned to reach narrow band transparency window corresponding to high Q-factor. Thus, next challenge is to create suitable structures with proper interaction for EIT-like phenomena.

One can demonstrate “transparency window” by inducing overlapping of electric and magnetic multipoles in plasmonic and all-dielectric particles [9]. This phenomenon called Fano-resonance arising due to the interference between different parts of constituent metamolecules [10, 11]. Another technique to observe the EIT in metamaterials is “trapping” incident electromagnetic wave and exciting destructive interference

between the same multipoles in metamaterials [12,13]. In addition, anapole mode can be defined as the third principal method of transparency produced by destructive interference between electric and toroidal multipole moments of the same amplitudes and angular momentum [14-16].

All aforementioned systems possess low radiative losses and exhibit transparency due to destructive interference of the main two multipoles of the same order and suppressing other multipoles. However, in this paper we propose metamaterial transparency effect due to nullifying of main excited dipole moments leading to almost zero radiative losses in all-dielectric metamaterials.

The unit cell of proposed metamaterial consists of clusters containing four identical subwavelength high-index dielectric cylinders and has rhombic shape. For the demonstration of well-pronounced effect in THz frequency range, we choose the cylinders made of LiTaO₃. This ionic crystal is known to exhibit strong polaritonic response in THz frequency range and can be practically realized by means of methods of crystal growth [17,18]. Complex dielectric permittivity of LiTaO₃ shows Lorentz-type dispersion $\varepsilon = \varepsilon_{\infty} \frac{\omega^2 - \omega_L^2 + i\omega\gamma}{\omega^2 - \omega_T^2 + i\omega\gamma}$. Here $\omega_T/2\pi = 26.7$ THz stands for frequency of the transverse optical phonons, $\omega_L/2\pi = 46.9$ THz is the frequency of longitudinal optical phonons, $\gamma/2\pi = 0.94$ THz is the damping factor due to dipole relaxation and $\varepsilon_{\infty} = 13.4$ is the limiting value of the permittivity for frequencies much higher than ω_L . At frequencies lower than phonon resonance one can consider permittivity of LiTaO₃ to be ~ 41.4 . At these frequencies LiTaO₃ crystals have negligible dissipation losses.

The height of each cylinder assumed to be infinitely elongated. Electromagnetic response of our system is characterized by the displacement currents induced in each cylinder by an incident electromagnetic wave. Displacement currents cause electromagnetic scattering that becomes resonant due to the accurately chosen radius and permittivity of cylinders and the polarization of incident electromagnetic wave. This resonant behavior corresponds to Mie resonance emerging on cylindrical all-dielectric particles. In our case, parallel-polarized electromagnetic wave excites resonant electric dipolar moment in each high-index dielectric cylinder as can be seen in Fig. 1 [19,20]. It gives an opportunity to use dielectric metamaterials as unique structure for artificial magnetism, magnetic and toroidal dipolar excitation as well as anapole mode [20-26].

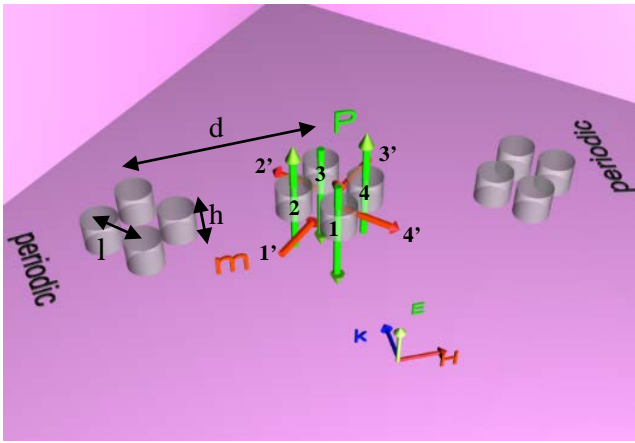


Fig.1. Illustration of the proposed metamolecule, consisting of 4 identical parallel dielectric cylinders. Electric component of incident wave polarized parallel to cylinder axis and induces counter-directed electric moments in each neighboring cylinder. Loops excite four magnetic dipolar moments that directed to and from the center of the metamolecule.

Incident electromagnetic wave with E-field polarized parallel to cylinder axis induces counter-directed \mathbf{P} electric moments in opposite cylinders (1 and 3; 2 and 4) and forms two pair of displacement current loops in each metamolecule (Fig. 1). Each pair of loops create two magnetic dipole ($2'$ and $4'$) moments \mathbf{m} directed out of center. Another pair of magnetic dipole moments emerges between cylinders from different pairs ($1'$ and $3'$) and directed to the center of the unit cell. All magnetic moments of the studied system are orthogonal to cylinder axis. Each pairs of magnetic dipolar moments \mathbf{m} have opposite direction and eliminate each other. Therefore, the total magnetic response of the system disappears. In addition, the total electric dipolar response of metamolecule damped as well. This can be explained by the cancellation of displacement currents of neighboring cylinders from different current loops.

In previous works transparency was suggested as consequence of destructive interference between multipole moments as in [10, 11, 15]. However, our findings presented here show that transparent metamaterials can be designed without any dipolar response just due to the nullifying of each kind of dipole moments. This provides zero radiation losses and such metamaterial becomes transparent in optical frequency range.

Our proposed system demonstrates strong electromagnetic interaction due to the near-field coupling of close located high-index dielectric cylinders. The unit cell parameters scaled to obtain pronounced dipolar response at the expense of higher order multipoles. Each cylinder has radius $r=5 \mu\text{m}$ and center-to-center distance with neighboring within metamolecule cylinders of $l=12 \mu\text{m}$. Period between clusters is $d=60 \mu\text{m}$. Clusters are surrounded by air or vacuum medium. We assume indefinitely elongated height h of cylinders allowing considering two-dimensional structure. Electromagnetic properties of metamaterial are calculated by commercial Maxwell's equation solver HFSS using standard modeling approach, where the whole structure described by replicating unit cell properties using periodic boundary conditions.

Transmission spectrum of our metamaterial is depicted on Fig 2. The sharp narrow transmission peak at 2.2446 THz

corresponds to the emergence of the transparency effect. This resonance has amplitude of 1 and width of 0.0017 THz and due to $Q = f_0/\Delta f$ one can obtain very high Q-factor value of 1320.

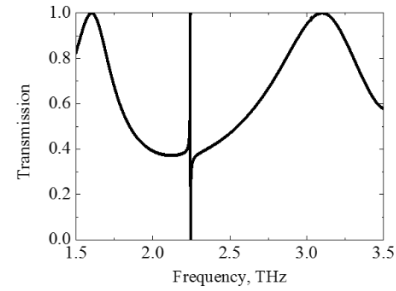


Fig. 2. Transmission spectrum for metamolecule in THz frequency range. Narrow sharp transparency peak corresponds to $f=2.2446 \text{ THz}$ with $Q=1320$.

Field map at this value shows opposite distributed electric field strengths in every neighboring cylinder indicating on occurrence of displacement currents loops in each pair of cylinders [Fig 3a]. Direction of field strength of cylinder 1 coincides with the direction in the cylinder 3, while both 2 and 4 cylinders have opposite field distribution directions. It means that these cylinders, let us say pair 1 and 2, 2 and 3, 3 and 4, 4 and 1, forms four loops of displacement currents in every unit cell of the system. In turn, two loops generate two magnetic dipolar responses directed out of center of the unit cell ($2'$ and $4'$ of Fig.3b). Other two loops of displacement currents generate other pair of magnetic dipole moments that directed toward center ($1'$ and $3'$ of Fig.3b). Field map on Fig. 3b shows magnetic field distribution in the unit cell. The strong magnetic field localization between cylinders indicates the near field coupling between them. These oppositely directed magnetic dipole moments have zero contribution in metamolecule dipolar response. There is also zero total electric dipolar response, since electric field strength of cylinders 1 and 3 cancels electric field strength of cylinders 2 and 4.

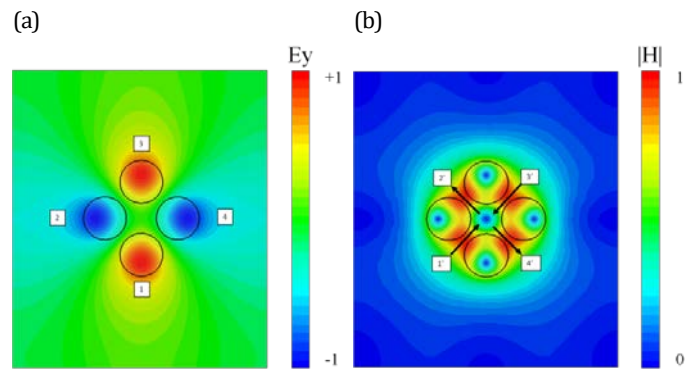


Fig. 3. Field maps of (a) y-component (along cylinders axis) of electric field and (b) absolute value of magnetic field intensities.

To confirm our assumption on nullifying the same kind multipoles we carried out multipolar decomposition up to second order of multipoles scattering by metamolecule at resonant frequency value from HFSS Transient solver method simulated displacement currents in each clusters with periodic boundary conditions [22]. Fig. 4 shows normalized power of near-field

distribution of metamolecule up to second order multipoles. Resonant frequency corresponds to the second peak of transmission spectrum. At the resonant frequency $f=2.2446$ THz one can see narrow dip of electric dipole moment accompanying with suppression of quadrupole moments and almost zero value of magnetic and toroidal dipoles. Accordingly, in this frequency range radiation losses tends to zero and this leads to full transparency at $f=2.2446$ THz. This confirms our assumption that transparency of metamaterial can be achieved due to nullifying of all multipoles contemporaneously. This effect accompanied by very high Q-factor, corresponding to narrow transparency window [Fig. 2].

We note, that high Q-factor value has been previously considered for all-dielectric metamaterial with broken symmetry in optical range. In particular, the symmetry of cubic dielectric metasurface was broken and placed on substrate to obtain resonance between two modes [27]. Authors of 28 presented high Q metamaterial due to overlapping of resonant modes of asymmetric metallic bars. As the result, very high-Q Fano resonances were obtained. Moreover, high Q response can be obtained due to toroidal mode excitation in planar toroidal metamaterials [29, 30]. Our metamaterial distinguished with avoidance of such sophisticated manufacturing by placing cylinders in defined sequence. This elegant approach is obviously technologically simple.

Let us note, that the first peak close to 1.5 THz corresponds to electric dipole contribution with strong influence of magnetic dipole and electric quadrupole modes. Moreover, the behavior of third peak which is close to $f=3$ THz gives the following outcomes: from 3 THz to 3.14 THz frequency range, which coincides to third peak of transmission spectra (Fig. 2,4), it becomes higher among other multipoles leading to the establishing of toroidal mode. It is obvious, that presented effect is the result of collective Mie resonance within each metaatom and specially chosen geometric parameters and permittivity of cylinders. Based on the Mie electric mode of the single cylinder, we may call magnetic modes induction in our observed metamaterial as collective Mie resonance, since these modes appear due to interparticle interaction. Owing to chosen unit cell geometry, these modes cancel each other leading to Fano-type transparency window.

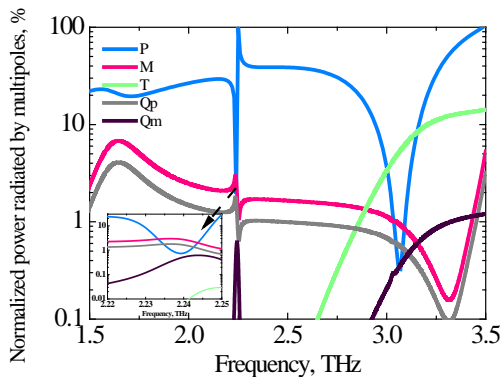


Fig. 4. Normalized power of near-field distribution of metamolecule up to second order multipoles. Sharp dip of electric dipole moment (light blue curve) corresponds to transparency peak at 2.2446 THz. There is also reduction of electric (grey curve) and magnetic (violet curve) quadrupole moments and magnetic (pink curve) and toroidal (green

curve) dipolar moments. Inset shows excited multipoles close to $f=2.2446$ THz.

In addition, we carried out simulations of various parameters of proposed THz metamaterials geometry exhibiting transparency. Fig. 5 contains five graphs for different parameters of our proposed metamaterials in a wide range of values. Figure 5 (a) illustrates transmission spectra for various radii r for cylinders and one can see that reduction of radius from initial 5 μm leads to blueshift of resonant frequency and for even smaller values $r=2$ μm resonance is widening. Figure 5 (b) corresponds to changing of transmission spectra depending on center-to-center distance between clusters d of metamaterial. Graph clearly shows that varying of center-to-center distance from initial value $d=60$ μm leads to widening of resonant peak, i.e. decrease of Q-factor. In figure 5 (c) transmission spectrum for various value of interparticle distance l is depicted. The larger value (initial interparticle distance is $l=12$ μm) results in blueshift of resonant frequency with following widening of resonance. Figure 5 (d) shows transmission spectra behavior in dependence of different permittivity ϵ of cylinders. From the graph, the smaller values of permittivity ($\epsilon=15$) corresponds to decrease of electric size of clusters terminating with the broadening of resonance. However, the increase of ϵ leads to redshifting of resonance with squeezing of peak. Moreover, figure 5 (e) shows transmission of metamaterial of realizable heights h of cylinders. In purpose of experimentally feasibility, we simulated proposed metamaterial height differing from 3 μm to 20 μm and for considered infinite $h=\infty$ height. Smaller heights of cylinders lead to redshift of resonant frequency with eventually damping of resonance.

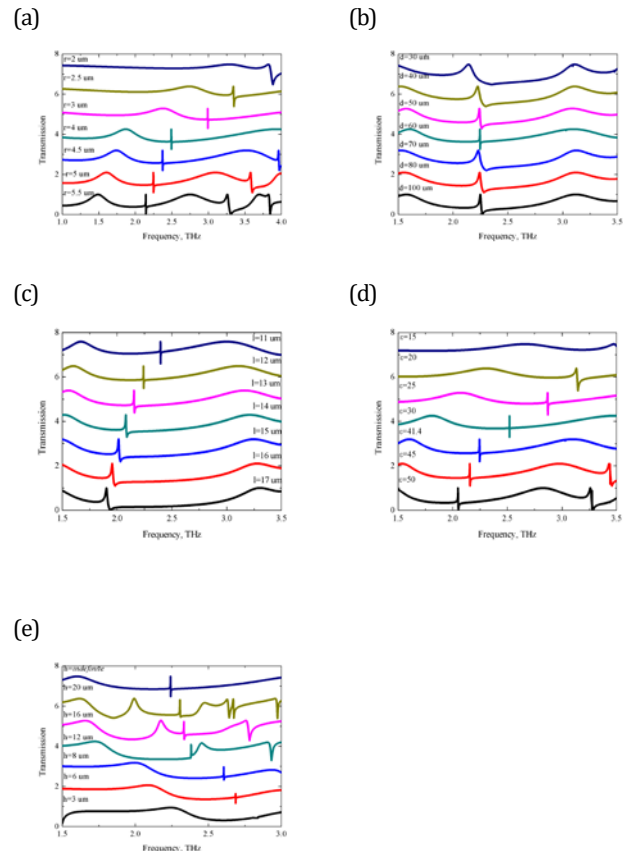


Fig. 5. Transmission versus frequency for various values of metamaterial parameters. (a) Transmission spectra for different radii r of cylinders of proposed THz metamaterial. (b) Transmission spectra for different center-to-center distance between clusters d . (c) Transmission spectra for different interparticle distance l . (d) Transmission spectra for different permittivity ϵ . (e) Transmission spectra for different height h .

In this work, we presented new metamaterial induced transparency effect arising from extinguishing of all kinds of multipoles at the same time. To confirm our assumption, we proposed metamaterial structure and carry out multipolar decomposition. Simulation results show that at transparency peak at $f=2.2446$ THz of $Q=1320$ there is suppression of all type of multipole moments up to zero pretending non-radiative system. Although, the Q-factor of metamaterials is described as $1/Q=1/Q_{\text{rad}}+1/Q_{\text{non}}$, where Q_{rad} is radiating losses and Q_{non} is nonradiating or dissipative losses. While the radiating losses in our case are low due to nullifying of multipoles, but not zero, the Q-factor is limited by radiating losses of multipoles and Joule losses in LiTaO₃. For ideal case of zero multipoles, lossless materials, we can expect extremely high Q-factor. These results are extremely important in designing invisible to external observer systems that could be used in many applicable fields as nanophotonics, plasmonic and applied optics.

As the summary, we would like to discuss the difference between proposed effect and wellknown EIT and Fano-type resonances. Indeed, Fano resonance is determined by interference between dark and bright modes. However, EIT is a special case of Fano-resonance if frequencies ω_1 and ω_2 of modes are coincided to each other. In this case, the Fano curve goes to the narrow peak of the Lorentz transparency form [31, 32]. In our case, the power scattered by multipoles is very low for all multipoles on the resonance frequency 2.24 THz. Indeed this behavior defined by contribution of electric, magnetic and electric quadropole dipole modes, which are very low, but uncoincided frequencies. Fig 4 shows that minimum deep of electric dipole goes ahead of maximum peak while for magnetic and toroidal dipoles vice versa and quadropoles. Indeed, our resonance can be characterized as Fano-like form. Then, we present here novel type of transparency effect due to mutual cancellation of multipoles of the same kind excited in each cylinder with coinciding resonant frequencies.

ACKNOWLEDGMENTS. This work was supported by the National University of Science and Technology MISiS, K4-2015-031, the RFBR (No. 16-32-50139 and No. 16-02-00789). In addition, this work has been partially supported by the Joint Projects for the internationalization of Research launched by the Politecnico di Torino with the financial support of the Compagnia di San Paolo, project title: "Advanced Non-radiating Architectures Scattering Tenuously And Sustaining Invisible Anapoles (ANASTASIA)". The work on the multipoles decomposition investigation of the metamolecules was supported by Russian Science Foundation (project 17-19-01786).

References

1. Stephen E. Harris, *Phys. Today* 50, 7, 36 (1997).
2. S. E. Harris, J. E. Field, and A. Imamoglu, *Phys. Rev. Lett.* 64, 1107 (1990).
3. N. Papasimakis, V. A. Fedotov, N. I. Zheludev, and S. L. Prosvirnin, *Phys. Rev. Lett.* 101, 253903 (2008).
4. D. R. Smith, J. B. Pendry, M. C. K. Wiltshire, *Science*, 305(5685), 788-92 (2004).
5. Yongmin Liu and Xiang Zhang, *Chem. Soc. Rev.*, 40, 2494-2507 (2011)
6. J.B. Pendry, *Phys Rev Lett*, 85, 3966, (2000).
7. P.D. Terekhov, K.V. Baryshnikova, A.S. Shalin, A. Karabchevsky, A.B. Evlyukhin *Optics Letters*, 42, 4, (2017)
8. C. M. Soukoulis and M. Wegener, *Nat. Photonics* 5, 523 (2011).
9. Boris Luk'yanchuk, Nikolay I Zheludev, Stefan A Maier, Naomi J Halas, Peter Nordlander, Harald Giessen, Chong Tow Chong, *Nature materials*, 9, 9, 707 (2010).
10. Shuang Zhang, Dentcho A. Genov, Yuan Wang, Ming Liu, and Xiang Zhang, *PRL* 101, 047401 (2008).
11. P Tassin, L Zhang, R Zhao, A Jain, T Koschny, CM Soukoulis, *Phys. Rev. Lett.* 109 (18), 187401 (2012).
12. Fedotov VA1, Rose M, Prosvirnin SL, Papasimakis N, Zheludev NI., *Phys. Rev. Lett.* 99(14), 147401 (2007).
13. Nikitas Papasimakis and Nikolay I. Zheludev, *Optics and Photonics News* 20, 10, 22-27 (2009).
14. N. Papasimakis, V. A. Fedotov, V. Savinov, T. A. Raybould and N. I. Zheludev, *Nature Materials* 15, 263–271 (2016).
15. V. A. Fedotov, A. V. Rogacheva, V. Savinov, D. P. Tsai and N. I. Zheludev, *Sci. Rep.* 3, 2967 (2013).
16. Nikita A. Nemkov, Alexey A. Basharin, and Vassili A. Fedotov, *Phys. Rev. B* 95, 165134 (2017).
17. Edward D. Palik *Handbook of Optical Constants of Solids, Five-Volume Set: Handbook of Thermo-Optic Coefficients of Optical Materials with Applications* Academic Press, 1997
18. A. Buzády, M. Unferdorben, G. Tóth et al. *J Infrared Milli Terahz Waves* 38: 963, (2017).
19. C. F. Bohren and D. R. Huffman, Wiley-Interscience, New York (1983).
20. Yuri Kivshar and Andrey Miroshnichenko, *Philos Trans A Math Phys Eng Sci.*, 375, 2090 (2017).
21. A.A. Basharin, I.V. Stenischev, Toroidal response in all-dielectric metamaterials based on water, *Sci. Rep.*, accepted (2017).
22. Alexey A. Basharin, Maria Kafesaki, Eleftherios N. Economou, Costas M. Soukoulis, Vassili A. Fedotov, Vassili Savinov and Nikolay I. Zheludev, *Phys. Rev. X* 5, 011036 (2015).
23. W Liu and Yuri S.Kivshar, *Philos Trans A Math Phys Eng Sci.* 375, 2090, (2017).
24. Boris Luk'yanchuk, Ramón Paniagua-Domínguez, Arseniy I. Kuznetsov, Andrey E. Miroshnichenko, Yuri S. Kivshar, *Philos Trans A Math Phys Eng Sci.* 375, 2090 (2017).
25. Wei Liu, Andrey E. Miroshnichenko, arXiv:1704.06049 [physics.optics].
26. Andrey E. Miroshnichenko, Andrey B. Evlyukhin, Ye Feng Yu, Reuben M. Bakker, Arkadi Chipouline, Arseniy I. Kuznetsov, Boris Luk'yanchuk, Boris N. Chichkov, and Yuri S. Kivshar, *Nat Commun.* 6, 8069 (2015).
27. Salvatore Campione, Sheng Liu, Lorena I. Basilio, Larry K. Warne, William L. Langston, Ting S. Luk, Joel R. Wendt, John L. Reno, Gordon A. Keeler, Igal Brener, and Michael B. Sinclair, *ACS Photonics*, 3 (12), 2362–2367 (2016).
28. Y. Fan, et al., *Phys. Rev. B* 87, 115417 (2013).
29. G. Sun, et al., *Sci Rep* 7, 8128, (2017).
30. A.A. Basharin, et al, *Phys. Rev. B* 3, 95, 035104 (2017)
31. Mikhail F. Limonov, et al., *Nature Photonics*, 11, 543–554 (2017)
32. B. Peng, S.K. Özdemir, W. Chen, F. Nori, L. Yang, *Nature Communications*, 5, 5082 (2014).

References

1. Stephen E. Harris "Electromagnetically induced transparency", *Phys. Today* 50, 7, 36 (1997).
2. S. E. Harris, J. E. Field, and A. Imamoglu "Nonlinear optical processes using electromagnetically induced transparency", *Phys. Rev. Lett.* 64, 1107 (1990).
3. N. Papasimakis, V. A. Fedotov, N. I. Zheludev, and S. L. Prosvirnin "Metamaterial Analog of Electromagnetically Induced Transparency", *Phys. Rev. Lett.* 101, 253903 (2008).
4. D. R. Smith, J. B. Pendry, M. C. K. Wiltshire "Metamaterials and Negative Refractive Index", *Science*, 305(5685), 788-92 (2004).
5. Yongmin Liu and Xiang Zhang "Metamaterials: A New Frontier of Science and Technology", *Chem. Soc. Rev.*, 40, 2494-2507 (2011).
6. J.B. Pendry "Negative Refraction Makes a Perfect Lens", *Phys Rev Lett*, 85, 3966, (2000).
7. P.D. Terekhov, K.V. Baryshnikova, A.S. Shalin, A. Karabchevsky, A.B. Evlyukhin "Resonant forward scattering of light by high-refractive-index dielectric nanoparticles with toroidal dipole contribution" *Optics Letters*, 42, 4, (2017)
8. C. M. Soukoulis and M. Wegener "Past achievements and future challenges in the development of three-dimensional photonic metamaterials", *Nat. Photonics*, 5, 523 (2011).
9. Boris Luk'yanchuk, Nikolay I Zheludev, Stefan A Maier, Naomi J Halas, Peter Nordlander, Harald Giessen, Chong Tow Chong "The Fano resonance in plasmonic nanostructures and metamaterials", *Nature materials*, 9, 9, 707 (2010).
10. Shuang Zhang, Dentcho A. Genov, Yuan Wang, Ming Liu, and Xiang Zhang "Plasmon-Induced Transparency in Metamaterials", *PRL* 101, 047401 (2008).
11. P Tassin, L Zhang, R Zhao, A Jain, T Koschny, CM Soukoulis "Electromagnetically induced transparency and absorption in metamaterials: The radiating two-oscillator model and its experimental confirmation", *Phys. Rev. Lett.* 109 (18), 187401 (2012).
12. Fedotov VA, Rose M, Prosvirnin SL, Papasimakis N, Zheludev NI. "Sharp trapped-mode resonances in planar metamaterials with a broken structural symmetry", *Phys. Rev. Lett.* 99(14), 147401 (2007).
13. Nikitas Papasimakis and Nikolay I. Zheludev "Metamaterial-Induced Transparency: Sharp Fano Resonances and Slow Light", *Optics and Photonics News* 20, 10, 22-27 (2009).
14. N. Papasimakis, V. A. Fedotov, V. Savinov, T. A. Raybould and N. I. Zheludev "Electromagnetic toroidal excitations in matter and free space", *Nature Materials* 15, 263–271 (2016).
15. V. A. Fedotov, A. V. Rogacheva, V. Savinov, D. P. Tsai and N. I. Zheludev "Resonant Transparency and Non-Trivial Non-Radiating Excitations in Toroidal Metamaterials", *Sci. Rep.* 3, 2967 (2013).
16. Nikita A. Nemkov, Alexey A. Basharin, and Vassili A. Fedotov "Nonradiating sources, dynamic anapole, and Aharonov-Bohm effect", *Phys. Rev. B* 95, 165134 (2017).
17. Edward D. Palik *Handbook of Optical Constants of Solids, Five-Volume Set: Handbook of Thermo-Optic Coefficients of Optical Materials with Applications* Academic Press, 1997
18. A. Buzády, M. Unferdorben, G. Tóth et al. "Refractive Index and Absorption Coefficient of Undoped and Mg-Doped Lithium Tantalate in the Terahertz Range" *J Infrared Milli Terahz Waves* 38: 963, (2017).
19. C. F. Bohren and D. R. Huffman "Absorption and Scattering of Light by Small Particles", Wiley-Interscience, New York (1983).
20. Yuri Kivshar and Andrey Miroshnichenko "Meta-Optics with Mie Resonances", *Philos Trans A Math Phys Eng Sci*, (2017).
21. A.A. Basharin, I.V. Stenischev, "Toroidal response in all-dielectric metamaterials based on water", *Sci. Rep.*, accepted (2017).
22. Alexey A. Basharin, Maria Kafesaki, Eleftherios N. Economou, Costas M. Soukoulis, Vassili A. Fedotov, Vassili Savinov and Nikolay I. Zheludev "Dielectric Metamaterials with Toroidal Dipolar Response", *Phys. Rev. X* 5, 011036 (2015).
23. W Liu and Yuri S.Kivshar "Multipolar interference effects in nanoparticles", *Philos Trans A Math Phys Eng Sci.* 375, 2090, (2017).
24. Boris Luk'yanchuk, Ramón Paniagua-Domínguez, Arseniy I. Kuznetsov, Andrey E. Miroshnichenko, Yuri S. Kivshar "Suppression of scattering for small dielectric particles: anapole mode and invisibility", *Philos Trans A Math Phys Eng Sci.* 375, 2090, (2017).
25. Wei Liu, Andrey E. Miroshnichenko "Scattering invisibility with free-space field enhancement of all-dielectric nanoparticles", arXiv:1704.06049 [physics.optics].
26. Andrey E. Miroshnichenko, Andrey B. Evlyukhin, Ye Feng Yu, Reuben M. Bakker, Arkadi Chipouline, Arseniy I. Kuznetsov, Boris Luk'yanchuk, Boris N. Chichkov, and Yuri S. Kivshar "Nonradiating anapole modes in dielectric nanoparticles", *Nat Commun.*6, 8069, (2015).
27. Salvatore Campione, Sheng Liu, Lorena I. Basilio, Larry K. Warne, William L. Langston, Ting S. Luk, Joel R. Wendt, John L. Reno, Gordon A. Keeler, Igal Brener, and Michael B. Sinclair "Broken Symmetry Dielectric Resonators for High Quality Factor Fano Metasurfaces", *ACS Photonics*, 3 (12), 2362–2367 (2016).
28. Y. Fan, et al., "Low-loss and high-Q planar metamaterial with toroidal moment" *PRB* 87, 115417, (2013).
29. G. Sun, et al., "Q-factor enhancement of Fano resonance in all-dielectric metasurfaces by modulating meta-atom interactions" *Sci Rep* 7, 8128, (2017).
30. A.A. Basharin, et al., "Extremely high Q-factor metamaterials due to anapole excitation", *PRB* 3, 95, 035104 (2017)
31. Mikhail F. Limonov, et al., "Fano resonances in photonics", *Nature Photonics*, 11, 543–554 (2017)
32. B. Peng, S.K. Özdemir, W. Chen, F. Nori, L. Yang, "What is and what is not electromagnetically induced transparency in whispering-gallery microcavities" *Nature Communications*, 5, 5082 (2014).