LOSSES IN OPTICAL WAVEGUIDES

Prof. Alina Karabchevsky, www.alinakarabchevsky.com

Integrated Photonics Course 377-2-5599

School of ECE

Ben-Gurion University of the Negev, Israel

1

OUTLINE

Introduction

Losses

- Attenuation coefficient
- Scattering loss
- Absorption loss
- Thermal generation
- Optical generation
- Molar concentration
- Free carriers absorption - intra-band
- Radiation losses

Semiconductor detector

- Characteristics in a detector
- Low-level injection
- Drift current
- Detector on a waveguide

THE MOTIVATION

So far we learned how light propagates in the optical waveguide and the **cut-off condition** for this propagation. The next important parameter to learn is the **losses** or the **attenuation** in optical waveguides.

Losses in optical waveguide can be characterized as

1. **Scattering** loss - by the particles much smaller than the wavelength of the incident light. The scattering losses is as small λ^{-4} due to the Rayleigh scattering effect.
2. **Absorption** loss inter-band and intra-band in semiconductors.
3. **Radiation** loss.

These losses constitute the *propagation loss* and *insertion loss* in optical waveguides. The main reason for the coupling losses is the modal mismatch between waveguides.

ATTENUATION COEFFICIENT - α

Attenuation coefficient α is related to the change in light intensity per unit length which is in turn proportional to the overall intensity as following:

$$\boxed{\frac{dI(z)}{dz} \propto I(z)}$$

where the light intensity I is $I(z)$ [W/cm^2]. Therefore, we define the attenuation coefficient α as: [1/cm]

$$\frac{dI(z)}{dz} = -\alpha I(z) \qquad (1)$$

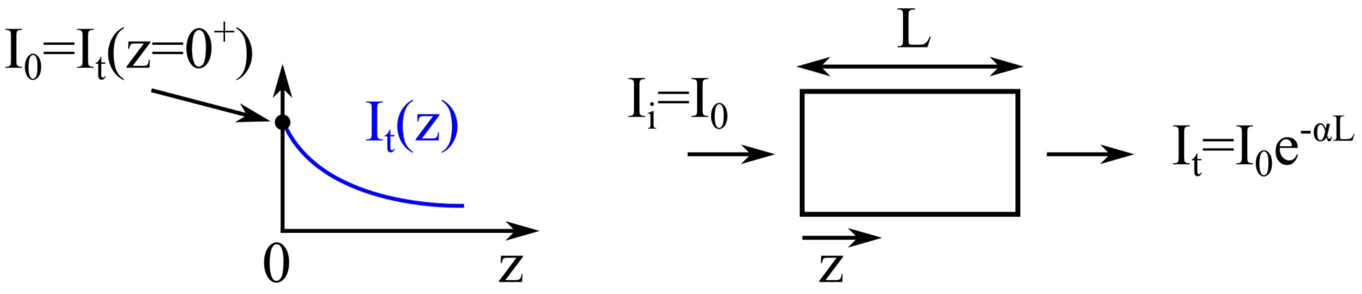
The solution of this differential equation of the first order is:

$$I(z) = I_0 e^{-\alpha z} \qquad (2)$$

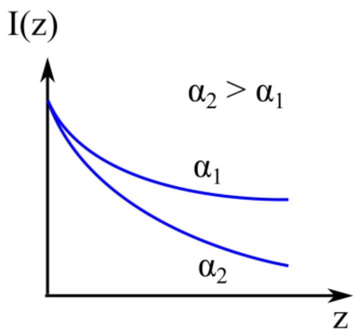
where $I_0 = I(z=0) = I_i$ (the input intensity)

ATTENUATION COEFFICIENT - α

- If in the input of the waveguide the intensity is I_i, we will obtain the transmitted intensity as following:



- The following graph depicts the relation between the radiation intensity in α showing that when α increases the attenuation is stronger.



LOSS COEFFICIENT - L

- We define the losses in units of [dB/cm] and designate the loss coefficient as L. The loss L is defined as the loss per unit length and is given by the formula:

$$\boxed{L = 10\log\left(\frac{I_{\text{in}}}{I_{\text{out}}}\right)_{L=1\text{cm}} \quad \left[\frac{\text{dB}}{\text{cm}}\right]} \tag{3}$$

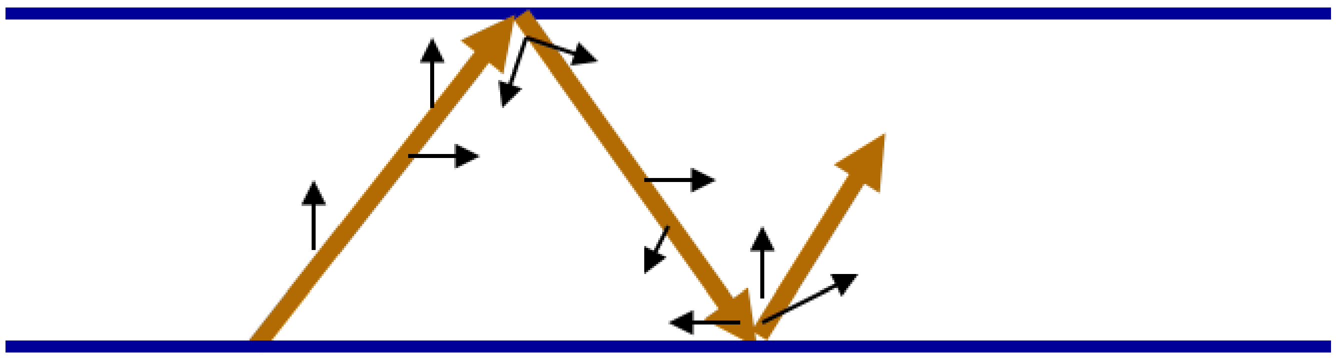
$$I_{\text{out}} = I_{\text{in}}e^{-\alpha z} \quad \Rightarrow \quad I_{\text{out}}(z = 1\text{cm}) = I_{\text{in}}e^{-\alpha 1\text{cm}}$$

we obtain

$$L\left[\frac{\text{dB}}{\text{cm}}\right] = 4.3\alpha \qquad L = 10\log\left(\frac{I_{\text{in}}}{I_{\text{in}}e^{-\alpha 1\text{cm}}}\right) = 10\log(e^{\alpha 1\text{cm}}) = 4.3\cdot\alpha$$

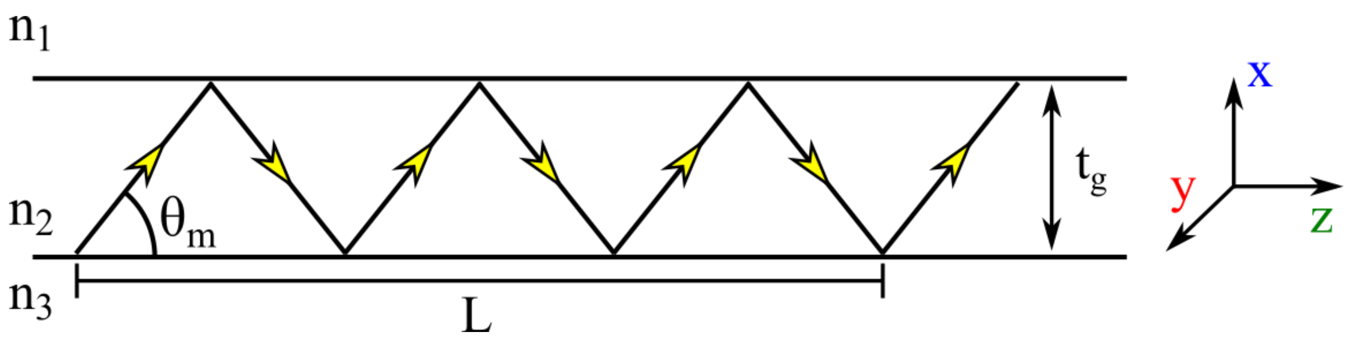
SCATTERING LOSS

- Caused by defects in the medium (such as glass with non-perfect transparency) resulting in volumetric scattering.
- Caused by reflections from defects in the surface resulting in a surface scattering.



SCATTERING LOSS

- Even a smooth surface is suffering from scattering, especially higher order modes exhibit more surface scattering:



<u>Ray optics explanation:</u>

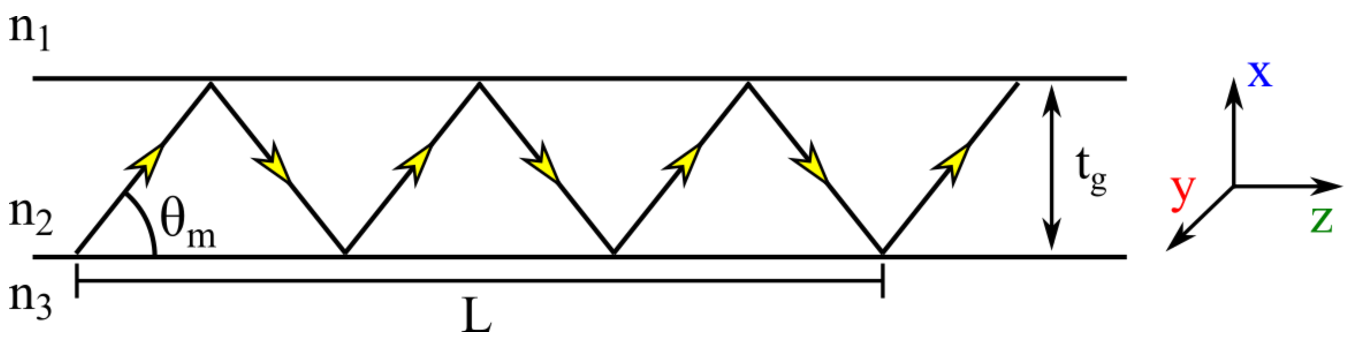
In planar waveguide, the ray has $N_{\text{reflections}} = \frac{Length}{2t_g \cot(\theta_m)}$ reflections from each surface.

Therefore, θ_m increases as m increases, the ray has more reflections for the same distance L. As a result, the scattering losses are more significant.

SCATTERING LOSS

$$N_{\text{reflections}} = \frac{\mathrm{L}}{2t_g \cot(\theta_m)}$$

Example:



1) For waveguide made of Ta_2O_5 ($n_2 = 2$), $t_g = 3\ \mu m$, $\lambda = 0.9\ \mu m$ and $\beta_m = 0.8$. How many reflections from the surfaces the ray will have for $\mathrm{L} = 1$ cm? (1250)
2) Will the number of reflections increase or decrease for higher order modes? (Since θ_m increases for higher order modes, the number of reflections will increase.)

SCATTERING LOSS

We define the attenuation loss (power to area) in waveguide at distance z as:

$$\alpha_s = A^2 \left[\frac{1}{2}\frac{\cos^3(\theta_m')}{\sin(\theta_m')}\right]\left[\frac{1}{t_g + 1/p + 1/q}\right] \qquad (4)$$

$$A = \frac{4\pi}{\lambda_2}\sqrt{{\sigma_{12}}^2 + {\sigma_{23}}^2}$$

σ - the variance of the surface roughness.

A - the ratio between the surface roughness and the wavelength.

λ_2 - the wavelength in medium (in the guiding layer)

SCATTERING LOSS

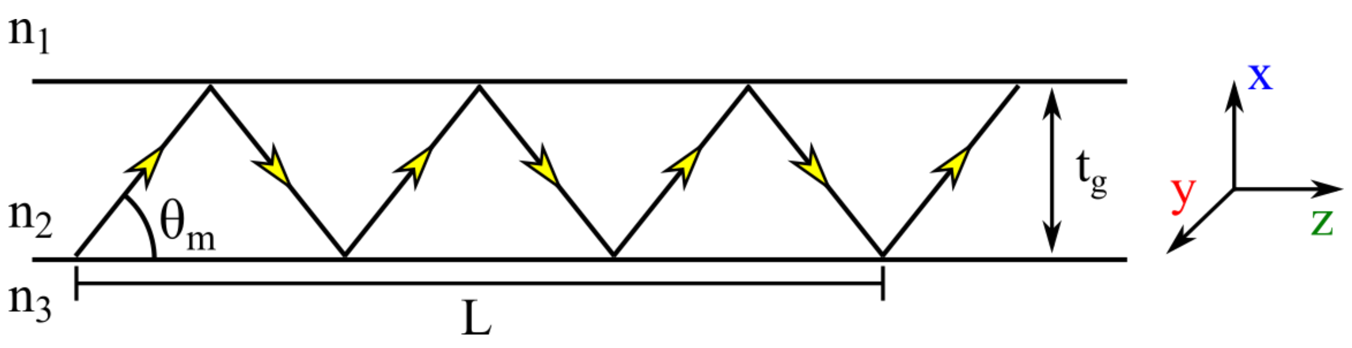
- $1/q$ and $1/p$ are related to the tails of the mode in regions 1 and 3, respectively.
- Highly confined mode will suffer less than less confined modes with large evanescent tails in regions 1 and 3.
- If $1/q$ and $1/p$ are smaller compared to the guiding layer thickness the scattering is significantly lower.

$\frac{\cos^3(\theta'_m)}{\sin(\theta'_m)}$ shows that the losses are significant as the mode order increases. It occurs due to the increasing the reflections per unit length.

It was shown experimentally that for Ta_2O_5 waveguide at wavelengths of $\lambda = 0.9\ \mu m$:

for $m = 0 \;\Rightarrow\; \alpha_s = 0.3\ \text{cm}^{-1}$ and for $m = 3 \;\Rightarrow\; \alpha_s = 2.8\ \text{cm}^{-1}$.

SCATTERING LOSS



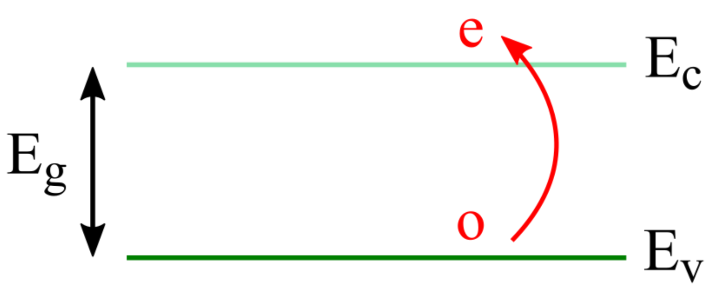
Based on the Rayleigh principle, a ray that incident a waveguide surface with power P_i will reflect from the surface with power P_r:

$$P_r = P_i \exp\left[-\left(\frac{4}{\lambda_2}\cos\theta'_m\right)^2\right] \qquad (5)$$

ABSORPTION LOSS

- Is caused by absorption of photons by the material itself.
- In crystalline material such as $TiNbO_3$, the absorption losses are very low compared to the scattering losses [1,2].
- The absorption losses are significant for semiconductors due to the inter-band absorption and intra-band absorption of free carriers.

ABSORPTION LOSS - THERMAL GENERATION



- In equilibrium state, the valence band is populated with electrons - e.
- By heating the material, an electron can get elevated from the valence band to the conduction band.
- An electron that is elevated from the valence band to the conduction band will leave a hole in the valence band.
- As the temperature increases, more electrons will populate the conduction band and holes the valence band.
- In intrinsic material: $N_e = N_o$

At the valence band E_v at temperature of 0°K there are plenty of energy levels which are populated with electrons and at the conduction energy level there are possible energy levels for population with electrons but there are free from electrons.

ABSORPTION LOSS - THERMAL GENERATION

Between the valence band and the conduction band there is a region called the band gap - E_g: $E_g = E_c - E_v$

- As E_g increases it's hard to generate electrons and holes.
- The conductivity depends on the electrons in the conduction band and not in the valence band.
- In metal - $E_g \rightarrow 0$

At room temperature:

silicon - $E_g = 1.1$ eV, GaAs - $E_g = 1.43$ eV, glass - $E_g = 8$ eV (1 eV $= 1.6 \cdot 10^{19}$ J)

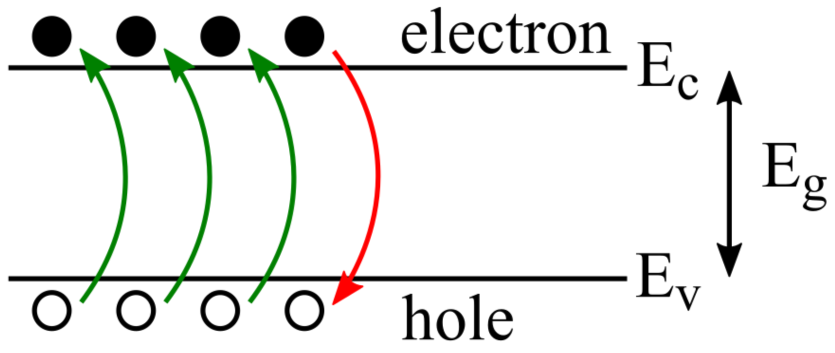
- Heating will make more electrons elevate to the conduction band. As a result, more electron-hole pairs (EHP) will be generated.

ABSORPTION LOSS - THERMAL GENERATION

- Raising electron from the valence band to the conduction band by heating the material is called "thermal generation" (GT).
- The electrons strive to descend back to the valence band - this process is called "recombination".
- In equilibrium state - recombination = generation.

In room temperature:

$n_0 = p_0 = n_i(\text{intrinsic}) \approx 1.55 \cdot 10^{10}\ \text{cm}^{-3}$ - for silicon

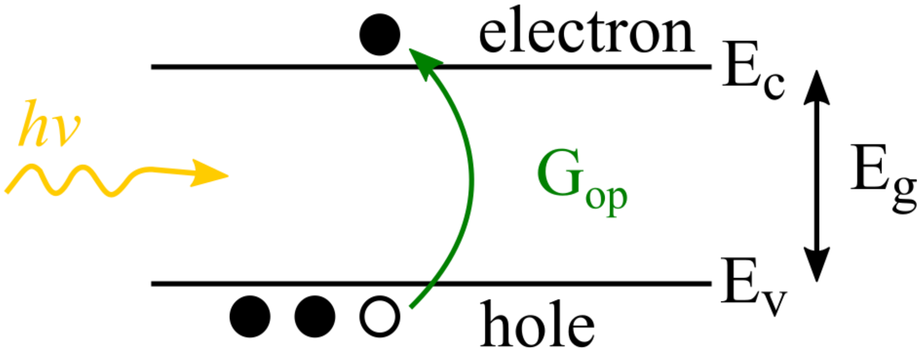


ABSORPTION LOSS - OPTICAL GENERATION

- Inter-band absorption occurs between the two bands - the valence band and the conduction band.
- The photon energy needs to be higher than the energy gap:

$$\boxed{h\nu > E_g} \quad \nu = c/\lambda$$

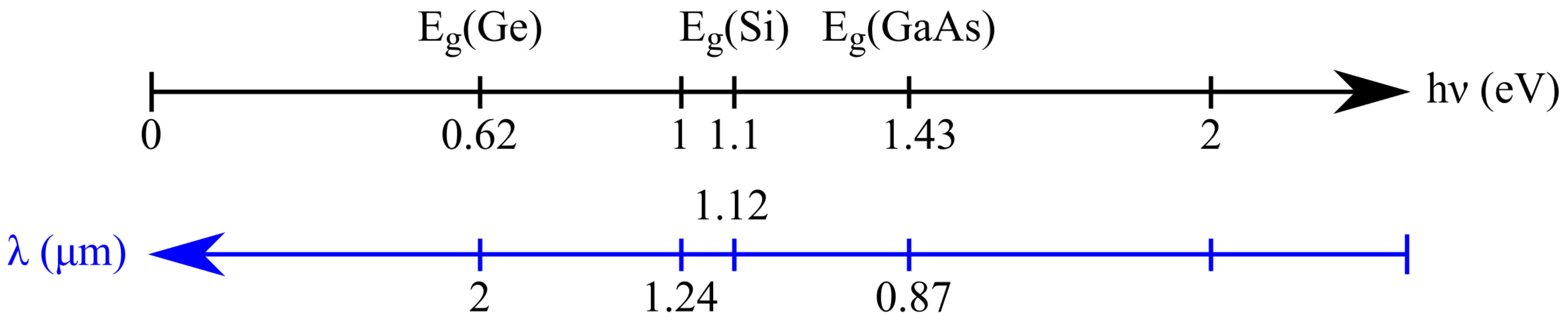
where h is the Planck constant.



ABSORPTION LOSS - OPTICAL GENERATION

- If a photon has enough energy - $h\nu > E_g$, it will be absorbed and the electron will "jump" to the conduction band. This process is called "optical generation" ($\boldsymbol{G}_{\mathbf{op}}$). ייצור אופטי
- If a photon doesn't have enough energy - $h\nu < E_g$, it won't be absorbed. The material is transparent and can be used as a waveguide for this wavelength. הולכה
- For designing waveguide: The E_g needs to be checked compared to $h\nu$.

ENERGY GAP



$$\lambda[\mu\text{m}] = \frac{1.24}{h\nu[\text{eV}]}$$

- For GaAs - $E_g = 1.43$ eV at $\lambda = 0.87\ \mu m$
- For $Ga_{1-x}Al_xAs$ - E_g depends on x. Therefore, we can control the band gap.

MOLAR CONCENTRATION

- For creating a monolithically integrated circuit on the same chip, GaAs can be used.

For $Ga_{1-x}Al_xAs$ when $0 < x < 0.38$, E_g depends on x. E_g is function of x:

$$E_g(x) = 1.439 + 1.042x + 0.468x^2 \quad (6)$$

$$\boxed{x \uparrow \Rightarrow \; E_g \uparrow; \; h\nu \downarrow}$$

For example:

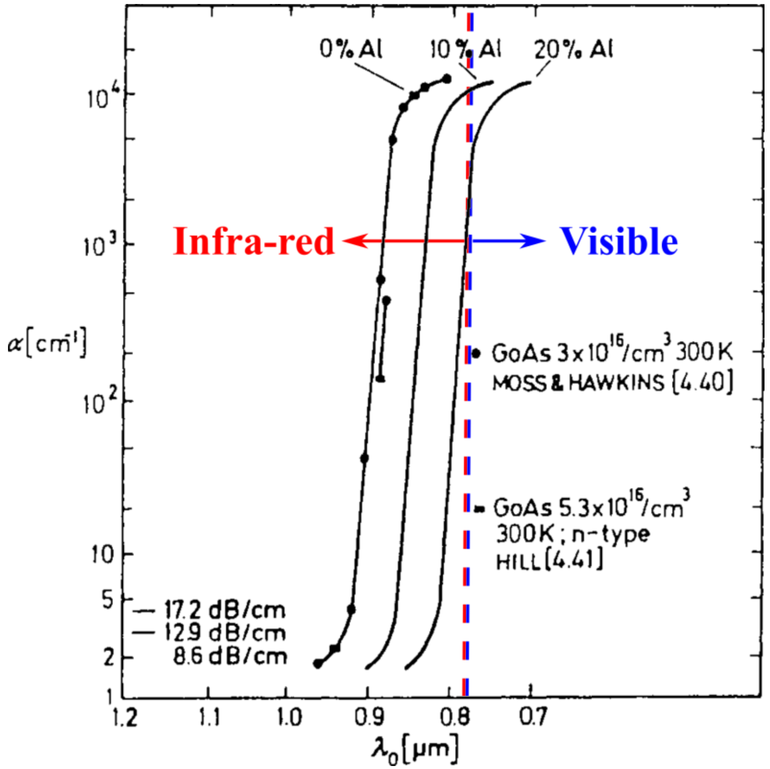
a) For $x = 0$ - $E_g(\mathrm{GaAs}) = 1.439\ \mathrm{eV}, \lambda = 1.24/1.439 = 0.8617\ \mu m$.

b) For $x = 0.2$ - $E_g(\mathrm{Ga_{0.8}Al_{0.2}As}) = 1.666\ \mathrm{eV}, \lambda = 1.24/1.666 = 0.744\ \mu m$.

By changing the molar concentration, we can create materials for different purposes: laser, waveguide or detector.

MOLAR CONCENTRATION

- The absorption coefficient depends on the wavelength for different concentration (x) of aluminum (Al) in $Ga_{1-x}Al_xAs$ [3]:



MOLAR CONCENTRATION

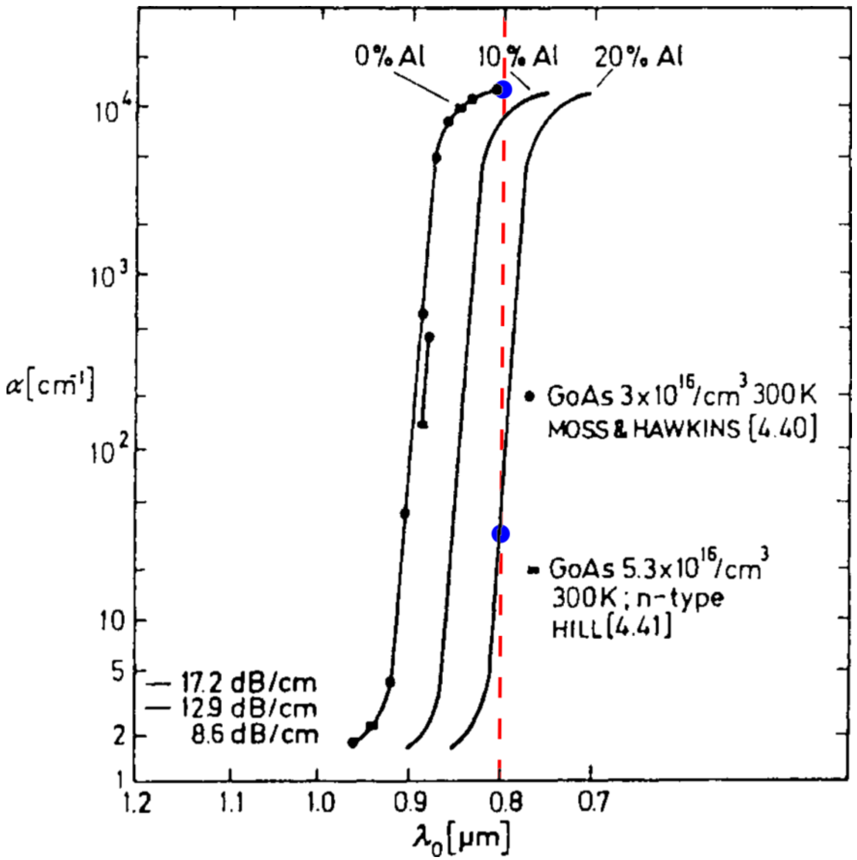
Example:

For $\lambda = 0.8\,\mu m$, assuming material $Ga_{1-x}Al_xAs$: can it be used as detector or waveguide [3]?

Solution:

1) For $x = 0$, α is big. The light is well absorbed (detector).

2) For $x = 0.2$, α is small. The light is propagating (waveguide).



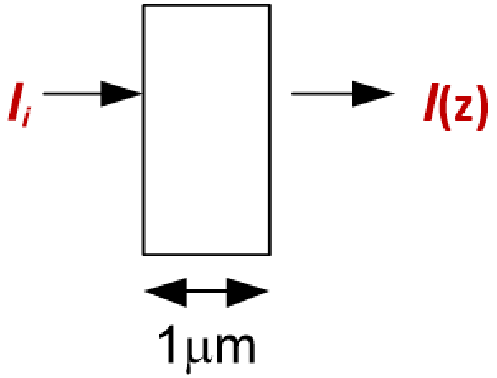
MOLAR CONCENTRATION

Example:

For GaAs illuminated with $\lambda = 0.8\ \mu m$, how much light will pass through the material with a thickness of 10^{-4} cm?

Solution:

$$I(z) = I_i e^{-\alpha z} = I_i \exp(-\overbrace{10^4}^{\alpha} \cdot \overbrace{10^{-4}}^{z}) = \frac{I_i}{e} = 0.37 I_i$$

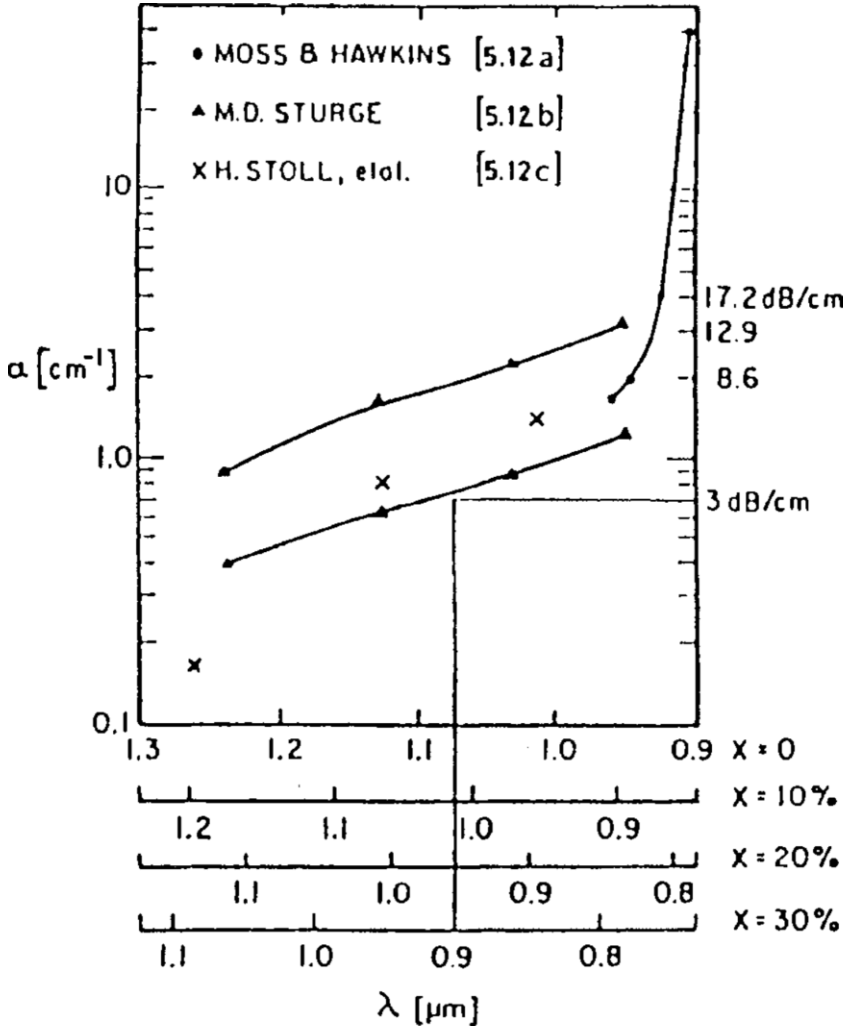


- Only 37% of the light will pass (63% absorption) - worst waveguide.
- In this case, we will use infra-red because then the absorption coefficient is smaller than 2.

MOLAR CONCENTRATION

To decrease the absorption, the appropriate wavelength needs to be chosen [3]:

- The graph shows that for higher molar concentration the absorption coefficient decreases.
- The four x axes show changes in the absorption coefficient at short wavelengths while increasing the Al concentration. The absorption losses decrease to 3 dB/cm for wavelength of 0.8 µm.
- Without aluminum, the absorption is around 50 cm^{-1} or 215 dB/cm.



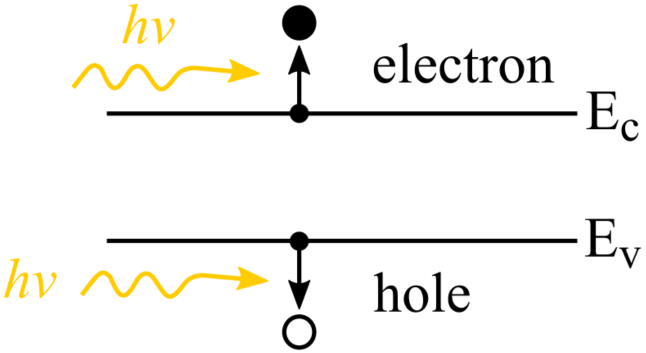
MOLAR CONCENTRATION

- The required aluminum concentration for decreasing the inter-band absorption for three common semiconductor lasers in integrated optics with loss of 3 dB/cm or loss of 8.6 dB/cm [3]:

	Required aluminum concentration in the guide	
Source wavelength	$\alpha = 2\ \mathrm{cm}^{-1}$ (8.6 dB/cm)	$\alpha = 0.7\ \mathrm{cm}^{-1}$ (3 dB/cm)
0.85 μm GaAlAs	17%	40%
0.90 μm GaAs	7%	32%
0.95 -1.0 μm Si:GaAs	0%	20%

FREE CARRIERS ABSORPTION - INTRA-BAND

- Absorption of photon by electron which is already in the conduction band. The electron will jump to higher energy level inside the conduction band.
- The absorption also possible for a hole in the valence band. The hole will descend to lower energy level inside the valence band.
- The intra-band absorption is also called free carriers absorption.



The intra-band absorption coefficient is defined as α_{FC} (**F**ree **C**arriers).

ABSORPTION COEFFICIENT - α_{FC}

- The movement of the electron due to the electric field $E_0 e^{j\omega t}$ should fulfill:

$$\underbrace{m^* \frac{d^2 x}{dt^2}}_{\text{I}} + \underbrace{m^* \Gamma \frac{dx}{dt}}_{\text{II}} = \underbrace{-eE_0 e^{j\omega t}}_{\text{III}}$$

where Γ is the damping coefficient, m^* is the mass of free carriers and e is the charge of an electron.

I - The force - $F = ma$.

II - The linear restraint caused by the interaction of the electron with the material.

III - The applied force.

ABSORPTION COEFFICIENT - α_{FC}

- The solution of the equation in equilibrium state:

$$x = \frac{(eE_0)/m^*}{\omega^2 - j\omega t} e^{j\omega t}$$

- The absorption coefficient α_{FC} is defined as:

$$\alpha_{\mathrm{FC}} = \frac{Ne^2{\lambda_0}^2}{4\pi^2 n(m^*)^2 \mu\varepsilon_0 c^3} \qquad (7)$$

Absorption Coefficient - α_{FC}

The imaginary part of the refractive index κ is related to the absorption. The amplitude of the electric field is proportional to:

$$\boxed{E(z) \propto e^{j\beta n z} e^{-\beta\kappa z}}$$

Therefore, the intensity is proportional to:

$$\boxed{I \propto |E|^2 \propto e^{-2\beta\kappa z} \propto e^{-\alpha_{\mathrm{FC}} z}}$$

α_{FC} is the absorption coefficient of the free carriers:

$$\boxed{\alpha_{\mathrm{FC}} = 2\beta\kappa}$$

ABSORPTION COEFFICIENT - α_{FC}

We saw in lecture of "Index control technologies":

$$\tilde{n} = \sqrt{\tilde{\varepsilon}} = \sqrt{\tilde{\varepsilon}_r + j\tilde{\varepsilon}_i} = \sqrt{\tilde{\varepsilon}_r}\sqrt{\tilde{\varepsilon}_r q + j\frac{\tilde{\varepsilon}_i}{\tilde{\varepsilon}_r}} = \sqrt{\tilde{\varepsilon}_r}\left(1 + j\frac{\tilde{\varepsilon}_i}{2\tilde{\varepsilon}_r}\right) =$$

$$\tilde{n} = \underbrace{\sqrt{\tilde{\varepsilon}_r}}_{n} + j\underbrace{\frac{\tilde{\varepsilon}_i}{2\sqrt{\tilde{\varepsilon}_r}}}_{\kappa}$$

Substitute $\tilde{\varepsilon}_r$ and $\tilde{\varepsilon}_i$ with:

$$\boxed{\tilde{\varepsilon}_r = {n_0}^2 - \frac{{\omega_p}^2}{\omega^2}} \qquad \boxed{\tilde{\varepsilon}_i = \frac{{\omega_p}^2}{\omega^3 \tau}} \qquad \boxed{\tau = \frac{m^* \mu}{q}}$$

ABSORPTION COEFFICIENT - α_{FC}

We obtain:

$$\alpha_{\mathrm{FC}} = \frac{Nq^3}{(m^*)^2 \varepsilon_0 \omega^3 \mu} \beta_0 \qquad (8)$$

where μ is the mobility and N is the concentration of the free carriers.

Substitute $\omega = 2\pi c/\lambda$ and $\beta_0 = 2\pi/\lambda_0$

$$\alpha_{\mathrm{FC}} = \frac{Nq^3 {\lambda_0}^2}{4\pi^2 n[(m^*)^2 \mu \varepsilon_0 c^3]} \qquad (9)$$

$$\alpha_{\mathrm{FC}} = AN{\lambda_0}^2 \qquad (10)$$

where A is the material constant.

The higher the free carriers in the material, the higher the absorption.

ABSORPTION COEFFICIENT - α_{FC}

Example:

For waveguide made of GaAs - n-type, what is the free carriers absorption - α_{FC}?

Given: $\lambda_0 = 1.15\ \mu m, n = 3.4, m^* = 0.08 m_e, \mu = 2000\ \left[\frac{\mathrm{cm}^3}{\mathrm{V \cdot sec}}\right]$.

Solution:

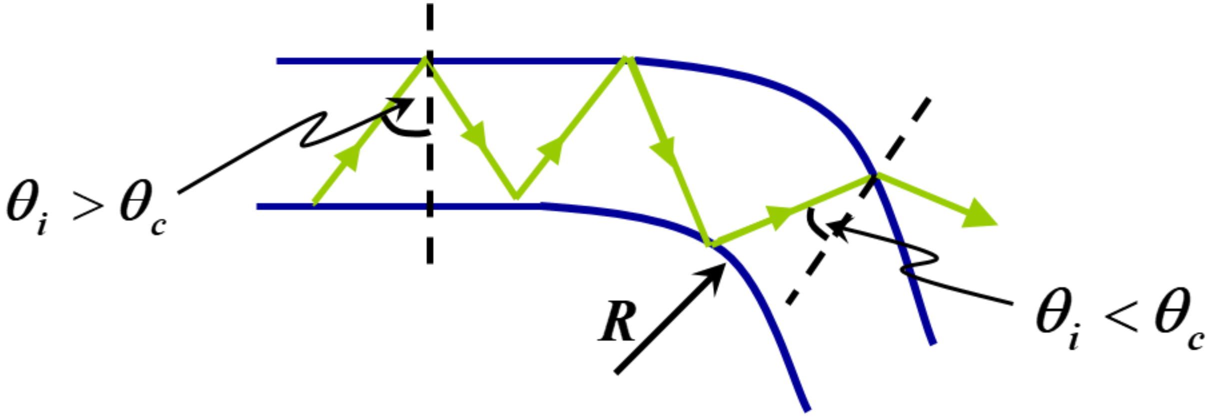
$\alpha_{\mathrm{FC}} = 10^{-18}\ \ [\mathrm{cm}^{-1}]$

Example:

1) For pure (intrinsic) material - $N \approx 10^7\ \mathrm{cm}^{-3}$. We get $\alpha_{\mathrm{FC}} = 10^{-11} \approx 0$. Means, no absorption due to free carriers.
2) For doped (extrinsic) material - $N \approx 10^{18} - 10^{20}\ \mathrm{cm}^{-3}$. We obtain $\alpha_{\mathrm{FC}} = 1 - 100$. This significant absorption is due to the free carriers and occurs only for high doping.

RADIATION LOSSES

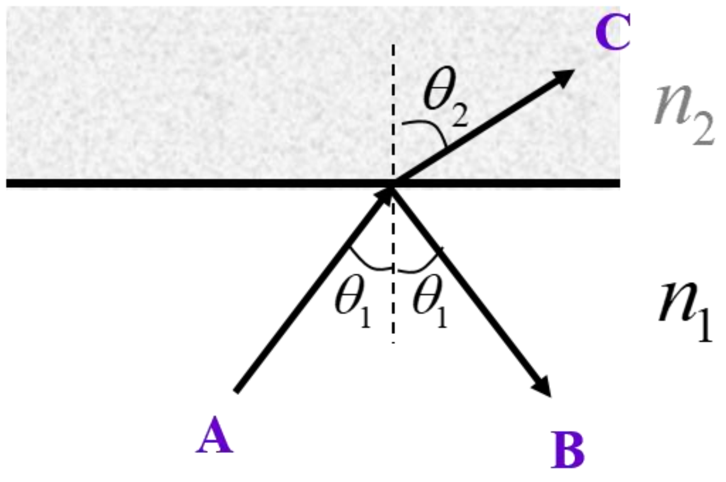
- Results from radiation of photons outside the guiding layer to the surrounding medium.
- The radiation losses are significant mainly when the guiding layer is bend due to local constraint.
- There is an allowed curvature radius - R. when the radius is smaller, the losses increase and number of modes decreases as shown below.



REFLECTION AND REFRACTION

- Assuming two dielectric, isotropic, loss-less and homogeneous materials with n_1 and n_2.
- Coherent wave incident the boundary in angle θ_1.
- Generally, the wave have reflection and refraction. The refraction angle can be calculated using Snell's law.

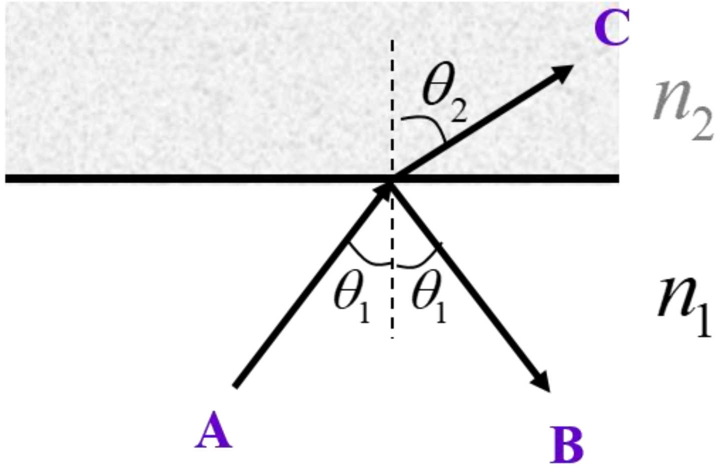
$$\boxed{n_1 \sin \theta_1 = n_2 \sin \theta_2}$$



REFLECTION AND REFRACTION

- R depends on the incidence angle and the light polarization.

$$B = RA$$



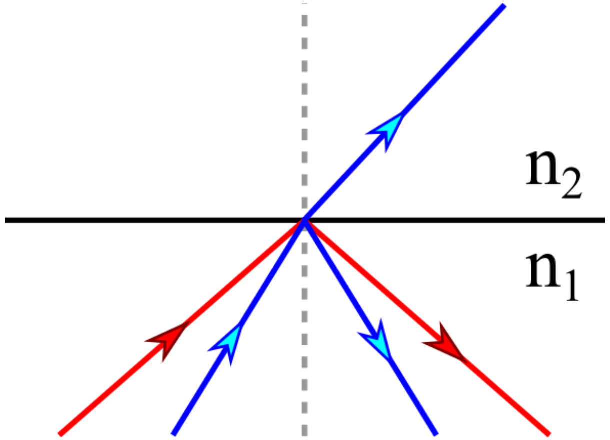
For TE (**s-polarized**):

$$R_s = \left|\frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}\right|^2 = \left|\frac{n_1 \cos\theta_1 - n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2}}{n_1 \cos\theta_1 + n_2\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2}}\right|^2$$

For TM (**p-polarized**):

$$R_p = \left|\frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1}\right|^2 = \left|\frac{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2} - n_2 \cos\theta_1}{n_1\sqrt{1-\left(\frac{n_1}{n_2}\sin\theta_1\right)^2} + n_2 \cos\theta_1}\right|^2$$

REFLECTION AND REFRACTION



We define the critical angle - θ_c as: $\boxed{\sin\theta_c = \frac{n_2}{n_1}}$

- $\theta_1 < \theta_c$ (blue) - partial reflection. R is real.
- $\theta_1 > \theta_c$ (red) - $|R| = 1$ total reflection. R is complex and the phase-shift is: $R = e^{2j\phi}$

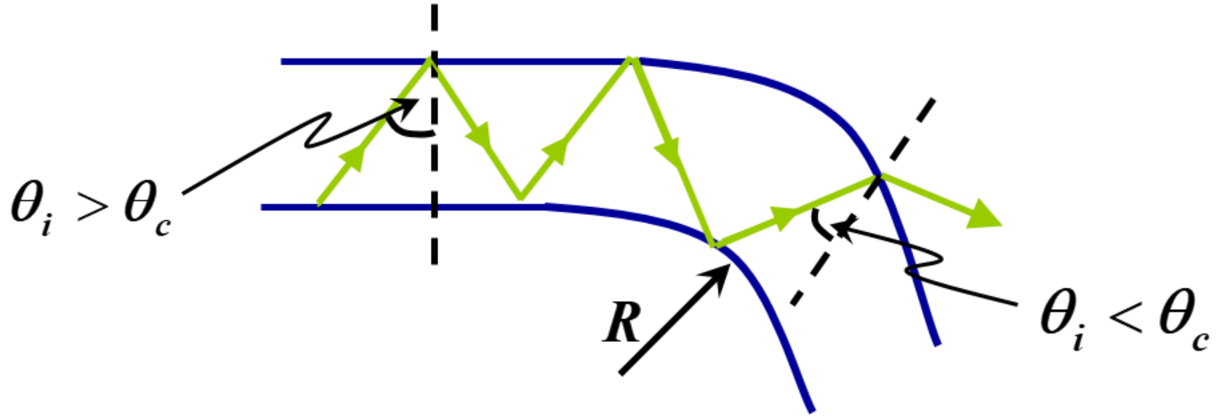
From Fresnel equations the phase-shift is:

$$\boxed{\text{TE} \Rightarrow \tan\phi_{12} = \frac{\sqrt{{n_1}^2 \sin^2\theta_1 - {n_2}^2}}{n_1\cos\theta_1}}$$

$$\boxed{\text{TM} \Rightarrow \tan\phi_{12} = \frac{{n_1}^2}{{n_2}^2}\frac{\sqrt{{n_1}^2 \sin^2\theta_1 - {n_2}^2}}{n_1\cos\theta_1}}$$

RADIATION LOSSES

- The Radiation losses change depends on the curvature radius.
- In the curvature, the incident angle is smaller than the critical angle so ray escapes.

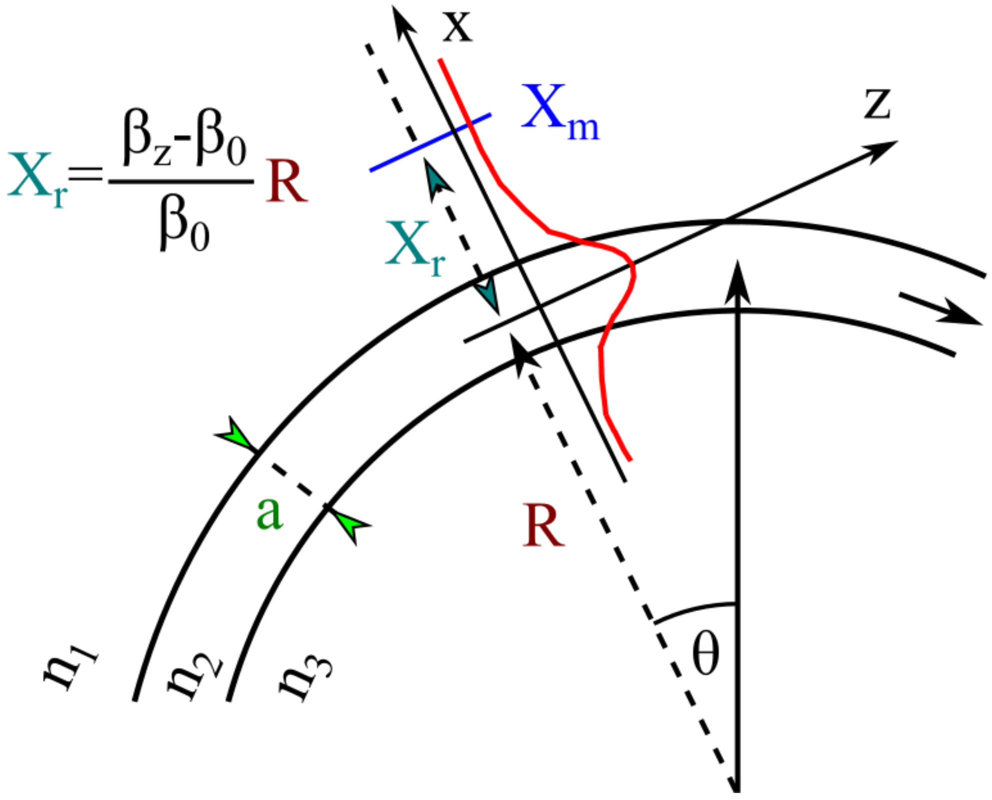


LOSSES COEFFICIENT DUE TO THE RADIATION LOSS

β_z - the propagation constant.

β_0 - the propagation constant of unguided light in medium n_1.

R - the radius curvature.



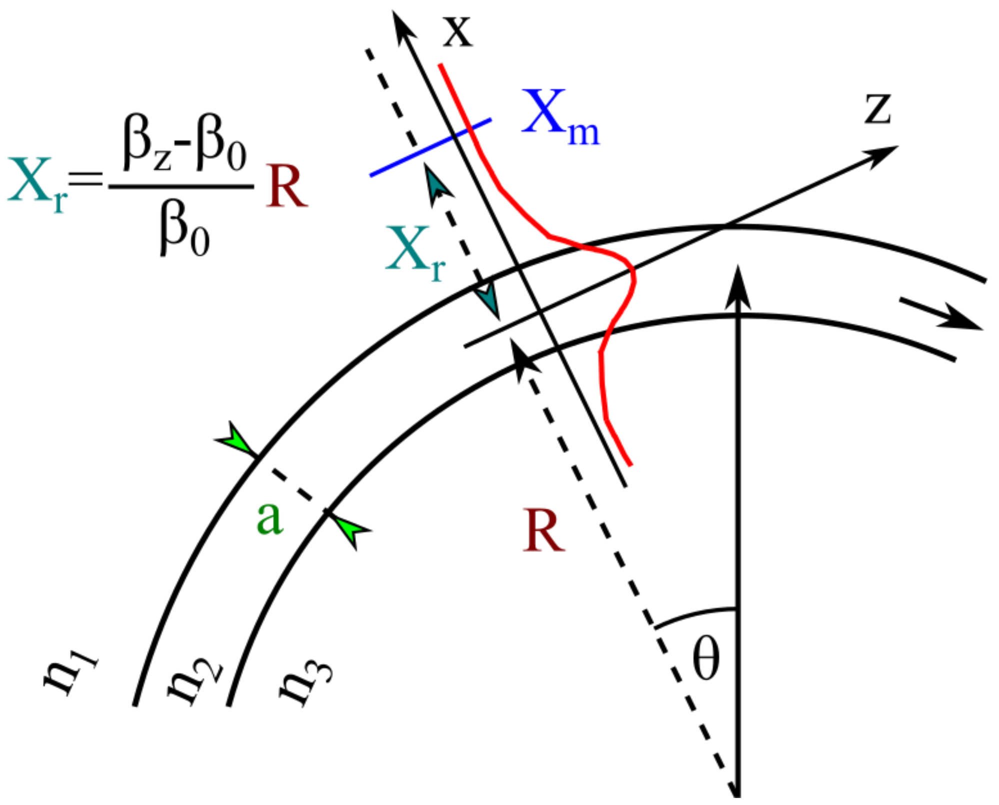
LOSSES COEFFICIENT DUE TO RADIATION LOSSES

For mode to exist, the angular velocity needs to be equal to preserve a wavefront. Therefore, the tangential phase velocity of the wave needs to be proportional to the distance from the center of the waveguide.

$$v = r\frac{d\theta}{dt} = x\frac{d\theta}{dt} = x\frac{\omega}{\beta x} = \frac{\omega}{\beta} \qquad (11)$$

The maximal velocity in n_1 is

$$v_1 = \frac{c}{n_1}$$



LOSSES COEFFICIENT DUE TO RADIATION LOSSES

For a certain distance X_m the velocity needs to be bigger than v_1 which is not physical. The photons will radiate into medium n_1.

$$X_m = R + X_r$$

In the center of the guiding layer (n_2):

$$v = \frac{\omega}{\beta_z} = R\frac{d\theta}{dt}$$

$$\boxed{\frac{d\theta}{dt} = \frac{\omega}{R\beta_z}} \qquad (12)$$

In medium 1 (n_1):

$$\boxed{\frac{d\theta}{dt} = \frac{\omega}{\underbrace{(R + X_r)}_{X_m}\beta_0}} \qquad (13)$$

LOSSES COEFFICIENT DUE TO RADIATION LOSSES

To preserve the wavefront, the angular velocities need to be equal.

$$\frac{\omega}{R\beta_z} = \frac{\omega}{(R + X_r)\beta_0} \quad \Rightarrow \quad R\beta_z = (R + X_r)\beta_0$$

$$X_m = \frac{\beta_z}{\beta_0} R \qquad X_r = \frac{\beta_z - \beta_0}{\beta_0} R$$

Till X_m, the wavefront is preserved.

Since $\beta_i = \beta_{\text{air}} n_i$, we get:

$$X_m = \frac{n_2}{n_1} R \qquad X_r = \frac{\Delta n}{n_1} R \qquad (14)$$

For $x > X_m$ the radiation is slower while disappearing after the distance of Z_c.

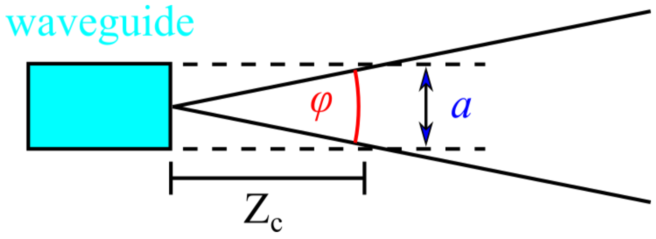
RADIATION LOSS COEFFICIENT - α_r

$$\frac{dP(z)}{dz} = -\alpha P(z) \quad \Rightarrow \quad \boxed{\frac{P_{\text{loss}}}{Z_c} = \alpha P_{\text{total}}}$$

where P_{loss} is the power in the tail of the mode beyond X_r (the power lost by radiation within a length Z_c) and P_{total} is the total power.

Z_c can be calculated by analogy to the emission of photons from an abruptly terminated waveguide. It is the distance for which the light emitted into a medium from an abruptly terminated waveguide remains collimated.

$$\boxed{Z_c = \frac{a}{\varphi} = \frac{a^2}{2\lambda_1}}$$



when $\lambda_1 = \frac{\lambda_0}{n_1}$, a is the near-field beam width and φ is the far-field angle

RADIATION LOSSES COEFFICIENT - α_r

$$\alpha_r = \frac{P_{\text{loss}}}{P_{\text{total}}} = \frac{\int_{X_r}^{\infty} E^2(x)dx}{\int_{-\infty}^{\infty} E^2(x)dx} \cdot \frac{1}{Z_c}$$

Substitute E of the modes in the different regions:

1) Inside the waveguide:

$$E(x) = \sqrt{C_0}\cos(hx), \qquad -\frac{a}{2} \leq x \leq \frac{a}{2}$$

2) Outside the waveguide:

$$E(x) = \sqrt{C_0}\cos\left(\frac{ha}{2}\right)\exp\left[-\left(\frac{|x| - a/2}{q}\right)\right], \qquad -\frac{a}{2} \leq x \leq \frac{a}{2}$$

RADIATION LOSSES COEFFICIENT - α_r

$$P_{\text{loss}} = \int_{X_r}^{\infty} E^2(x)dx = C_0 \frac{q}{2} \cos^2\left(\frac{ha}{2}\right) \exp\left[-\frac{2}{q}\left(X_r - \frac{a}{2}\right)\right]$$

and

$$P_{\text{total}} = \int_{-\infty}^{\infty} E^2(x)dx = C_0 \left[\frac{a}{2} + \frac{1}{2h}\sin(hx) + q\cos^2\left(\frac{ha}{2}\right)\right]$$

$$\alpha_R = \frac{P_{\text{loss}}}{P_{\text{total}}} \cdot \frac{1}{Z_c} = \frac{\frac{q}{2}\cos^2\left(\frac{ha}{2}\right) \exp\left(-\frac{2}{q} \overbrace{\frac{\beta_z - \beta_0}{\beta_0}}^{C_2} R\right) 2\lambda_1 \exp\left(\frac{a}{q}\right)}{\left[\frac{a}{2} + \frac{1}{2h}\sin(hx) + q\cos^2\left(\frac{ha}{2}\right)\right] a^2}$$

$$\boxed{\alpha_r = C_1 \cdot \exp(-C_2 R)}$$

RADIATION LOSSES COEFFICIENT - α_r

$$\boxed{\alpha_r = C_1 \cdot \exp(-C_2 R)} \tag{15}$$

where C_1 and C_2 are constants that depend on the dimensions of the waveguide and on the shape of the mode.

Case	Index of refraction		Width a [μm]	C_1 [dB/cm]	C_2 [cm^{-1}]	R For L =0.1dB/cm
	Waveguide	surrounding				
1	1.5	1.00	0.198	2.23×10^5	3.47×10^4	4.21 μm
2	1.5	1.485	1.04	9.03×10^3	1.46×10^2	0.78 μm
3	1.5	1.4985	1.18	4.69×10^2	0.814	10.4 cm

To conclude:

$$\boxed{\alpha = \alpha_{abs} + \alpha_{fc} + \alpha_r + \alpha_S} \tag{16}$$

SEMICONDUCTOR DETECTOR

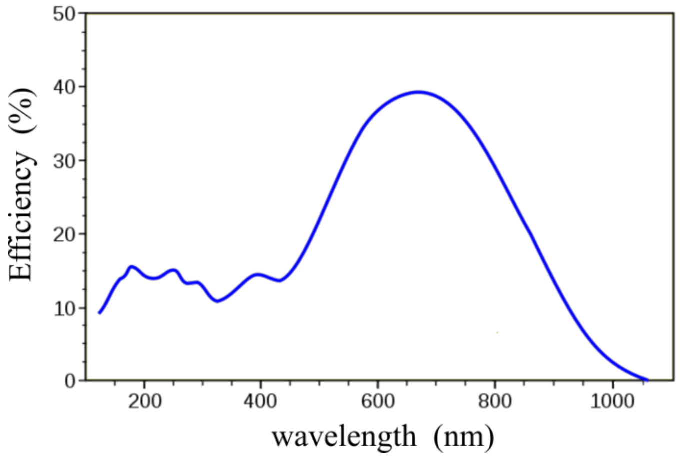
Types of detectors:

- Photoconductor.
- Photodiode - there are three types: $P^{+}N$, $N^{+}P$ and PIN.
- Schottky diode.
- Avalanche photodiode.
- Heterojunction photodiode.

RELEVANT CHARACTERISTICS IN A DETECTOR

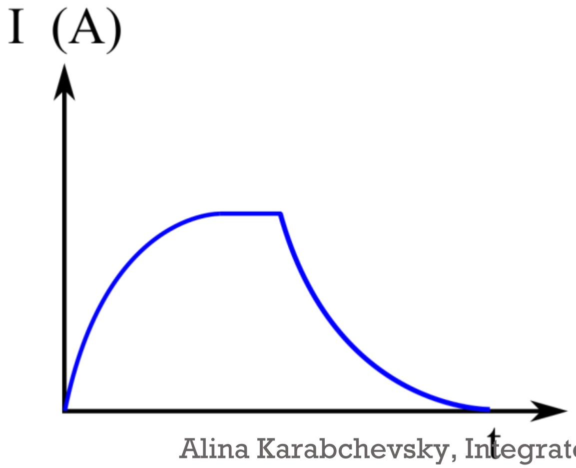
Spectral response and quantum efficiency

The output signal from the detector as a function of the radiation wavelength.



Velocity and response time

Velocity and response time of the detectors to radiation.



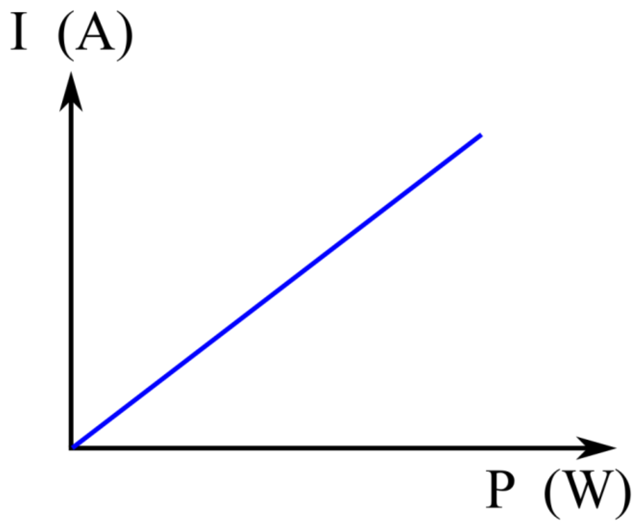
RELEVANT CHARACTERISTICS IN A DETECTOR

Noises (Thermal noise and Shot noise)

The noises are factors which limit the size and the quality of the signal.

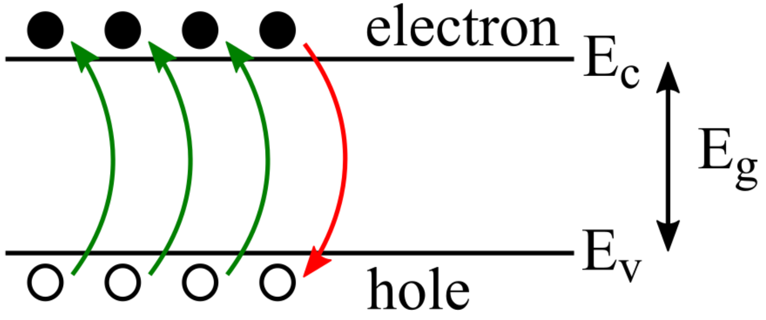
Biasing and linearity

Linearity in the output current as function of the input power, as shown in the graph.



SEMICONDUCTOR

The free carriers are electrons in the valence band and holes in the conduction band.



- Elevating electron from the valence band to the conduction band is called "**generation**".
- Descending of electron back to the valence band is called "**recombination**".
- The **recombination** is slower compared to the **generation** order of 10^{-6} – 10^{-9} seconds.

SEMICONDUCTOR

In equilibrium (darkness) state:

$$n_0 p_0 = n_i{}^2 \qquad (17)$$

where n_0 and p_0 are the concentration of electrons and holes at their bands, respectively, at equilibrium state.

In illumination state, there is an addition δ:

$$n_t = n_0 + \delta n \qquad\qquad p_t = p_0 + \delta p$$

$$n_t p_t \neq n_i{}^2 \qquad (18)$$

SEMICONDUCTOR

- The recombination rate is relative to the electron-hole-pairs (EHP):

$$R = \alpha_R np \left[\frac{\text{EHP}}{\text{cm}^{-3} \cdot \text{sec}}\right] \quad (19)$$

where α_R is the loss is due to the radiation.

- In equilibrium state, there is only **thermal generation** and its equal to the **recombination**.

$$G_{th} = R = \alpha_R n_0 p_0 \quad (20)$$

- In steady state, (not equilibrium state), we obtain:

$$G = G_{th} + G_{op} = R = \alpha_R np \quad (21)$$

LOW-LEVEL INJECTION

Assuming illumination we obtain small amount of free carriers:

$$\boxed{n_0 \text{ or } p_0 \gg \delta n \text{ or } \delta p}$$

Example:

Assuming silicon n-type with $n_0 = 10^{17}\ \text{cm}^{-3}$ and $p_0 = 2.25 \cdot 10^3\ \text{cm}^{-3}$.

If we illuminate it, we get an addition δ of electrons and holes of $\delta n = 10^{14}\ \text{cm}^{-3}$ and $\delta p = 10^{14}\ \text{cm}^{-3}$.

This is a **low-level injection** because the number of free carriers created is smaller compared to the electrons concentration but significant compared to the holes concentration.

LOW-LEVEL INJECTION

- From the previous equations we get at low-level injection:

$$G = G_{th} + \alpha_R n_0 \delta p + \alpha_R p_0 \delta n + \underbrace{\alpha_R \delta p \delta n}_{\text{negligible}} \qquad (22)$$

- The lifetime of the minority free carriers:

$$\tau_n = \frac{1}{\alpha_R n_0} \qquad \tau_p = \frac{1}{\alpha_R p_0}$$

where τ is the time when the minority free carrier moves from the conduction band back to the valence band, also known as **recombination**.

- The rate of optical generation is:

$$G_{op} = \frac{\delta n}{\tau_n} + \frac{\delta p}{\tau_p} \qquad (23)$$

LOW-LEVEL INJECTION

- For p-type, we are interested in the addition of electrons as free carriers.

$$G_{op} = \frac{\delta n}{\tau_n}$$

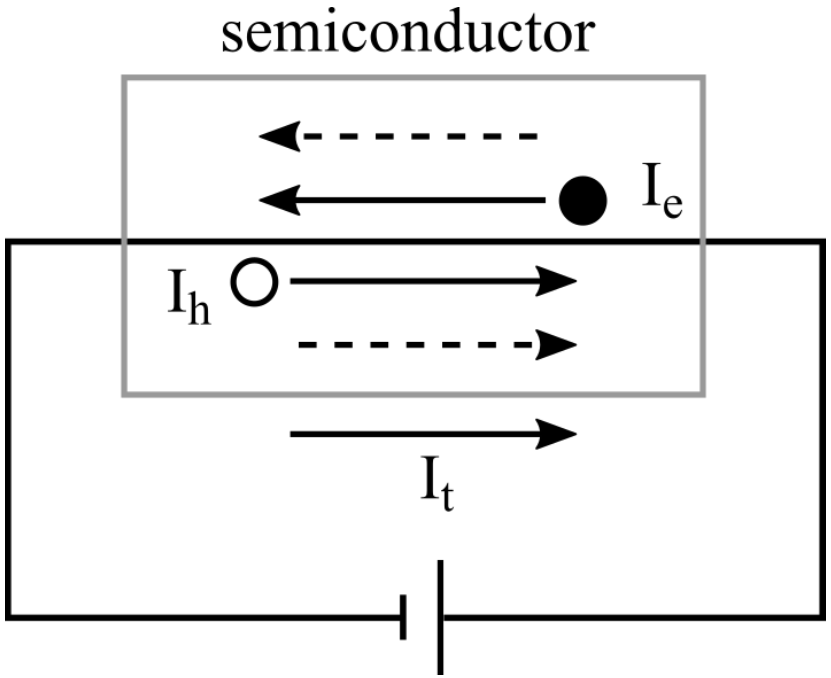
- For n-type, we are interested in the addition of holes as free carriers.

$$G_{op} = \frac{\delta p}{\tau_p}$$

- When illuminating material, we can find the change in the free carriers (δn and δp) since τ_n and τ_p are known for each material.

DRIFT CURRENT

- Given an electric circuit with a semiconductor, the drift direction of the electrons and the holes is:



DRIFT CURRENT

- The electrons and holes move in opposite directions however the total current in the circuit is in the original direction. The electric field accelerates the free carriers in the circuit.
- The current density in the circuit is: $J = J_e + J_h$

where J_e and J_h are the current density of electrons and holes, respectively.

- The total current is: $I = J \cdot A$

where A is the area where the current flows.

- In addition, J is defined as: $J = \sigma \cdot E$

where E is the electric field (units of [V/cm]) and σ is the electrical conductivity (units of [1/Ω·cm])

DRIFT CURRENT

- Therefore, J can be defined as:

$$\boxed{J = J_e + J_h = qnv_e + qpv_h}$$

- where v_e is the velocity of electron, v_h is the velocity of hole and q is the charge of electron.

- Substitute $v = \mu \cdot E$ and get:

$$J = qn\mu_n E + qp\mu_p E \tag{24}$$

where μ is the mobility for material (in [cm^2/V·sec])

- The total electrical conductivity is:

$$\sigma = q\left(n\mu_n + p\mu_p\right) \tag{25}$$

- The electrical conductivity increases as the electrons and holes concentration increases.

DETECTOR OF A WAVEGUIDE

- Detectors can be also monolithically integrated on the same planar platform. Evanescently-coupled germanium (Ge) waveguide photodetector on a SOI platform can be used for photodetector at the telecommunication windows.
- The substrate is locally implanted with boron to minimize both free carrier loss and parasitic capacitance between the substrate and contact metal pads in the detector region.
- The Si rib waveguide is then fabricated.
- A 1.3 μm thick film of Ge is grown by a selective epitaxial process.

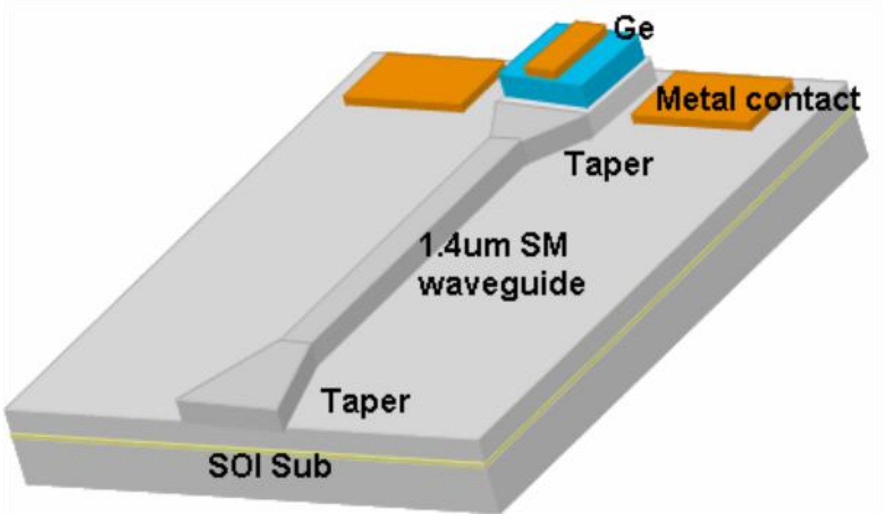


Figure 1: Schematic layout for the Ge-on-SOI waveguide detector [4].

DETECTOR OF A WAVEGUIDE

- The Ge is planarize using a chemical-mechanical polishing (CMP) step.
- Phosphorous is implanted into the top of the Ge film to form a vertical n-i-p junction, with the boron-implanted Si substrate serving as the anode.
- Boron is localized implant into the Si to improve the ohmic contact between the metal contacts and implanted Si substrate.

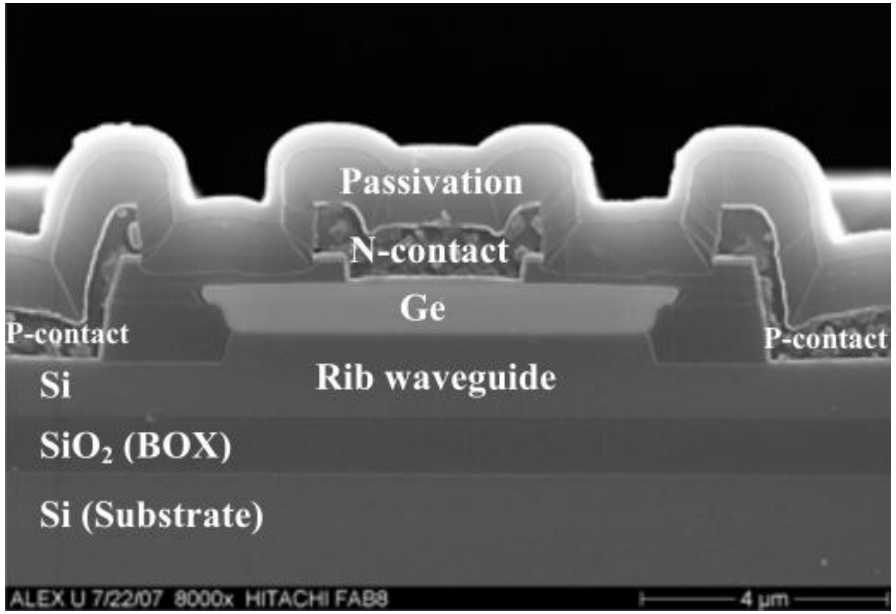


Figure 2: Cross-section SEM image of the Ge waveguide photodetector (7.4 µm x 50 µm) after processing [4].

BIBLIOGRAPHY

[1] N.K. Uzunoglu, J.G. Fikioris: J. Opt. Soc. Am. 72, 628 (1982)

[2] F. Lu, G. Fu, C. Jia, K. Wang, H. Ma, D. Shen: Lithium niobate channel waveguide at optical communication wavelength formed by multienergy implantation, Opt. Express 13, 9143 (2005)

[3] R. G. Hunsperger, Integrated Optics: Theory and Technology, Vol. 33, Springer-Verlag, 3rd ed., 1995, Chapter 6.2.2.

[4] Yin, Tao, et al. "40gb/s ge-on-soi waveguide photodetectors by selective ge growth." OFC/NFOEC 2008-2008 Conference on Optical Fiber Communication/National Fiber Optic Engineers Conference. IEEE, 2008.