LECTURE 5 - NONLINEAR EFFECTS IN OPTICAL FIBERS

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OUTLINE

Introduction

The Nonlinear Schrodinger Equation

- Derivation of the Nonlinear Schrodinger Equation
- Dispersion and Nonlinear Length

Nonlinear phase modulation

- Self-phase modulation
- Cross-phase modulation

Four-Wave Mixing (FWM)

Solitons

Scattering

- Stimulated Raman scattering
- Stimulated Brillouin scattering

INTRODUCTION

The response of any dielectric material to light such as silica glass of optical fibers, becomes nonlinear. Even though silica is intrinsically not a highly nonlinear material, the waveguide geometry that confines light to a small cross section over long fiber lengths makes nonlinear effects quite important in the design of modern lightwave systems. The nonlinear phenomena that are most relevant for fiber-optic communications are:

1 Nonlinear phase modulation

1.1 Self-phase modulation

1.2 Cross-phase modulation

2 Four-wave mixing

3 Solitons

4 Stimulated light scattering

4.1 Stimulated Brillouin scattering

4.2 Stimulated Raman scattering

THE NONLINEAR SCHRODINGER EQUATION

The nonlinear Schrodinger equation (NSE) is of particular importance in the description of nonlinear effects in optical fibers.

The nonlinear term of the NSE leads to a spectral broadening of the pulses which can interact with the dispersion in different ways.

The nonlinear wave equation is:

$$\nabla^2 \mathbf{E} - \frac{n^2}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\left(\chi^{(2)}EE + \chi^{(3)}EEE + \cdots\right) \tag{1}$$

where $\mathbf{P} = \varepsilon_0 \chi \mathbf{E}$

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E

Relation between P and E

$$P(r,t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi(r, t - t')E(r, t')\mathrm{d}t'$$

where χ is the linear susceptibility which is in general a second-rank tensor but a scalar for an isotropic medium such as silica glass.

Dimensionless proportionality constant, electric susceptibility χ, indicates the degree of polarization of a dielectric material in response to an applied electric field. The greater the electric susceptibility χ, the greater the ability of a material to polarize in response to the field, and thereby reduce the total electric field E inside the material (and store energy). It is in this way that the electric susceptibility χ influences the electric permittivity ε of the material.

WAVE EQUATION

From Lecture 3:

$$\nabla \times \nabla \times \vec{E} = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} - \mu_0 \frac{\partial^2 P}{\partial t^2} \qquad (2)$$

where $c = \sqrt{\mu_0 \varepsilon_0}$

$$\nabla \times \nabla \times \vec{E} = \nabla(\nabla \times \vec{E}) - \nabla^2 \vec{E} = -\nabla^2 \vec{E} \qquad (3)$$

THE NONLINEAR SCHRODINGER EQUATION

For the simplification, few assumptions will be made:

- The waveguide is a single mode.
- The material is perfectly transparent and the wavelength is far away from any material resonances.
- All scattering effects in the waveguide are neglected.
- The amplitude of the considered wave packet changes very slowly with respect to its carrier.
- The field strength of the applied field is, compared to the inner atomic field, relatively small.
- The fields are linearly polarized in the same direction and the polarization status remains the same during the propagation.
- The nonlinearity has no influence on the field components perpendicular to the propagation direction.

THE NONLINEAR SCHRODINGER EQUATION

Under this simplifications, the quasi-monochromatic propagating wave is (c.c. is complex conjugate) propagating in z direction:

$$E = \frac{1}{2}\left[\hat{E}(r)e^{j(kz-\omega t)} + c.c.\right] \tag{4}$$

The slowly varying envelope, $E(r)$, consists of two parts: one in the propagation direction (longitudinal, $A(z)$) and perpendicular to the propagation direction (transverse, $F(r,\varphi)$) as $\hat{E}(r) = F(r,\varphi)A(z)$

In fiber, the intensity decreasing quadratically with an increase of the radius r and defined as: $I = \frac{1}{2}\varepsilon_0 cn\left|\hat{E}\right|^2$ and the power is

$$P = \iint I \cdot r \, \mathrm{d}r\mathrm{d}\varphi \tag{5}$$

THE NONLINEAR SCHRODINGER EQUATION

The nonlinear Schrodinger equation (NSE) can be represented as:

$$j\frac{\partial B}{\partial z} = -\gamma|B|^2 B + \frac{k_2}{2}\frac{\partial^2 B}{\partial T^2} \tag{6}$$

The first term on the right side describes the influence of the nonlinearity on the pulse, whereas the second term can be addressed to the linear effect of dispersion in the waveguide.

In case that the absorption in the material is included

$$j\frac{\partial B}{\partial z} + j\frac{\alpha}{2}B = -\gamma|B|^2 B + \frac{k_2}{2}\frac{\partial^2 B}{\partial T^2} \tag{7}$$

where B is the amplitude envelope, T is a moving frame and α is the attenuation constant.

DISPERSION AND NONLINEAR LENGTH

Assume the time scale is normalized to the pulse width ($\tau = T/\tau_0$). The normalized amplitude (A) is defined as:

$$B(z,\tau) = \sqrt{P}A(z,\tau) \tag{8}$$

The NSE becomes

$$j\frac{\partial A}{\partial z} = -\gamma P|A|^2 A + \mathrm{sgn}(k_2)\frac{|k_2|}{2{\tau_0}^2}\frac{\partial^2 A}{\partial \tau^2} \tag{9}$$

where $\mathrm{sgn} = \pm 1$ describes the sign of the dispersion parameter k_2.

The dispersion length can be written as

$$L_D = \frac{{\tau_0}^2}{|k_2|} = \frac{{\tau_0}^2 2\pi c}{|D|\lambda^2} \tag{10}$$

where D is the dispersion parameter.

DISPERSION AND NONLINEAR LENGTH

The nonlinear length is

$$L_{\mathrm{NL}} = \frac{1}{\gamma P} \tag{11}$$

The NSE becomes

$$j\frac{\partial A}{\partial z} = -\frac{1}{L_{\mathrm{NL}}}|A|^2 A + \frac{\mathrm{sgn}(k_2)}{2L_D}\frac{\partial^2 A}{\partial \tau^2} \tag{12}$$

DISPERSION AND NONLINEAR LENGTH

- For a large Nonlinear Length – the fiber behaves linearly.
- It happens when the nonlinearity or the power is small.
- For a large dispersion length – the broadening of the pulses can be neglected.
- It happens when the initial pulse is large or the dispersion is small.

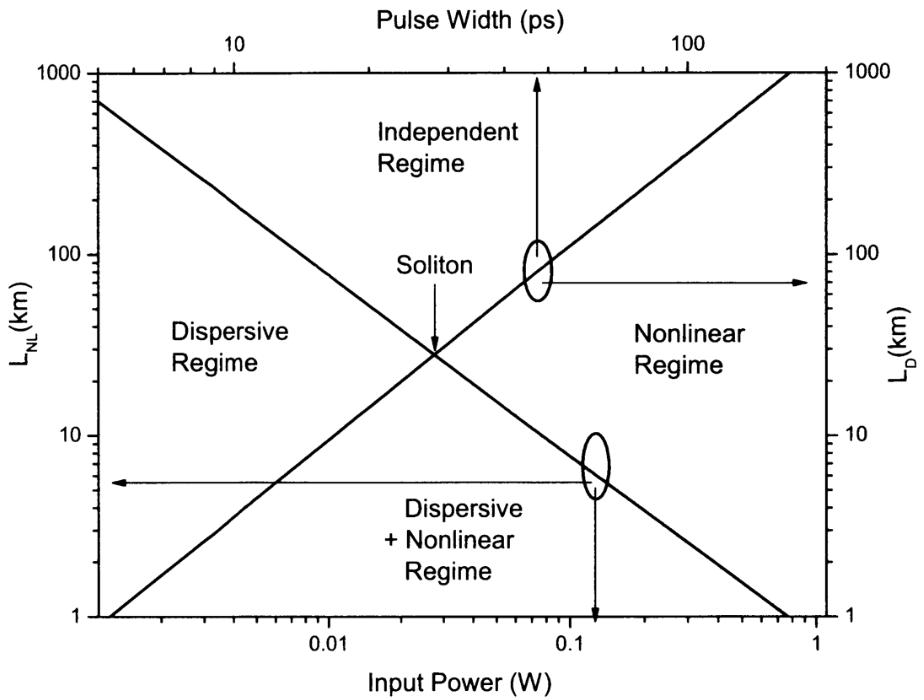


Figure 1: Nonlinear and dispersion length for a standard single mode fiber at a wavelength of 1.55 μm [1].

FIBER NONLINEARITIES

The response of any dielectric to light becomes nonlinear for intense electromagnetic fields, and optical fibers are no exception. On a fundamental level, the origin of nonlinear response is related to the anharmonic motion of bound electrons under the influence of an applied field. As a result, the total polarization P induced by electric dipoles is not linear in the electric field E, but satisfies the more general relation

$$\mathbf{P} = \varepsilon_0\left(\chi^{(1)}\mathbf{E} + \chi^{(2)}\mathbf{EE} + \chi^{(3)}\mathbf{EEE} + \cdots\right) = \mathbf{P}_{\mathrm{L}} + \mathbf{P}_{\mathrm{NL}}^{(2)} + \mathbf{P}_{\mathrm{NL}}^{(2)} + \cdots$$

where ε_0 is the vacuum permittivity and $\chi^{(j)}$ is j^{th} order susceptibility.

- As SiO_2 is a symmetric molecule, $\chi^{(2)}$ vanishes for silica glasses.
- The electric-quadrupole and magnetic-dipole moments can generate weak second-order nonlinear effects.

FIBER NONLINEARITIES

The lowest-order nonlinear effects in optical fibers originate from the third-order susceptibility $\chi^{(3)}$, which is responsible for phenomena such as third-harmonic generation, four-wave mixing, and nonlinear refraction. Most of the nonlinear effects in optical fibers therefore originate from nonlinear refraction, a phenomenon referring to the intensity dependence of the refractive index.

$$\tilde{n} = n_0 + n_2 I = n_0 + \bar{n}_2 |E|^2 \qquad (13)$$

where $\bar{n}_2$ is the nonlinear-index coefficient defined as

$$\bar{n}_2 = \frac{3}{8n} \Re\{\chi^{(3)}_{xxxx}\}$$

SELF- AND CROSS-PHASE MODULATION

Self-phase modulation (SPM) and cross-phase modulation (XPM or CPM) are two of the most important nonlinear effects in optical telecommunications. Both effects lead to a phase alteration of the pulses and are frequently called carrier-induced phase modulation (CIP). The alteration of the phase leads to a change of the pulse spectrum.

- In the case of SPM, the pulse changes its spectrum due to its own intensity; it will be broadened. The broadening causes, of course, a degradation of system performance
- The XPM is similar to the SPM but the origin of the spectral broadening of the pulses are other pulses propagating at the same time in the waveguide; they will mutually influence each other.

SELF-PHASE MODULATION

In the case of SPM, the pulse changes its spectrum due to its own intensity; it will be broadened. Together with the dispersion of the material, this spectral broadening can lead to an alteration of the temporal width of the pulse.

$$\frac{\partial B}{\partial z} = j\gamma PB \tag{14}$$

The envelope of the wave (B) after distance z is:

$$B(z) = B(0)\exp(j\gamma Pz) \tag{15}$$

Due to Eq. (15), the slowly varying amplitude $B(z)$ changes its phase during the propagation in the waveguide and the phase alteration is proportional to the propagation distance z. The total transmission in the fiber:

$$E(z,t) = \frac{1}{2}\left[\hat{E}(r,0)e^{j([\gamma P+k]z-\omega t)} + c.c.\right] \tag{16}$$

SELF-PHASE MODULATION

Using $I = P/A_{\text{eff}}$ and $\gamma = k_0 n_2/A_{\text{eff}}$, the phase of the wave can be represented as:

$$\Phi(z,t) = (n_0 + n_2 I)k_0 z - \omega t \qquad (17)$$

The pulse modulates, due to the intensity-dependent refractive index, its own phase with its intensity. Hence, the name self-phase modulation is used to describe this phenomenon.

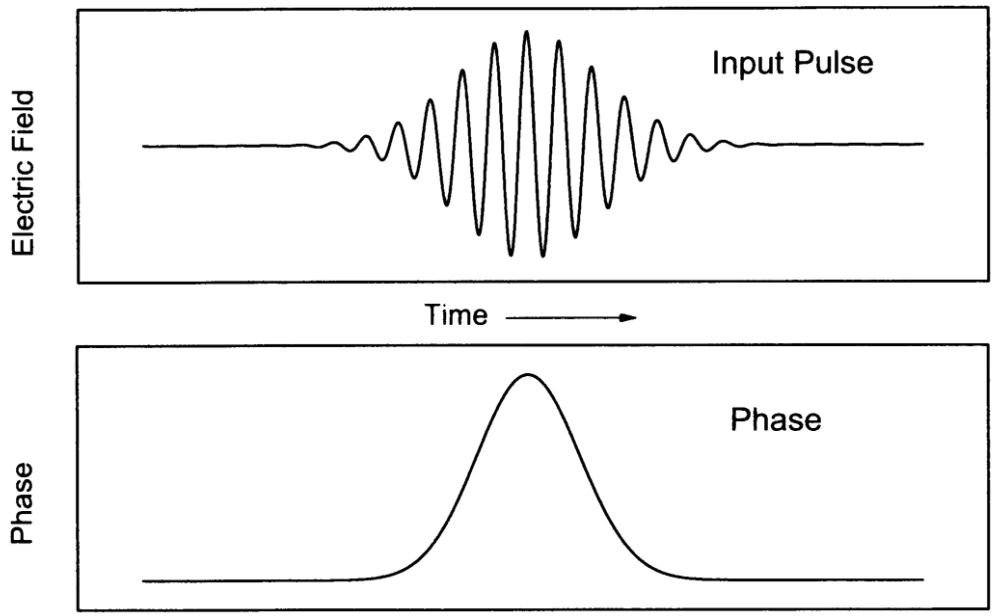


Figure 2: Electric field of a Gaussian pulse at the fiber input (top) and phase shift due to SPM (bottom) [1].

SELF-PHASE MODULATION

The temporal derivation of the phase corresponds to the frequency of the wave. Hence, the frequency is:

$$\omega(z) = -\frac{\partial \Phi(z,t)}{\partial t} = \omega_0 - n_2 k_0 \frac{\partial I}{\partial t} z \tag{18}$$

Therefore, it will decrease if the pulse intensity increases, reach the original frequency in the pulse maximum, and will increase for the decreasing wing.

SELF-PHASE MODULATION

Due to SPM, the leading edge is stretched. The peak maintains its original frequency and the tailing edge is compressed.

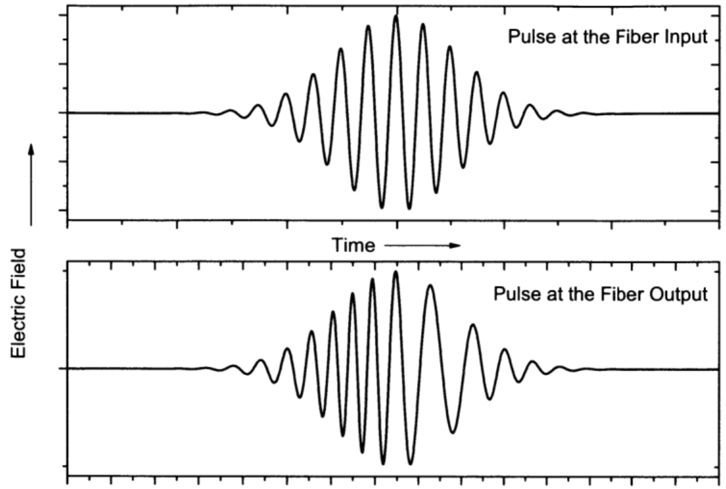


Figure 3: Electric field of a pulse at the fiber input and output. Due to the SPM in the fiber the frequency of the pulse is changed. Note that the pulse here propagates from the left to the right [1].

SPM'S IMPACT ON COMMUNICATION SYSTEMS

The nonlinear part of the phase from Eq. (17) is

$$\Phi_{\mathrm{NL}} = n_2 I k_0 z \tag{19}$$

using $\gamma = k_0 n_2 / A_{\mathrm{eff}}$,we get the phase change as function of the length. Considering the influence of attenuation

$$\Phi_{\mathrm{NL}} = \gamma P L_{\mathrm{eff}} \tag{20}$$

where $L_{\mathrm{eff}} = \frac{1-\exp(-\alpha z)}{\alpha}$ and α is units of km^{-1}.

SPM'S IMPACT ON COMMUNICATION SYSTEMS

- For an input power of 1 W, one yields a phase shift of $\pi/2$ after a distance of ~1.2 km in a standard single mode fiber (nonlinearity coefficient $\gamma = 1.3\ \mathrm{W^{-1}km^{-1}}$, attenuation $\alpha = 0.2\ \mathrm{dB/km}$).
- For the same phase shift, an effective length of 1200 km is required in systems with an input power of 1 mW.
- This is much more than the limiting value of the effective length in single mode fibers (~22 km).
- If no optical amplifiers are present in the system it follows a phase shift of at most ~0.03 rad for a signal power of 1 mW.

SPM'S IMPACT ON COMMUNICATION SYSTEMS

- If optical amplifiers are present in the system, the relations are completely different.
- The effective length depends on the number of amplifiers.
- If the system is long enough small input powers can cause a strong phase shift.

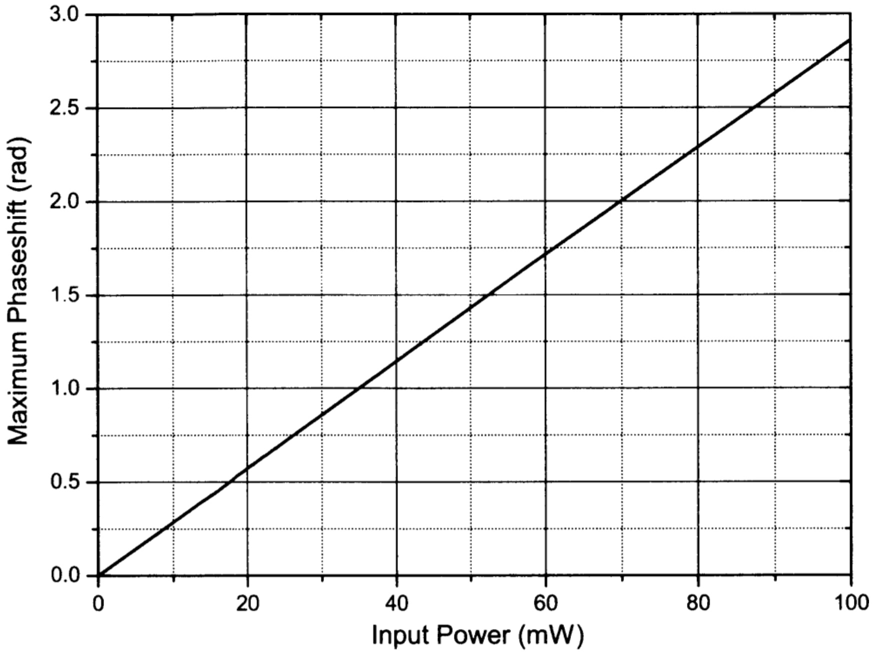


Figure 4: Maximum phase shift for an unamplified system versus input power. (Attenuation constant $\alpha = 0.2\ \mathrm{dB/km}$, nonlinearity coefficient $\gamma = 1.3\ \mathrm{W^{-1}km^{-1}}$ [1].

SPM'S IMPACT ON COMMUNICATION SYSTEMS

An alteration of the phase causes an alteration of the pulse frequency as well. The spectral broadening of the pulse at the fiber output is

$$\Delta B = \frac{\partial \Phi_{\mathrm{NL}}}{\partial t} = \gamma L_{\mathrm{eff}} \frac{\partial P}{\partial t} \quad (21)$$

The spectral broadening depends not only on the effective length but on the temporal alteration of the power of the input pulse as well. Short pulses are much more affected than long pulses due to SPM.

SPM'S IMPACT ON COMMUNICATION SYSTEMS

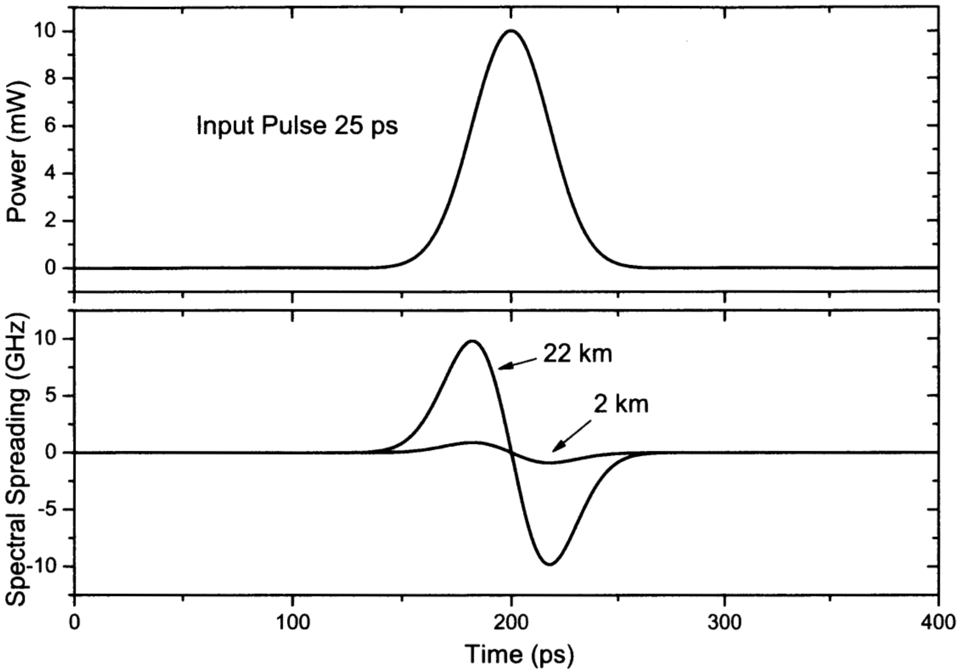


Figure 5: Spectral broadening of a pulse with temporal durations of 25 ps after a propagation distance of 2 and 22 km (effective length) in a standard single mode fiber [1].

SPM'S IMPACT ON COMMUNICATION SYSTEMS

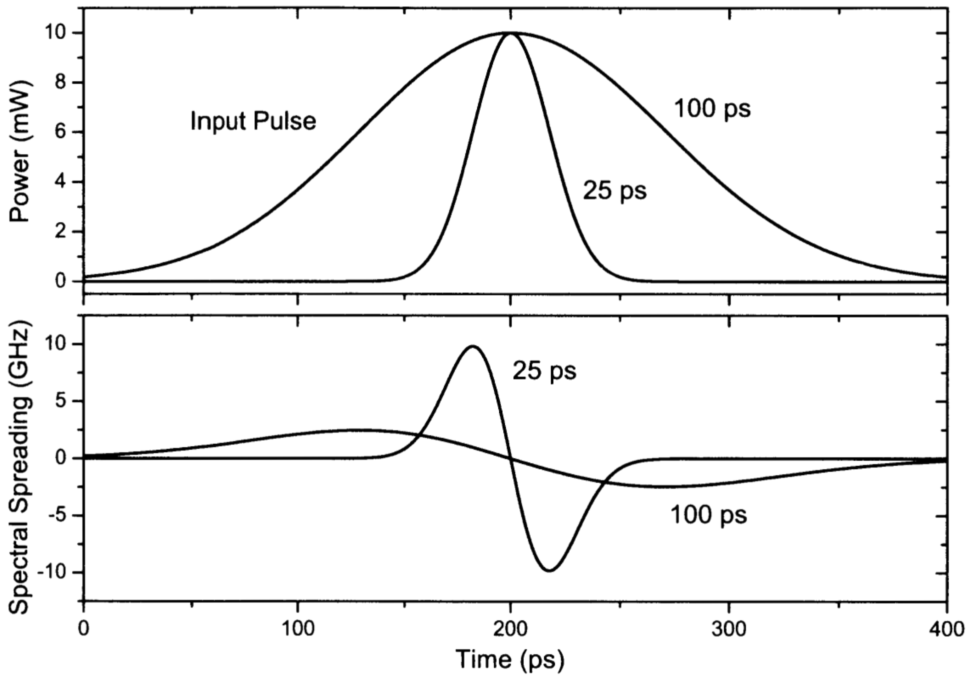


Figure 6: Spectral broadening of two pulses with temporal durations of 25 and 100 ps. The propagation distance in a standard single mode fiber is 22 km (effective length) [1].

CROSS-PHASE MODULATION

Cross-phase modulation (XPM, or sometimes, CPM) is similar to SPM, but contrary to SPM, different channels can interact with each other via the alteration of the intensity dependent refractive index.

$$\frac{\partial B}{\partial z} = j\gamma|B|^2 B \qquad (22)$$

For simplicity, only two channels propagating at the same time in the fiber will be considered. The entire field consists of two parts

$$B = B_1 \exp[j(k_1 z - \omega_1 t)] - B_2 \exp[j(k_2 z - \omega_2 t)] \qquad (23)$$

therefore

$$j\gamma|B|^2 B = j\gamma B B^* B \qquad (24)$$

where B^* is the complex conjugate of B.

CROSS-PHASE MODULATION

These terms can be separated by the arguments of the exponential functions:

$$\frac{\partial B_m}{\partial z} = j\gamma\left[(|B_m|^2 + 2|B_n|^2)B_m + {B_m}^2 {B_n}^* e^{j(\Delta kz - \Delta\omega t)}\right] \quad (25)$$

where $\gamma = \frac{\overline{\omega} n_2}{cA_{\text{eff}}}$ is the nonlinearity constant and $\overline{\omega}$ is the mean frequency.

Assuming that the fiber shows dispersion and therefore, the phase matching condition is not fulfilled and the generation of mixing frequencies is suppressed.

$$\frac{\partial B_m}{\partial z} = j\gamma[(|B_m|^2 + 2|B_n|^2)B_m] \quad (26)$$

Assuming that the brackets is constant for a distance z and $m = 1$.

$$B_1(z) = B_1(z)\exp[j\gamma(P_1 + 2P_2)z] \quad (27)$$

CROSS-PHASE MODULATION

Hence, the phase of the first wave is altered due to its own power - this is the SPM effect - but at the same time the power of the other wave has an influence on the phase of wave 1 as well. The influence is twice that of SPM.

The total refractive index in the fiber is:

$$n_{\mathrm{ges}} = n_0 + n_2 I_1 + 2n_2 I_2 \tag{28}$$

The second wave alters the refractive index experienced by the other wave via its intensity and hence, changes the phase of the other wave. The phase of the wave is:

$$\Phi = \left[\frac{n_2}{A_{\mathrm{eff}}}(P_1 + 2P_2) + n_0\right] k_{01} z - \omega_1 t \tag{29}$$

CROSS-PHASE MODULATION

The nonlinear part caused by XPM can be written as:

$$\Phi_{1\mathrm{XPM}} = 2\frac{n_2}{A_{\mathrm{eff}}}P_2 k_{01} z = 2\gamma P_2 z \qquad (30)$$

The frequency change at the output of a fiber with length L is:

$$\Delta B_{\mathrm{XPM}} = 2\gamma L\frac{\partial P_2}{\partial t} \qquad (31)$$

When the attenuation is considered $L = L_{\mathrm{eff}}$. The XPM spectral broadening is twice compared to SPM spectral broadening (compare to Eq. (21)).

XPM AND SPM EFFECTS

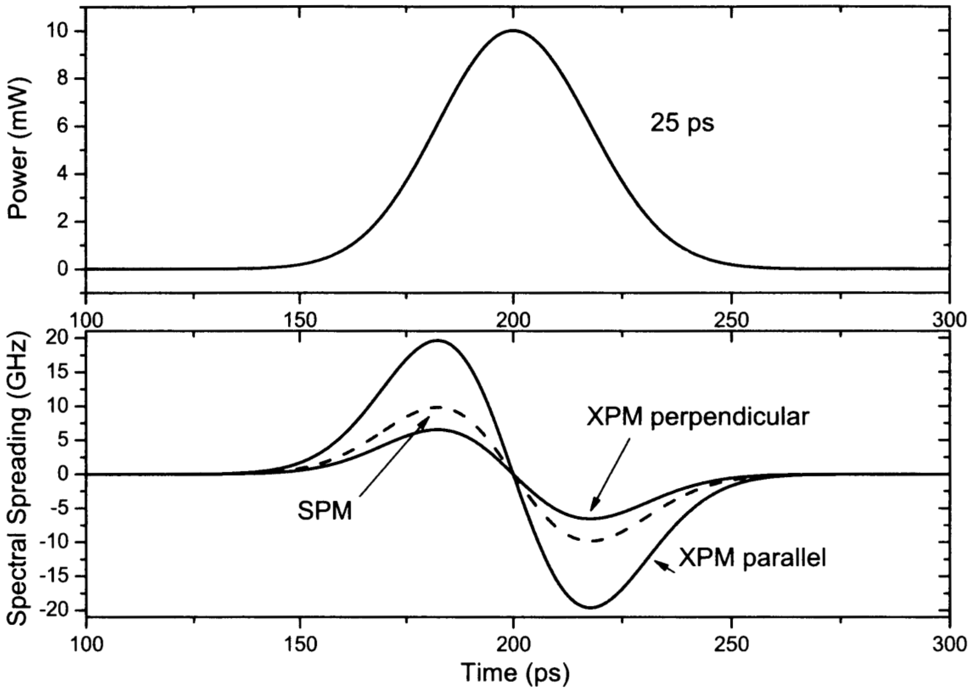


Figure 7: Spectral broadening of a pulse due to SPM and XPM in a standard single mode fiber after a propagation distance of 22 km (effective length). Both effects are considered independently [1].

SPM AND XPM

A common method for studying the impact of SPM and XPM uses a numerical approach. The equation

$$\frac{\partial B}{\partial z'} + \frac{j\beta_2}{2}\frac{\partial^2 B}{\partial t'^2} + \frac{\beta_3}{6}\frac{\partial^3 B}{\partial t'^3} = 0 \quad (32)$$

(β_2 is the GVD coefficient and β_3 is related to the dispersion slope S) can be generalized to include the SPM and XPM effects by adding a nonlinear term.

The resulting equation is known as the nonlinear Schrodinger equation and has the form:

$$\frac{\partial B}{\partial z} + \frac{j\beta_2}{2}\frac{\partial^2 B}{\partial t^2} = -\frac{\alpha}{2}B + j\gamma|B|^2 B \quad (33)$$

where we neglected the third-order dispersion and added the term containing to account for fiber losses.

XPM'S IMPACT ON COMMUNICATION SYSTEMS

- On the other hand, modern WDM systems consist of a huge number of channels at different carrier wavelengths.
- Each wavelength delivers a contribution to the nonlinear refractive index that acts back on the distinct channels.
- Hence, all channels can influence each other via their intensities.

The equations for M channels are

$$\begin{aligned} \frac{\partial B_1}{\partial z} &= j\gamma\left(|B_1|^2 + 2\sum_{i=2}^{M}|B_i|^2\right)B_1 \\ &\vdots \\ \frac{\partial B_M}{\partial z} &= j\gamma\left(|B_M|^2 + 2\sum_{i=1}^{M-1}|B_i|^2\right)B_M \end{aligned} \tag{34}$$

XPM'S IMPACT ON COMMUNICATION SYSTEMS

If all channels coincide temporally, the total refractive index in the first channel of a WDM system with M channels experiences is

$$n_{\text{ges}} = n_0 + n_2 I_1 + 2\sum_{i=2}^{M} n_2 I_i \tag{35}$$

and the phase of wave 1 is

$$\Phi_1 = \left[\frac{n_2}{A_{\text{eff}}}\left(P_1 + 2\sum_{i=2}^{M} P_i\right) + n_0\right] k_0 z - \omega t \tag{36}$$

- The influence of XPM is much more important than the influence of SPM in WDM systems.
- However, XPM is the most important factor that determines the transmission capacity of optical fibers.

XPM'S IMPACT ON COMMUNICATION SYSTEMS

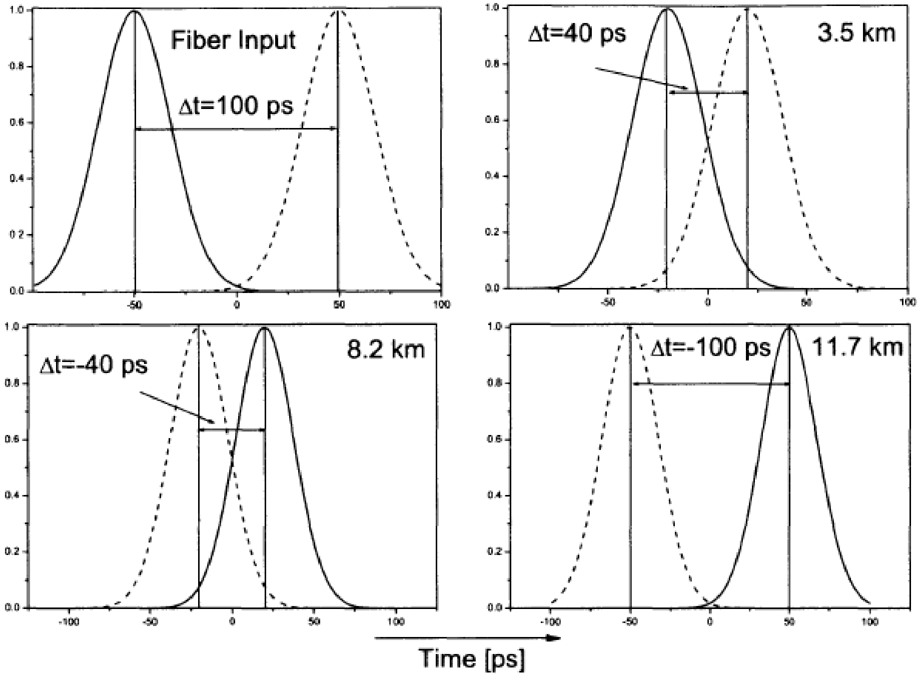
- The influence of XPM should be considered in phase-modulated systems in particular because an arbitrary phase alteration leads directly to a deterioration of the signal to noise ratio.
- If the systems are intensity modulated, the XPM has no influence on the system performance if the dispersion is neglected because only a phase modulation takes place.

XPM'S IMPACT ON COMMUNICATION SYSTEMS

New
Need to check

The red pulse was injected into the fiber 100 ps before the blue one. For convenience, the influence of dispersion and nonlinearity to the pulse width should be neglected.

In a standard single mode fiber, the propagation of pulses with a carrier wavelength above 1.3 μm is determined by the anomalous dispersion. The GVD parameter, and therefore the group velocity, decreases with increasing wavelength. Hence, the blue pulse injected into the fiber after the red one moves faster. After a distance of 3.5 km, both pulses overlap halfway and overlap completely after 5.9 km. If the pulses propagate further, they walk apart and after a distance of 12 km, they are again completely separated. But now the blue pulse is in front of the red one.



Gaussian pulses with a temporal width of 41.6 ps (FWHM) and a carrier wavelength of 1.577 and 1.578 μm for different propagation distances in a standard single mode fiber $D = 17\ ps/(nm \cdot km)$ [1]

XPM'S IMPACT ON COMMUNICATION SYSTEMS

If the interaction length between the pulses is defined as the distance between the beginning and the end of the overlap at half the maximum power, then the time for the interaction is $2\tau_{\mathrm{FWHM}}$, with τ_{FWHM} as the FWHM pulse width. The interaction length is

$$L_{\mathrm{I}} = \frac{2\tau_{\mathrm{FWHM}}}{D\Delta\lambda}$$

where D is the dispersion parameter and $\Delta\lambda$ is the difference between the carrier wavelengths of the pulses.

For example, two adjacent pulses of the C-band (channel spacing 50 GHz, $\lambda_1 = 1544.92\ nm$, $\lambda_2 = 1544.53\ nm$) with a FWHM width of 25 ps:

- A standard single mode fiber

$$D = 17\ \frac{ps}{nm \cdot km} \quad \rightarrow \quad L_{\mathrm{I}} = 7.54\ km$$

- A nonzero dispersion-shifted fiber

$$D = 2\ \frac{ps}{nm \cdot km} \quad \rightarrow \quad L_I = 64.1\ km$$

FOUR-WAVE MIXING (FWM)

Four-wave-mixing (FWM), or sometimes four-photon-mixing (FPM), describes a nonlinear optical effect at which four waves or photons interact with each other due to the third order nonlinearity of the material.

As a result, new waves with sum and difference frequencies are generated during the propagation in the waveguide.

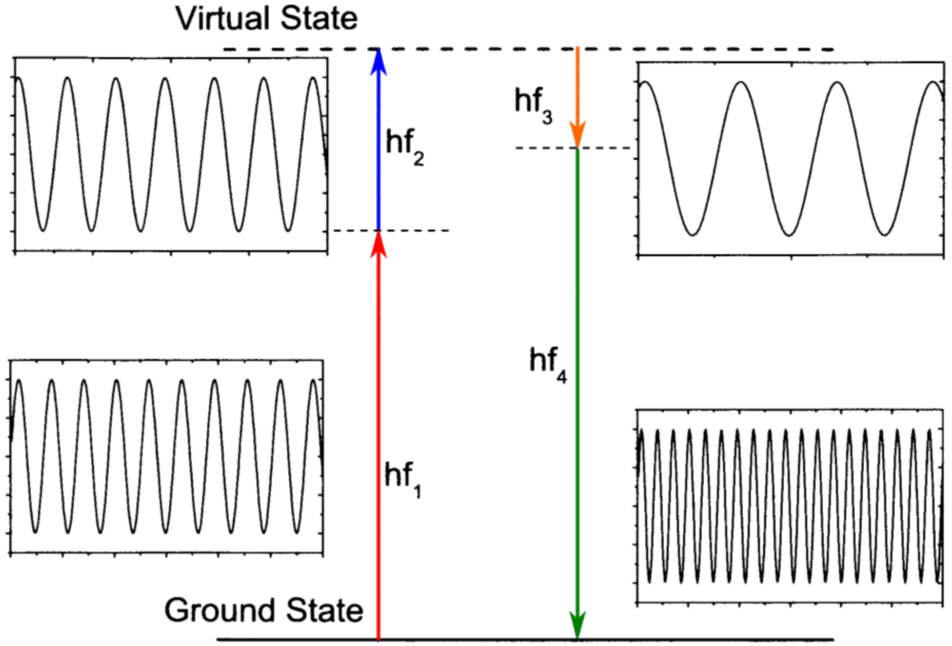


Figure 8: Energy diagram of Four-Wave Mixing (FWM) [1].

FOUR-WAVE MIXING (FWM)

If three optical waves with frequencies f_i, f_j and f_k are propagating in the fiber, they can interact via the third order susceptibility $\chi^{(3)}$ of the material and new waves with the frequencies

$$f_{i,j,k} = f_i + f_j - f_k \qquad (37)$$

can be generated by the FWM process, where i, j and k can have the values 1, 2, and 3.

- Three elements arranged in three classes can lead to 27 possible variations.
- If the third frequency (f_k) in equals the first, or second (f_i, f_j) frequency no new frequency is generated, resulting in f_i or f_j , respectively.
- If the first two frequencies are changing their places as well, no new frequencies are generated.
- If $k \neq i, j, k$, 9 residual combinations are left from the 27 variations.

Therefore, three waves with different frequencies are able to generate 9 new waves.

FOUR-WAVE MIXING (FWM)

The table below shows the possible combinations for the generation of new waves due to the FWM process for channels with equidistant wavelength (left column) and frequency spacing (right column).

i, j, k	$\lambda_{i,j,k}$/nm ($\Delta\lambda = 1$ nm)	$f_{i,j,k}$/THz ($\Delta f = 120$ GHz)
1,1,2	1557.001	189.99
1,1,3	1576.005	189.87
1,2,3	1577.002	189.99
1,3,2	1578.999	190.23
2,2,1	1580.001	190.35
2,2,3	1578.001	190.11
2,3,1	1581.003	190.47
3,3,1	1582.005	190.59
3,3,2	1581.001	190.47

FOUR-WAVE MIXING (FWM)

The frequency spacing depends on the wavelength. Therefore, if the channels have an equidistant wavelength spacing, their frequency differences are not completely equal.

If the original channels have an equal frequency spacing, the new generated waves will have the same spacing and a large number of them falls exactly into the original channels.

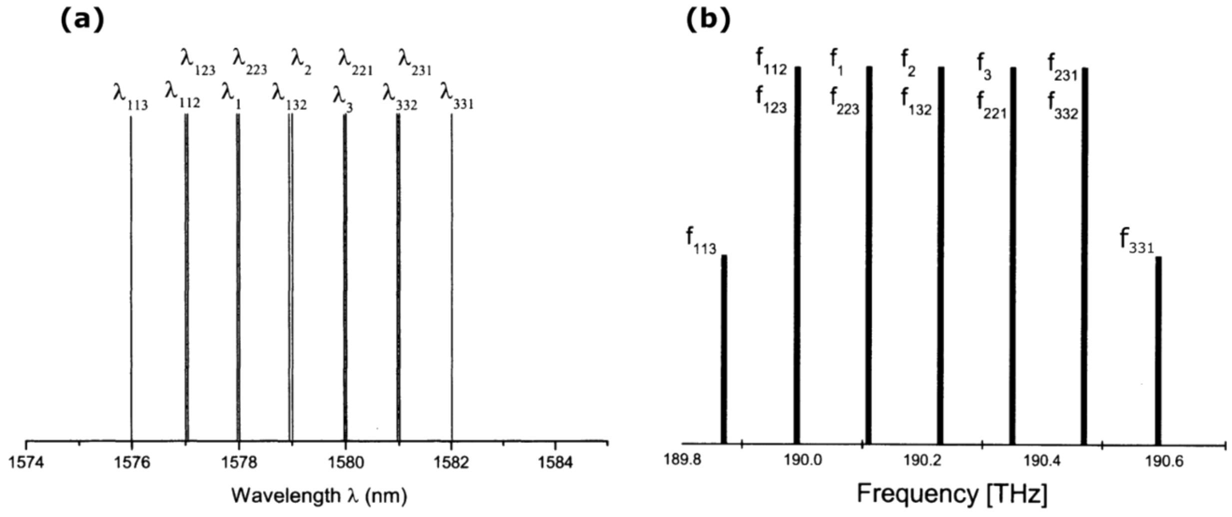


Figure 9: (a) Mixing frequencies due to FWM for an equal frequency spacing between the original channels. (b) Mixing frequencies due to FWM for equidistant wavelength spacing [1].

FOUR-WAVE MIXING (FWM)

For N original channels, the number of possible mixing products is

$$M = \frac{N^3 - N^2}{2} \qquad (38)$$

If the intensities of the new generated waves are strong enough, they can interact again with each other or with the original channels and will produce additional mixing products.

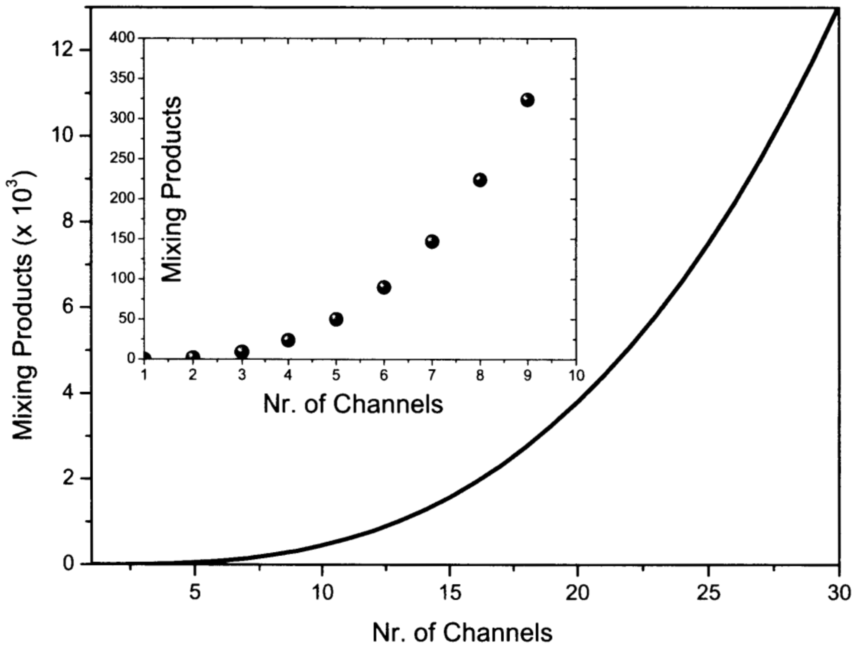


Figure 10: Number of possible mixing products due to FWM versus the number of channels [1].

FOUR-WAVE MIXING (FWM)

- The effectiveness of the generation of new mixing products depends strongly on the phase matching condition between the distinct waves. In a dispersive material, a larger frequency spacing results in a higher refractive index difference, and therefore, a higher phase mismatch between the channels. Hence, the FWM efficiency decreases for channels that are farther away.
- As a result, only adjacent channels generate mixing products effectively in dispersive fibers, leading to a decrease of the signal to noise ratio in these channels. The intensity of the mixing products between channels farther away is negligibly small.

MATHEMATICAL DESCRIPTION OF FWM

If the attenuation in the fiber is neglected as well, the NSE becomes

$$j\frac{\partial B}{\partial z} = -\gamma|B|^2B \tag{39}$$

The superposition propagating through the fiber is represented by

$$B = B_1e^{j(k_1z-\omega_1t)} + B_2e^{j(k_2z-\omega_2t)} + B_3e^{j(k_3z-\omega_3t)} + B_4e^{j(k_4z-\omega_4t)} \tag{40}$$

For convenience, the consideration is restricted to the case

$$\omega_4 = \omega_1 + \omega_2 - \omega_3 \tag{41}$$

where ω_1 and ω_2 are called the pump waves whereas ω_3 and ω_4 are called the idler waves.

MATHEMATICAL DESCRIPTION OF FWM

In analogy to stimulated Raman scattering (SRS), the lower sideband can be seen as the Stokes and the higher generated sideband as the anti-Stokes wave.

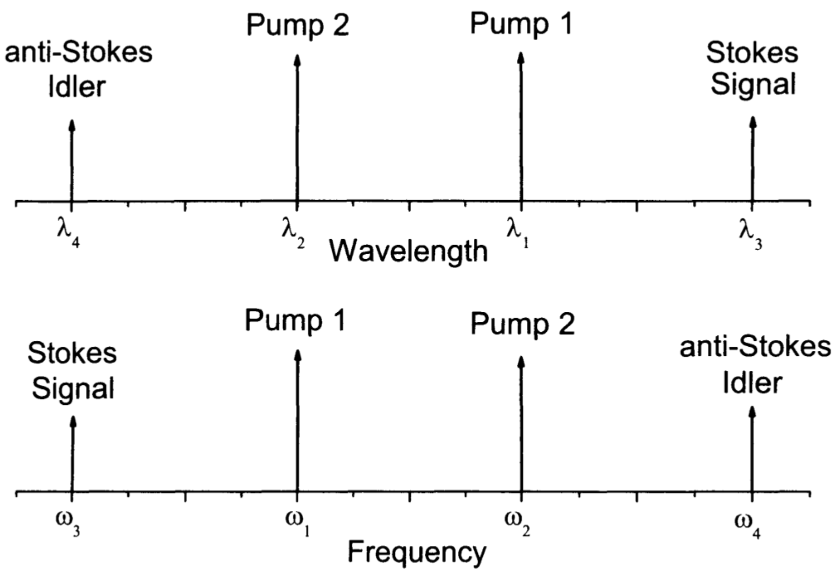


Figure 11: Position and notation of the distinct frequencies for $\omega_4 = \omega_1 + \omega_2 - \omega_3$ [1].

MATHEMATICAL DESCRIPTION OF FWM

There are two special cases:

1) If all three original waves have the same frequency ($\omega_1 = \omega_2 = \omega_3$), the process is called a degenerate FWM where the new generated mixing product has the same frequency ω. It can be distinguished by its propagation direction.

2) If only two of the three waves are equal ($\omega_1 = \omega_2 \neq \omega_3$), the process is called partly degenerate. The frequency of the mixing product is lower than the pump frequencies by $\Delta\omega$.

The second case is important for applications like the FWM wavelength converter and the parametric amplifier.

MATHEMATICAL DESCRIPTION OF FWM

A large number of terms can be obtained:

- $|B_i|^2 B_i$ with $i = 1,2,3,4$ - responsible for the self phase modulation of the waves.
- $|B_i|^2 B_j$ with $i \neq j$ - responsible for the cross-phase modulation of the waves.

All other terms are responsible for FWM but an effective generation of new waves can only happen if the phase matching condition is fulfilled.

The distinct FWM term

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 \quad (42)$$

where $\omega_i = 2\pi f_i$

MATHEMATICAL DESCRIPTION OF FWM

f_1 can be obtain from few combinations: $2f_3 + f_4 - f_2$, $2f_2 - f_4$ and $f_2 + f_3 - f_1$. NSE for the first pump wave (f_1) in the fiber:

$$\frac{\partial B_1}{\partial z} = j\gamma\left[\left(|B_1|^2 + 2\sum_{i\neq 1}|B_1|^2\right)B_1 + \text{FWM}\right] \qquad (43)$$

where $i = 1,2,3,4$ and

$$\text{FWM} = 2B_3B_4B_2{}^*e^{j\Delta k_{3,4,-2,-1}z} + 2B_2B_3B_1{}^*e^{j\Delta k_{2,3,-1,-1}z} + 2B_2{}^2B_4{}^*e^{j\Delta k_{2,2,-4,-1}z} \qquad (44)$$

The distinct amplitudes are normalized to the input power of the pump wave

P_1: $B_m = \sqrt{P_1} \cdot A_m$

- The first term inside the brackets responsible for SPM.
- The second term describes the effect of XPM.

MATHEMATICAL DESCRIPTION OF FWM

Figure shows the conditions for the four waves in an optical fiber if the phases between the distinct waves are matched.

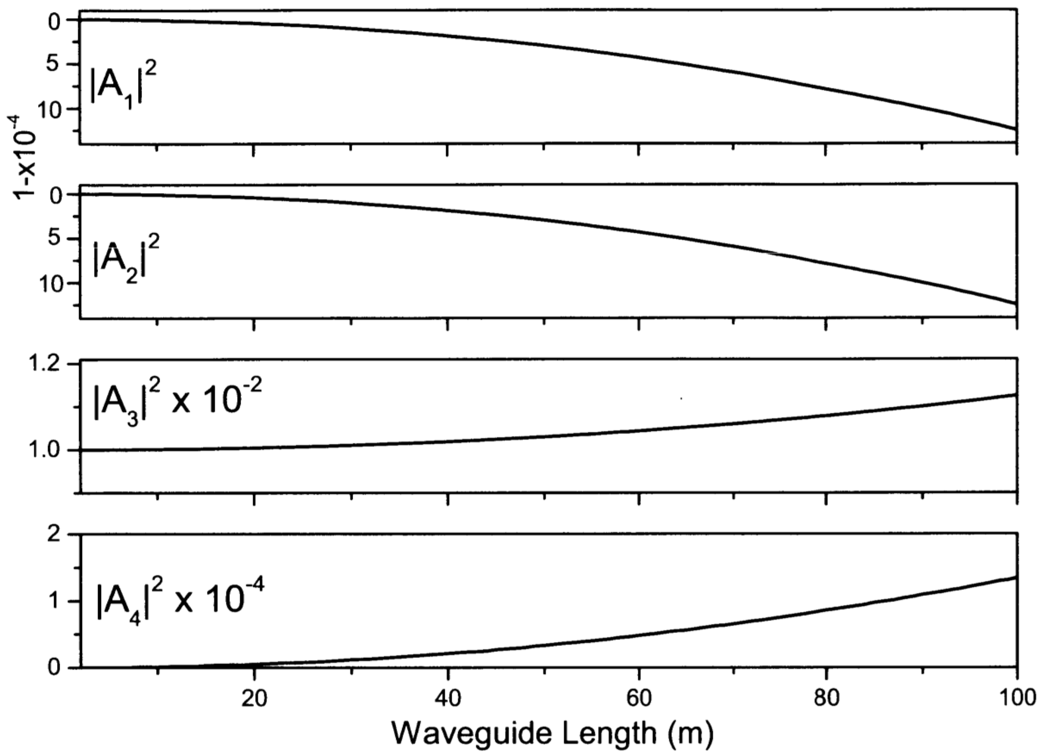


Figure 12: Power exchange between pump ($|A_1|^2$, $|A_2|^2$), signal ($|A_3|^2$), and idler waves ($|A_4|^2$) during the propagation in the fiber [1].

MATHEMATICAL DESCRIPTION OF FWM

- If the intensity of the pump waves increases the behavior differs.
- In optical telecommunications, relatively small input powers propagate in the fibers and only the periodic behavior will be seen.

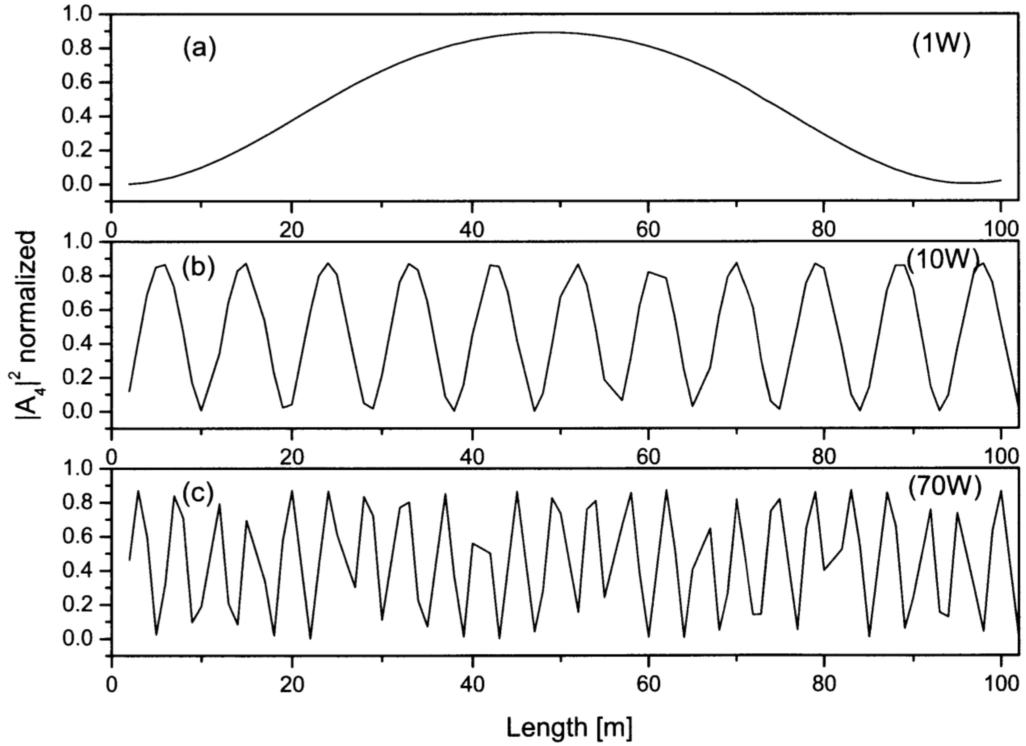


Figure 13: Mixing product versus fiber length for different input powers P_i [1].

PHASE MATCHING

In FWM, new waves with new frequencies are generated, contrary to SPM and XPM where only the phases are changed. the rules of conservation of energy and momentum must be fulfilled, leading to the phase matching condition.

Neglecting the phase alteration by SPM and XPM, We obtain

$$\frac{\partial A_4}{\partial z} = 2j\gamma P_1 A_1 A_2 {A_3}^* e^{j\Delta k_{1,2,-3,-4} z} \tag{45}$$

Assuming that the amplitudes of the pump waves remain nearly the same during propagation due to phase mismatch.

Integrating from $z = 0$ to $z = l$ and $A_4 = 0$ in the fiber input

$$A_4 = \frac{2\gamma P_1}{\Delta k_{1,2,-3,-4}} A_1(0) A_2(0) {A_3}^*(0) \left[e^{j\Delta k_{1,2,-3,-4} l} - 1\right] \tag{46}$$

PHASE MATCHING

By using $\sin(x/2) = \sqrt{(1-\cos x)/2}$, the intensity change of the idler depends on the propagation distance in the fiber.

$$I_4 \sim A_4 {A_4}^* \sim C I_1 I_2 I_3 l^2 \left[\frac{\sin\left(\Delta k_{1,2,-3,-4} l/2\right)}{\Delta k_{1,2,-3,-4} l/2}\right]^2 \qquad (47)$$

where I_n is the intensity of the waves, C is the sum of all constant and $\Delta k_{1,2,-3,-4}$ is the phase mismatch. The phase matching condition is

$$\begin{aligned} \Delta k_{1,2,-3,-4} &= k_1 + k_2 - k_3 - k_3 \\ &= \frac{1}{c}(n_1\omega_1 + n_2\omega_2 - n_3\omega_3 - n_4\omega_4) \\ &= 2\pi\left(\frac{n_1}{\lambda_1} + \frac{n_2}{\lambda_2} - \frac{n_3}{\lambda_3} - \frac{n_4}{\lambda_4}\right) \end{aligned} \qquad (48)$$

PHASE MATCHING

- The intensity of the idler increases quadratically with the length of the waveguide if the phases are matched ($\Delta k = 0$).
- If the phases are not matched the intensity shows a periodic function along the fiber.

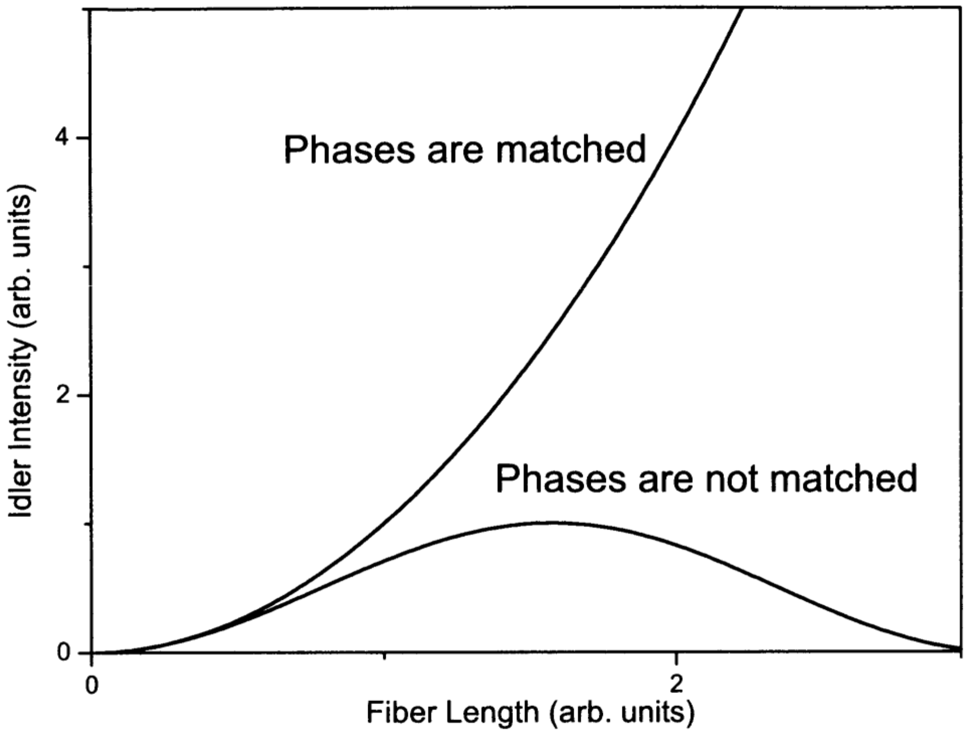


Figure 14: Intensity of the idler wave versus fiber length for phase matching and a phase mismatch between the waves [1].

PHASE MATCHING

The coherence length for FWM is

$$L_{\text{coh}} = \frac{\pi}{\left|\Delta k_{1,2,-3,-4}\right|} \tag{49}$$

The origin of the phase mismatch lies in the frequency dependence of the refractive index and the dispersion.

Table 1: Coherence length in fused silica for different average wavelengths and channel spacings

Δf **[GHz]**	$L_{\text{coh}}(1.3\ \mu m)$ **[m]**	$L_{\text{coh}}(1.55\ \mu m)$ **[m]**	$L_{\text{coh}}(1.6\ \mu m)$ **[m]**
1000	28	2.8	2.3
100	3293	286	233
10	13304	1144	1144
1	33557972	2863923	2338930

PHASE MATCHING

- The refractive index difference and therefore the phase mismatch is weak for a small channel spacing.
- Large coherence lengths for rather large channel spacings are possible in the range of the fibers zero dispersion.

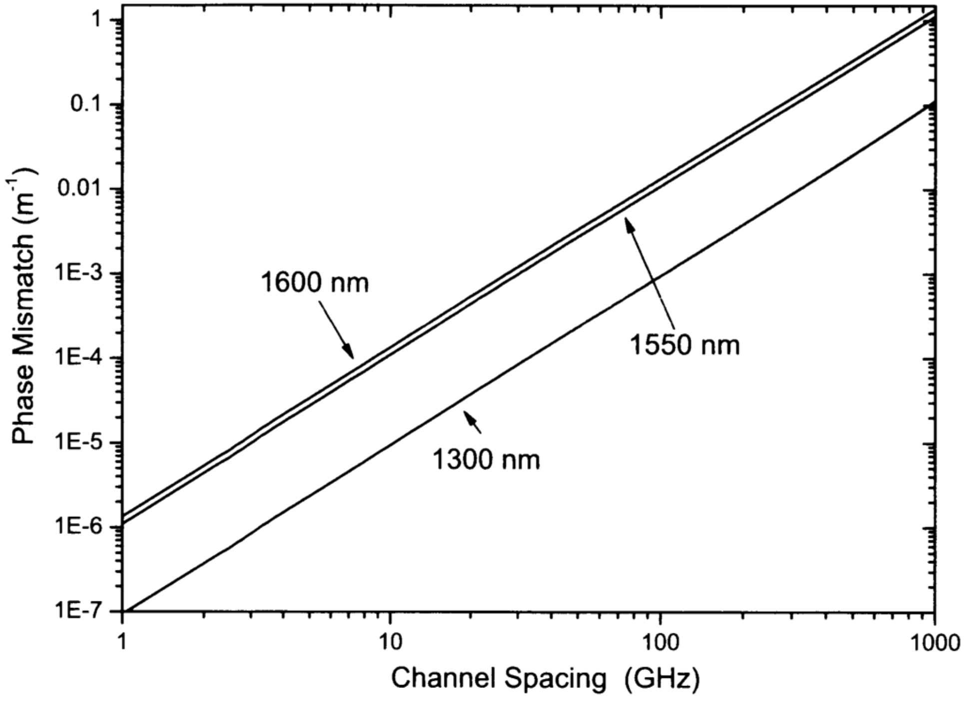


Figure 15: Phase mismatch against the channel spacing in fused silica for an average wavelength of 1600 nm (L-band), 1550 nm (C-band) and 1300 nm (S-band) [1].

PHASE MATCHING

- The intensity in the fiber increases until it reaches a maximum at the coherence length.
- It decreases until the fiber length corresponds to two times the coherence length.

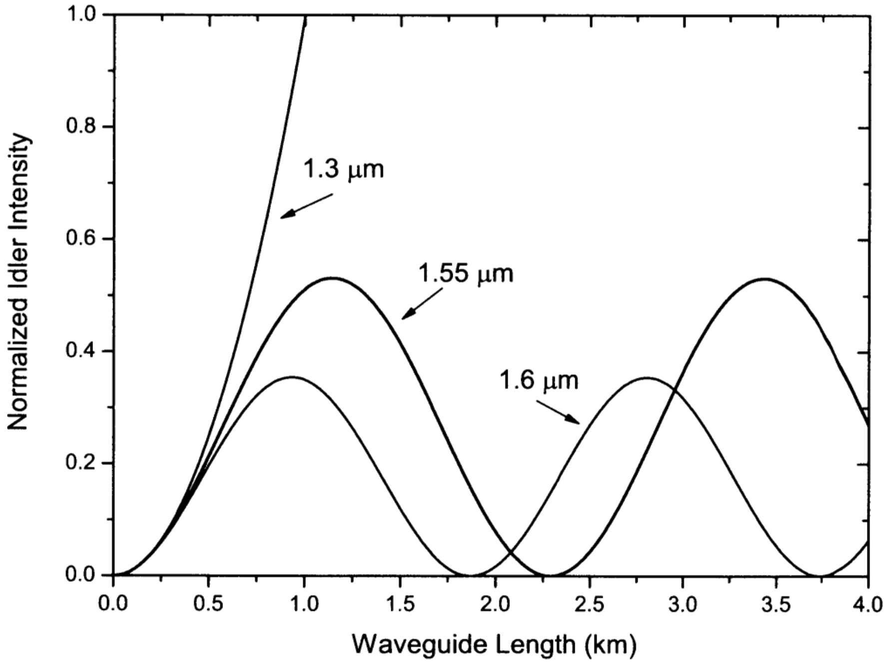


Figure 16: Normalized idler intensities for a 4 km fiber when the attenuation is neglected (channel spacing = 50 GHz) [1].

PHASE MATCHING COMPENSATION

- The FWM not only leads to a degradation of the system performance, but it can also be exploited for a number of applications as well. In this case, the phase mismatch has to be compensated.
- The easiest method for a compensation is using only small channel spacings but this method is limited due to the spectral width of the distinct channels.
- Birefringence is a possibility for phase matching. However, the refractive index difference in standard optical fibers is too small to compensate and also arbitrary. On the other hand, the birefringence is constant in polarization-maintaining fibers and the refractive index difference is much higher than in standard single mode fibers.
- The most often used method for phase mismatch compensation is the introduction of devices with a very small dispersion in the required wavelength range.

FWM'S IMPACT ON COMMUNICATION SYSTEMS

- The advantages of DSF relating to their dispersion properties lead to a broad insertion of these fibers for high bit-rate transmission systems. On the other hand, the dispersion advantage is a disadvantage in relation to the FWM. Hence, the incorporation of WDM into these systems is rat her difficult.
- The main effect that is responsible for the degradation of the system performance in WDM systems due to FWM is the coherent superposition between the original and the new generated waves at the receiver.

FWM'S IMPACT ON COMMUNICATION SYSTEMS

- A WDM system consisting of 10 channels with equal frequency spacing can generate 450 new waves in 28 frequency slots.
- For example, the two channels in the middle of the WDM band are superimposed with 29 new waves.

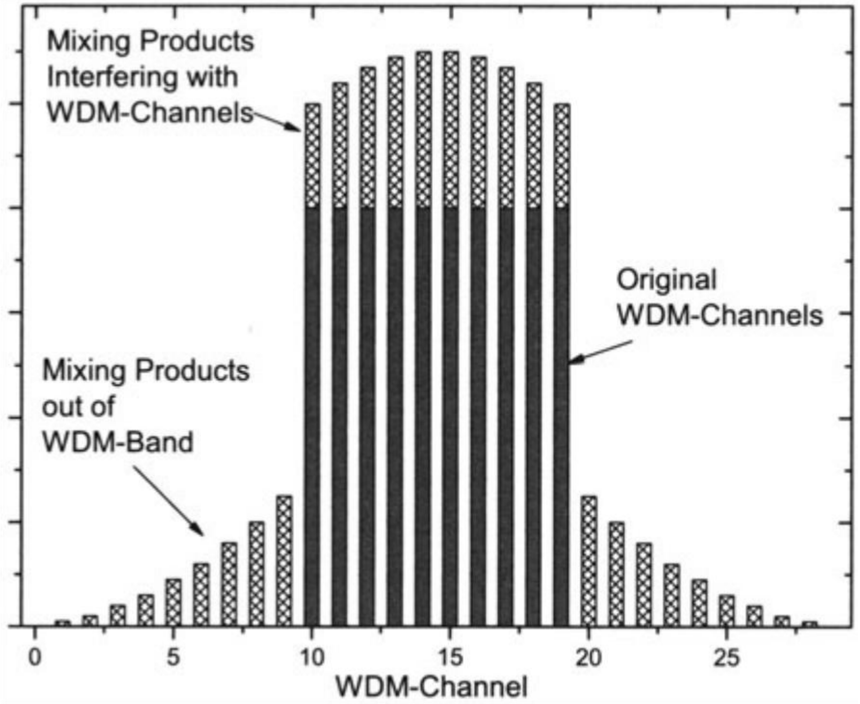


Figure 17: FWM products for a 10 channel WDM system. The out-of-band products can be eliminated by optical filtering [1].

FWM'S IMPACT ON COMMUNICATION SYSTEMS

The mixing products that fall together with the original WDM channels are responsible for a degradation of the system performance.

Assume only one wave generated by the FWM process and one wave in the WDM channel, both with the same frequency and wavenumber.

$$E_{\mathrm{FWM}} = \hat{E}_{\mathrm{FWM}} e^{j(kz-\omega t+\varphi_1)} \qquad E_{\mathrm{WDM}} = \hat{E}_{\mathrm{WDM}} e^{j(kz-\omega t+\varphi_2)} \tag{50}$$

where $\hat{E}$ is a slowly varying amplitude and φ is the phase.

The WDM wave superimposes with the FWM wave

$$E_S = E_{\mathrm{FWM}} + E_{\mathrm{WDM}} = (\hat{E}_{\mathrm{FWM}} e^{j\varphi_1} + \hat{E}_{\mathrm{WDM}} e^{j\varphi_2}) e^{j(kz-\omega t)} \tag{51}$$

The photodiode in the receiver can only detect the intensity of the wave.

$$I_S = \left|\hat{E}_S\right|^2 = \hat{E}_S \hat{E}_S{}^* = I_{\mathrm{FWM}} + I_{\mathrm{WDM}} + 2\sqrt{I_{\mathrm{FWM}} I_{\mathrm{WDM}}} \cos(\varphi_1 - \varphi_2) \tag{52}$$

FWM'S IMPACT ON COMMUNICATION SYSTEMS

$$I_S = \left|\hat{E}_S\right|^2 = \hat{E}_S\hat{E}_S{}^* = I_{\mathrm{FWM}} + I_{\mathrm{WDM}} + 2\sqrt{I_{\mathrm{FWM}}I_{\mathrm{WDM}}}\cos(\varphi_1 - \varphi_2)$$

- The received signal in the photodiode is either reinforced or weakened, depending on the relative phase between the original channel and the mixing product.
- The relative phase depends on the superposition of all the phases of the mixing products which fall into each WDM channel.
- The bit pattern transmitted in the channels is random and thus the relative phase is random as well. As a result, the receivers' output current shows a random fading.
- It responsible for an increase of the BER in the system.

SOLITONS

- The existence of solitons in optical fibers is the result of a balance between the group velocity dispersion (GVD) and self-phase modulation (SPM).
- The pulse envelope for solitons not only propagates undistorted but also survives collisions just as particles do.
- Solitons are not only fundamental interest but they have also found practical applications in the field of fiber-optic communications.

THE DISCOVERY OF SOLITONS

- In 1834, Scottish engineer and scientist John Scott Russell observed a water wave propagated in a river over several kilometers with a constant velocity and without significant change of its shape.
- The wave can walk through each other and keep their shape after a collision, behaves like a particle. Russel called them solitary waves.

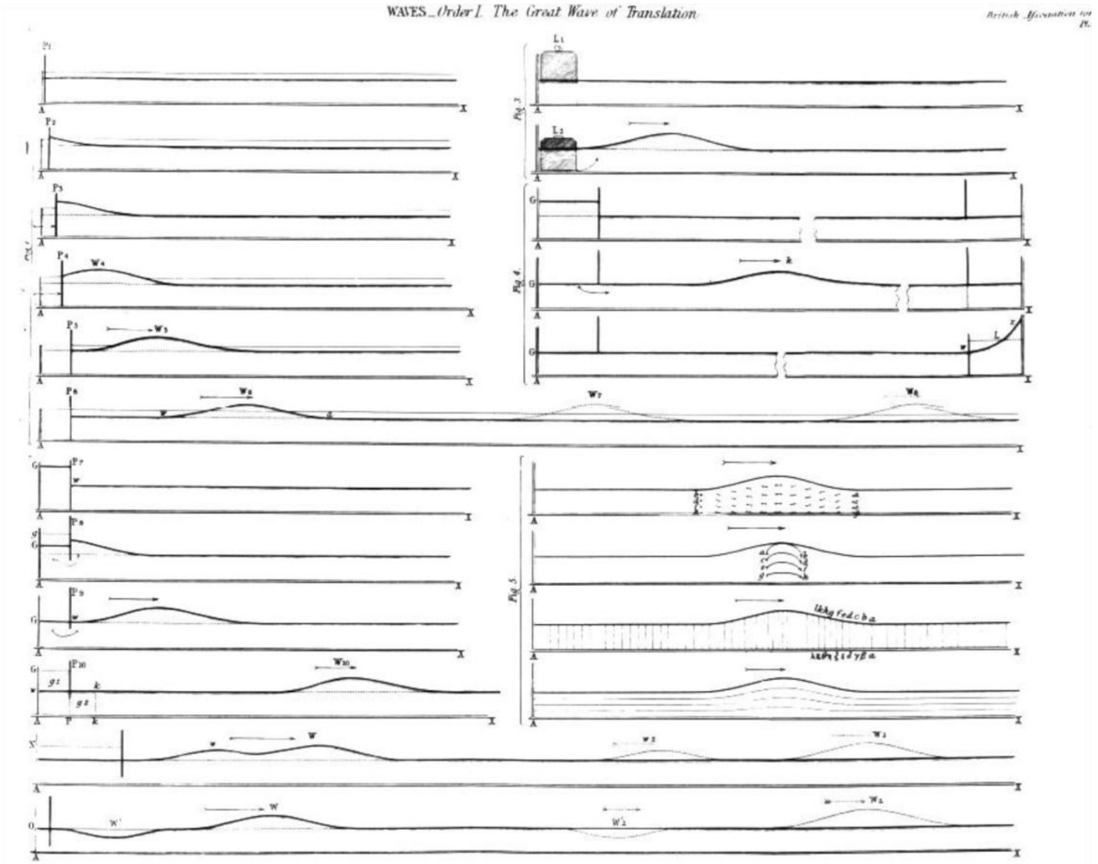


Figure 18: Russel solitons experiment [2].

THE DISCOVERY OF SOLITONS

1834 - Russell observed a solitary wave in a river.

1965 - The word soliton was coined to describe the particle-like properties of pulses propagating in a nonlinear medium.

1973 - the existence of solitons in fibers was suggested and demonstrated using numerical simulations.

1980 - solitons was observed experimentally in a fiber.

1988 - the potential of solitons was first demonstrated for long-haul communication.

MATHEMATICAL DESCRIPTION

Optical soliton can be described by the nonlinear Schrodinger equation (NSE)

$$j\frac{\partial B}{\partial z} = -\gamma|B|^2 + \frac{k_2}{2}\frac{\partial^2 B}{\partial T^2} \tag{53}$$

Assume that the attenuation is negligible. If the soliton is stable, the linear part must be compensated by the nonlinear part. Otherwise, the pulse will spread in time (due to linear dispersion) of frequency (due to SPM).

By using the standard NSE and using the following solitons units

$$A = \frac{B}{\sqrt{P_0}} \qquad \xi = \frac{z}{L_D} \qquad \tau = \frac{T}{\tau_0}$$

we get

$$\frac{j}{L_D}\frac{\partial A}{\partial \xi} = -\gamma P_0|A|^2 A + \frac{k_2}{2\tau_0{}^2}\frac{\partial^2 A}{\partial \tau^2} \tag{54}$$

MATHEMATICAL DESCRIPTION

The ratio between the dispersive and nonlinear length is

$$\frac{L_D}{L_{\mathrm{NL}}} = \frac{\gamma P_0 {\tau_0}^2}{|k_2|} = N^2 \tag{55}$$

This ratio shows which effect have stronger effect on the fiber. If the ration equal 1, dispersive and nonlinear effects can compensate each other and a soliton is formed.

The ratio above and $L_{\mathrm{NL}} = 1/\gamma P$, the NSE can be written as

$$j\frac{\partial A}{\partial \xi} = -N^2|A|^2 A + \frac{1}{2}\operatorname{sgn}(k_2)\frac{\partial^2 A}{\partial \tau^2} \tag{56}$$

The standard form of a soliton can only be formed in the anomalous dispersion region of the optical fiber

$$u = NA = \sqrt{\frac{\gamma {\tau_0}^2 P_0}{|k_2|}}\frac{B}{\sqrt{P_0}} \tag{57}$$

BRIGHT SOLITONS

For anomalous dispersion $\mathrm{sgn}(k_2) = -1$ and

$$j\frac{\partial u}{\partial \xi} + |u|^2 u + \frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} = 0 \qquad (58)$$

This equation corresponds to the time-dependent, dimensionless, nonlinear Schrodinger equation of quantum mechanics.

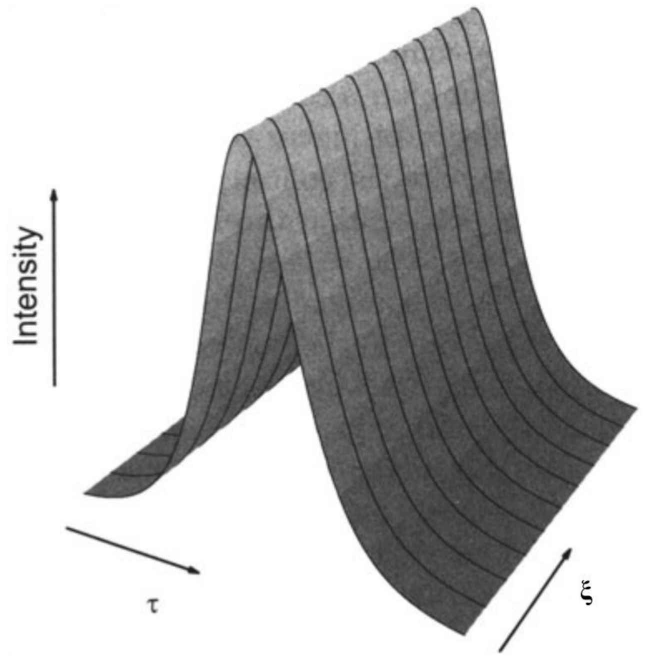


Figure 19: Soliton in a nonlinear waveguide [1].

BRIGHT SOLITONS

An unlimited possible solutions can be found but the most important one is solution of a hyperbolic secant shape. The input pulse having an initial amplitude of

$$u(0,\tau) = N\text{sech}\left(\frac{T}{\tau_0}\right) = N\text{sech}(\tau) = \frac{2N}{e^{\tau} + e^{-\tau}} \qquad (59)$$

where $\text{sech} = 1/\cosh$.

- Fundamental soliton ($N = 1$) - shape remains unchanged during the propagation.
- Higher order solitons ($N > 1$) - follows a periodic pattern when the input shape recovers at $\xi = m\pi/2$ where m is integer.

The soliton period is

$$z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{{\tau_0}^2}{k_2} \qquad (60)$$

BRIGHT SOLITONS

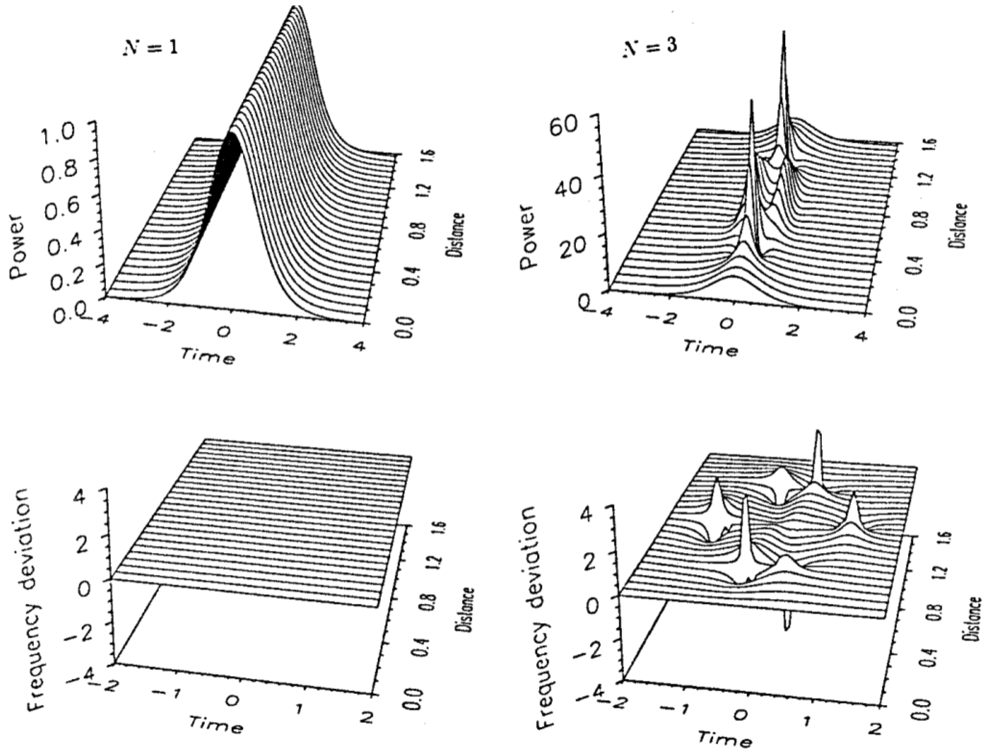


Figure 20: Evolution of the first-order (left column) and third-order (right column) solitons over one soliton period. Top and bottom rows show the pulse shape and chirp profile, respectively [3].

BRIGHT SOLITONS

Solving Eq. (58), we get the well-known 'sech' solution for the fundamental soliton by integrating the NLS equation directly

$$u(\xi,\tau) = N\text{sech}(\tau)\exp(j\xi/2) \tag{61}$$

It shows that the input pulse acquires a phase shift $\xi/2$ as it propagates inside the fiber, but its amplitude remains unchanged. It is this property of a fundamental soliton that makes it an ideal candidate for optical communications. In essence, the effects of fiber dispersion are exactly compensated by the fiber nonlinearity when the input pulse has a 'sech' shape and its width and peak power are related in such a way that $N = 1$.

PULSE EVOLUTION

- An important property of optical solitons is that they are remarkably stable against perturbations.
- The fundamental soliton requires a specific shape and a certain peak power corresponding to $N = 1$.
- However, it can be created even when the pulse shape and the peak power deviate from the ideal conditions.

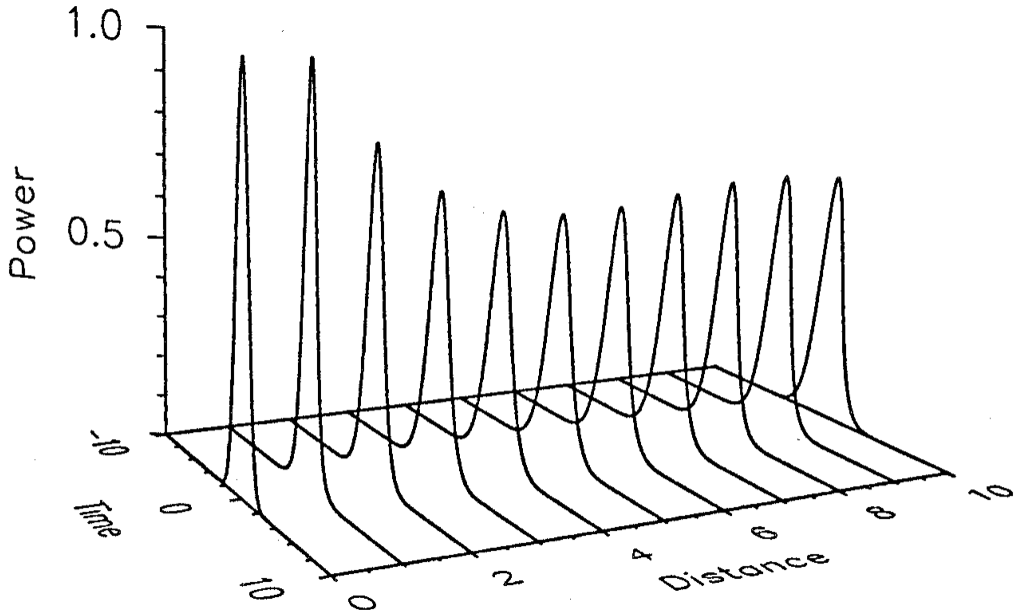


Figure 21: Evolution of a Gaussian pulse with $N = 1$ over the range $\xi = 0 - 10$. The pulse evolves toward the fundamental soliton by changing its shape, width, and peak power [3].

PULSE EVOLUTION

- A similar behavior is observed when N deviates from 1 (in the range of $N - 0.5 < N < N + 0.5$).
- The pulse width and the peak power oscillate initially but eventually become constant after the input pulse has adjusted itself to satisfy the condition $N = 1$.

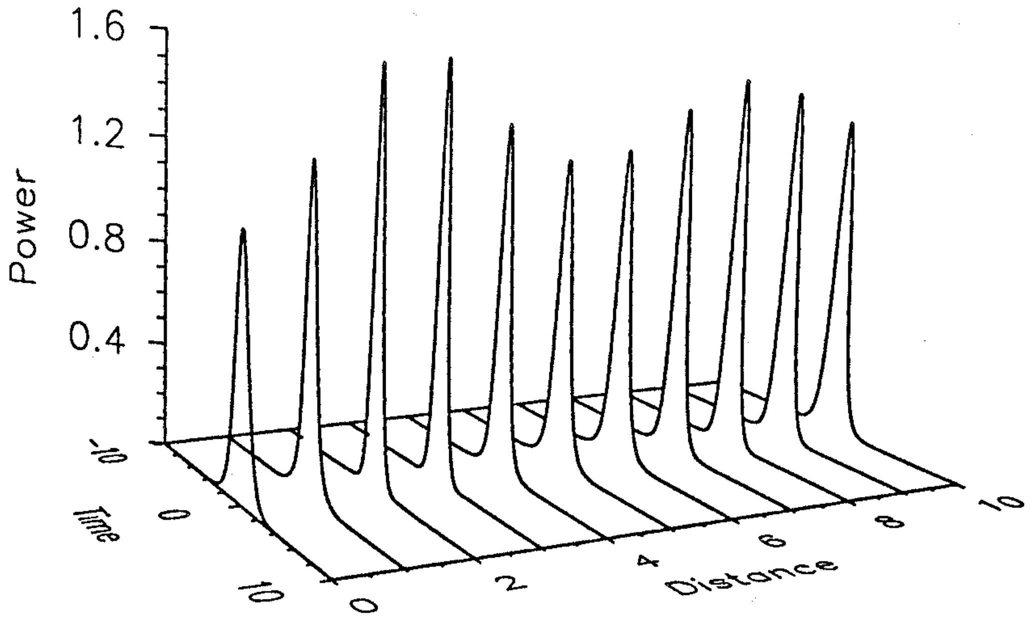


Figure 22: Pulse evolution for a 'sech' pulse with $N = 1.2$ over the range $\xi = 0 - 10$. The pulse evolves toward the fundamental soliton ($N = 1$) by adjusting its width and peak power [3].

PULSE EVOLUTION

- Higher intensities in the pulse center create a temporal waveguide by increasing the refractive index only in the central part of the pulse.
- Such a waveguide supports temporal modes just as the core-cladding index difference led to spatial modes.
- Most of the pulse energy can still be coupled into that temporal mode even when it does not match the temporal mode precisely.
- The rest of the energy spreads in the form of dispersive waves.

DARK SOLITONS

- The NSE can be solved even in case on normal dispersion $(\mathrm{sgn}(k_2) = 1)$.
- The intensity profile of the resulting solutions exhibits a dip in a uniform background, and it is the dip that remains unchanged during propagation inside the fiber.
- Those solitons are called 'dark solitons’.

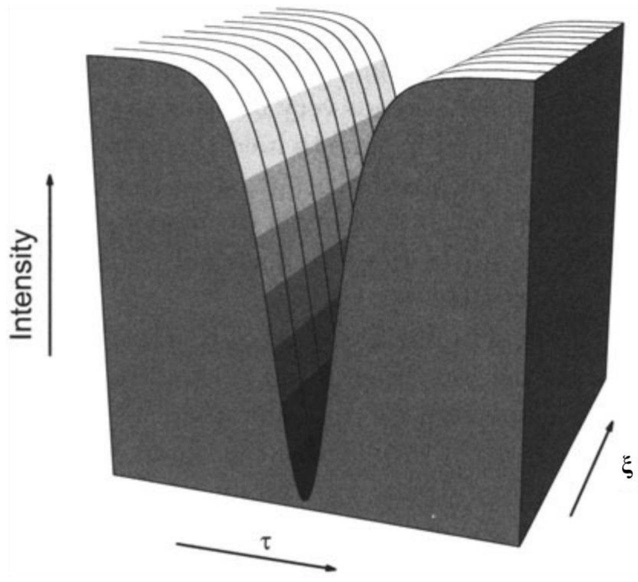


Figure 23: Dark soliton in a waveguide with normal dispersion [1].

DARK SOLITONS

The general solution can be written as

$$u_d(\xi,\tau) = (\eta \tanh\zeta - j\kappa)\exp(ju_0{}^2\xi)$$

and

$$\zeta = \eta(\tau - \kappa\xi), \qquad \eta = u_0\cos\phi\,, \qquad \kappa = u_0\sin\phi$$

where u_0 is the amplitude of the continuous-wave (CW) background and ϕ is an internal phase angle in the range 0 to $\pi/2$.

An important difference between the bright and dark solitons is that the speed of a dark soliton depends on its amplitude η through ϕ. For $\phi = 0$,

$$u_d(\xi,\tau) = u_0\tanh(u_0\tau)\exp(ju_0{}^2\xi)$$

- $\phi = 0$ - The dip drops to zero. This soliton is called black soliton.
- $\phi \neq 0$ - The dip does not drop to zero. This soliton is called gray soliton.

TYPES OF DARK SOLITONS

In contrast with bright solitons which have a constant phase, the phase of a dark soliton changes across its width.

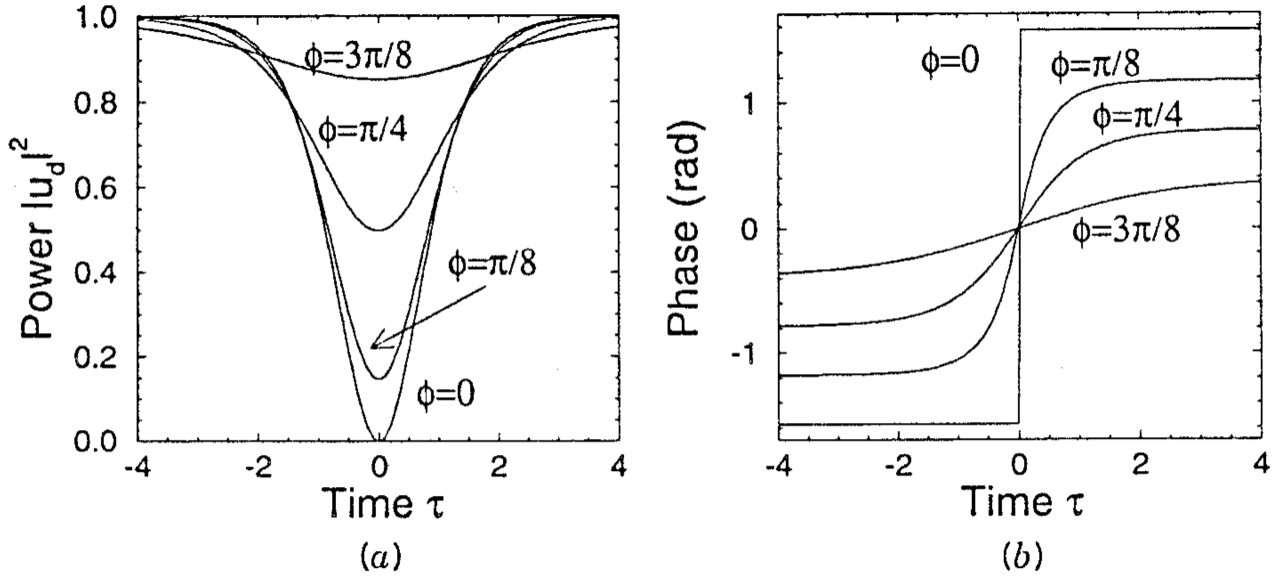


Figure 24: (a) Intensity and (b) phase profiles of dark solitons for several values of the internal phase ϕ. The intensity drops to zero at the center for black solitons [3].

STIMULATED LIGHT SCATTERING

- **Rayleigh** scattering is **elastic scattering** for which the frequency (or the photon energy) of scattered light remains unchanged. By contrast, the frequency of scattered light is shifted downward during inelastic scattering.
- Two examples of **inelastic scattering** are **Raman** scattering and **Brillouin** scattering. Both of them can be understood as scattering of a photon to a lower energy photon such that the energy difference appears in the form of a phonon. The main difference between the two is that optical phonons participate in Raman scattering, whereas acoustic phonons participate in Brillouin scattering. Both scattering processes result in a loss of power at the incident frequency. However, their scattering cross sections are sufficiently small that loss is negligible at low power levels.
- At high power levels, the nonlinear phenomena of stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS) become important. The intensity of the scattered light in both cases grows exponentially once the incident power exceeds a threshold value. SRS and SBS were first observed in optical fibers during the 1970s.

OPTICAL AND ACOUSTIC PHONONS

- In physics, a **phonon** is a collective excitation in a periodic, elastic arrangement of atoms or molecules in condensed matter, like solids and some liquids. Often designated a quasiparticle, it represents an excited state in the quantum mechanical quantization of the modes of vibrations of elastic structures of interacting particles. Phonons play a major role in many of the physical properties of condensed matter, like thermal conductivity and electrical conductivity. The study of phonons is an important part of condensed matter physics.
- The concept of phonons was introduced in 1932 by Soviet physicist Igor Tamm. The name phonon comes from the Greek word $\varphi\omega\upsilon\eta$ (phone), which translates to sound or voice because long-wavelength phonons give rise to sound. Shorter-wavelength higher-frequency phonons give rise to heat.

OPTICAL AND ACOUSTIC PHONONS

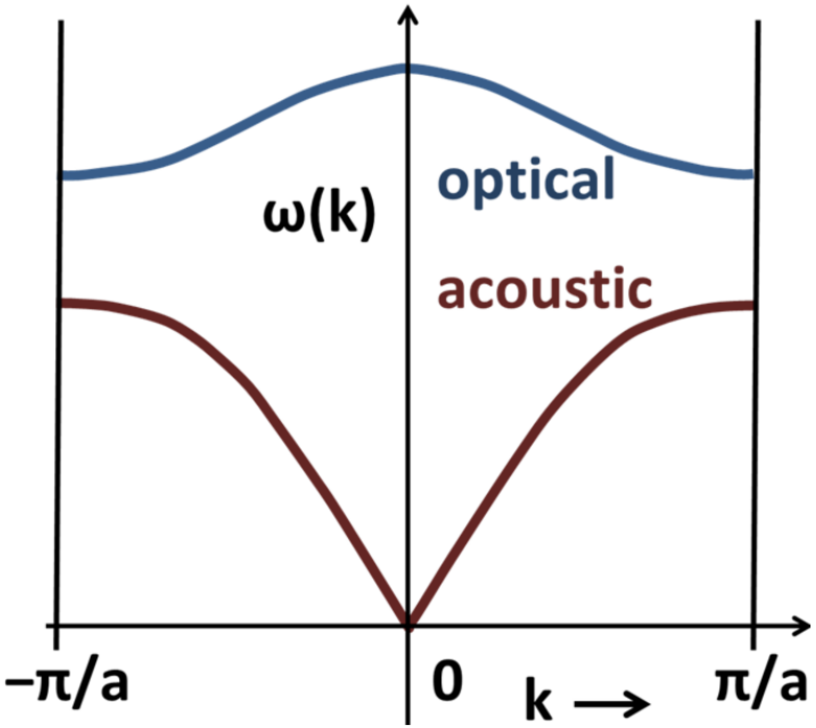


Figure 25: Optical phonons arise from out of phase vibrations between neighboring atoms within the unit cell, while in phase vibrations give rise to acoustic phonons.

STIMULATED LIGHT SCATTERING

Even though SRS and SBS are quite similar in their origin, different dispersion relations for acoustic and optical phonons lead to the following differences between the two in single mode fibers:

1) SBS occurs only in the backward direction whereas SRS can occur in both directions.
2) The scattered light is shifted in frequency by about 10 GHz for SBS but by 13 THz for SRS (this shift is called the Stokes shift)
3) The Brillouin gain spectrum is extremely narrow (bandwidth < 100 MHz) compared with the Raman-gain spectrum that extends over 20-30 THz.

The origin of these differences lies in a relatively small value of the ratio $v_A/c \sim 10^{-5}$, where v_A is the acoustic velocity in silica and c is the velocity of light.

RAMAN SCATTERING

- The spontaneous Raman scattering was discovered, long before the invention of the laser, in the year 1924 by the Indian physicist Sir Chandrasekhara Raman (knighted 1929).
- During a ship travel, after a visit to a congress in England, Raman admired the wonderful blue color of the Mediterranean sea. He found the origin of this effect in the scattering of the sun light at the molecules of the water.
- For the discovery of the effect, Raman won the Nobel Prize in physics in 1930.

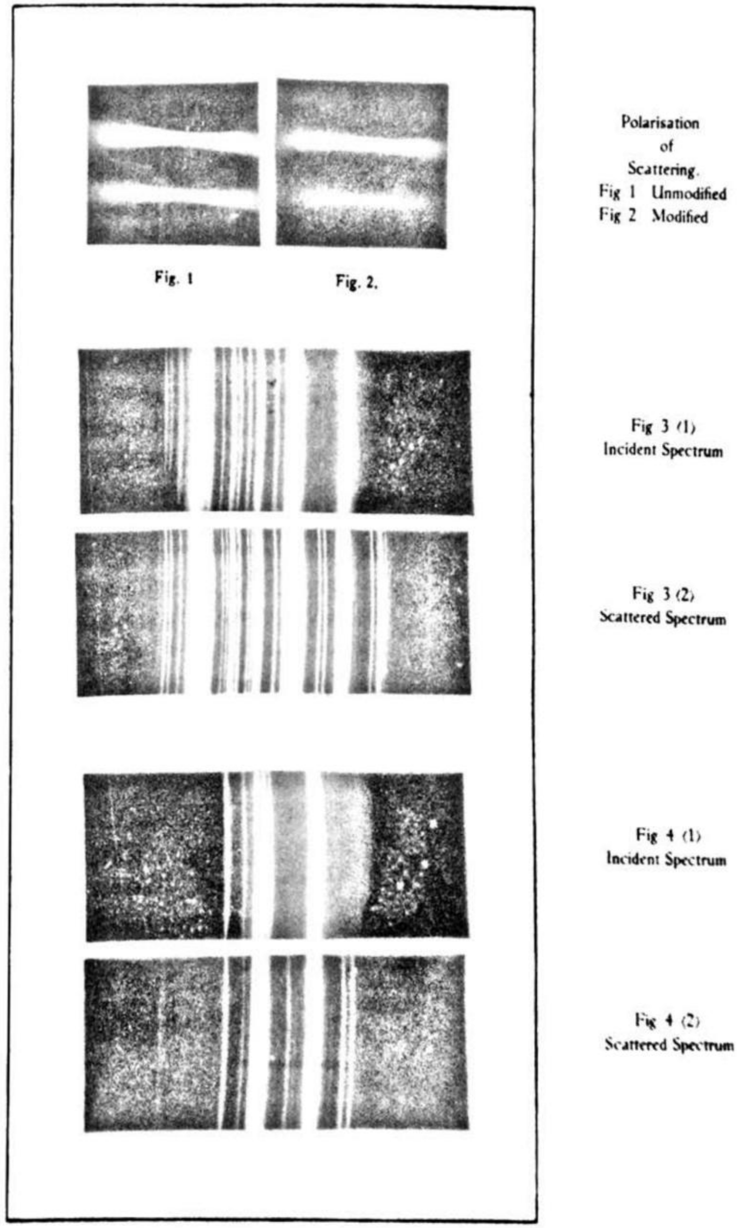


Figure 26: [4]

RAMAN SCATTERING

- When photons hit material, they mostly scatter elastically and the energy is conserved which called "Rayleigh scattering".
- Very small fraction of the photons (1 part in a million) are inelastic scattered and the energy is not conserved, this is called "Raman scattering".

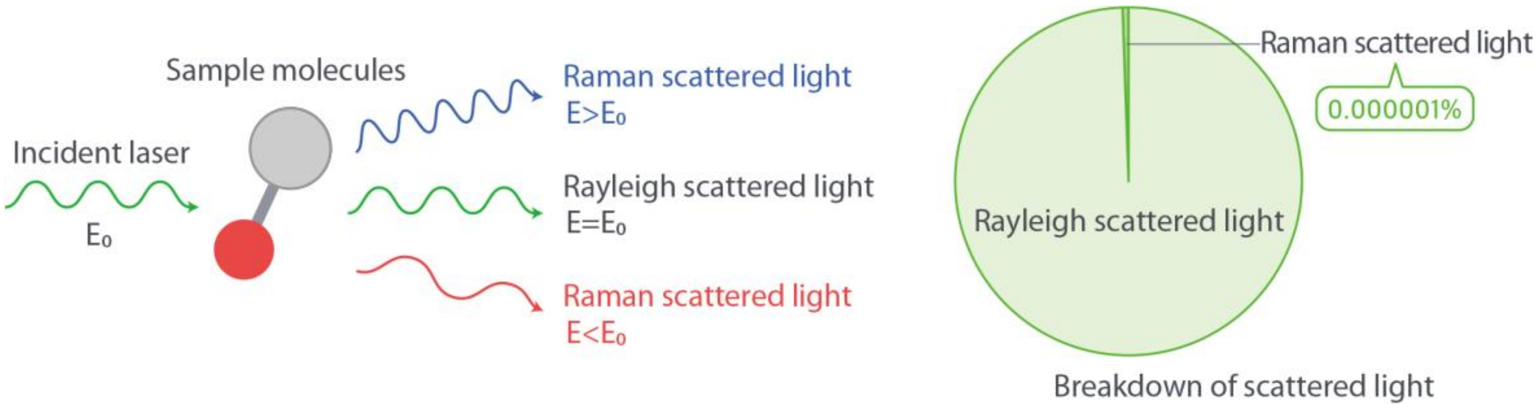


Figure 27: Schematics of Raman effect.

RAMAN SCATTERING

In Raman scattering the molecule is excited to virtual energy level and fall back to different energy level. There are two types of Raman scattering:

1) **Stokes scattering** - when a molecule in the ground state excite and fall to a higher energy level, a photon with a lower energy (as a result a longer wavelength) is emitted.

2) **Anti-Stokes scattering** - when a molecule in excited state absorb photon the fall to the ground state, a photon with a higher energy (as a result a shorter wavelength) is emitted.

RAMAN SCATTERING

The energy difference is converted to vibration of the molecule atoms.

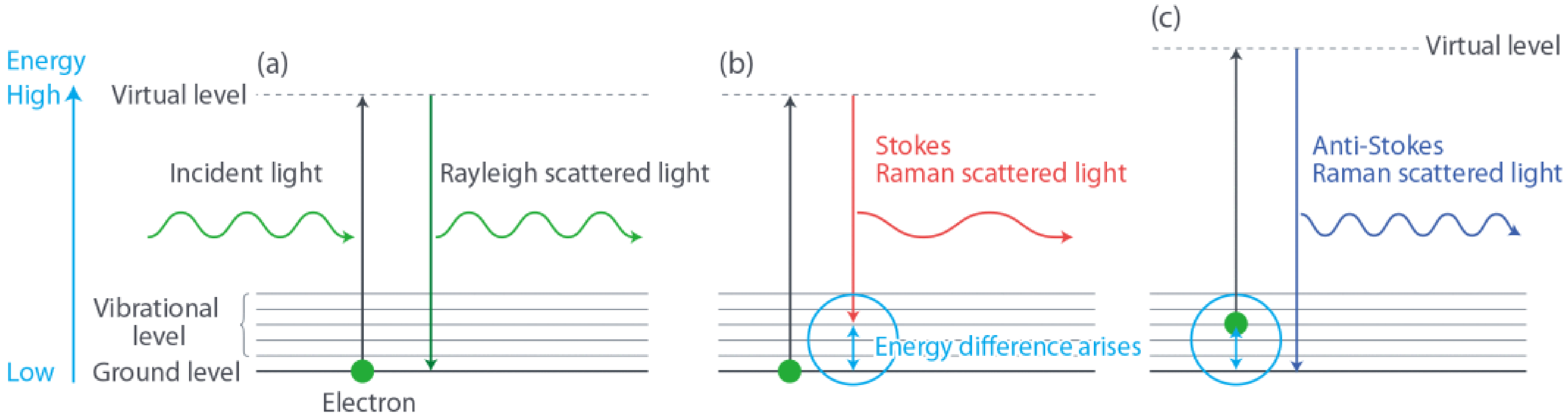


Figure 28: Diagram for the description of (a) Rayleigh scattering, (b) Stokes Scattering and (c) Anti-Stokes Scattering.

RAMAN SCATTERING SPECTRA

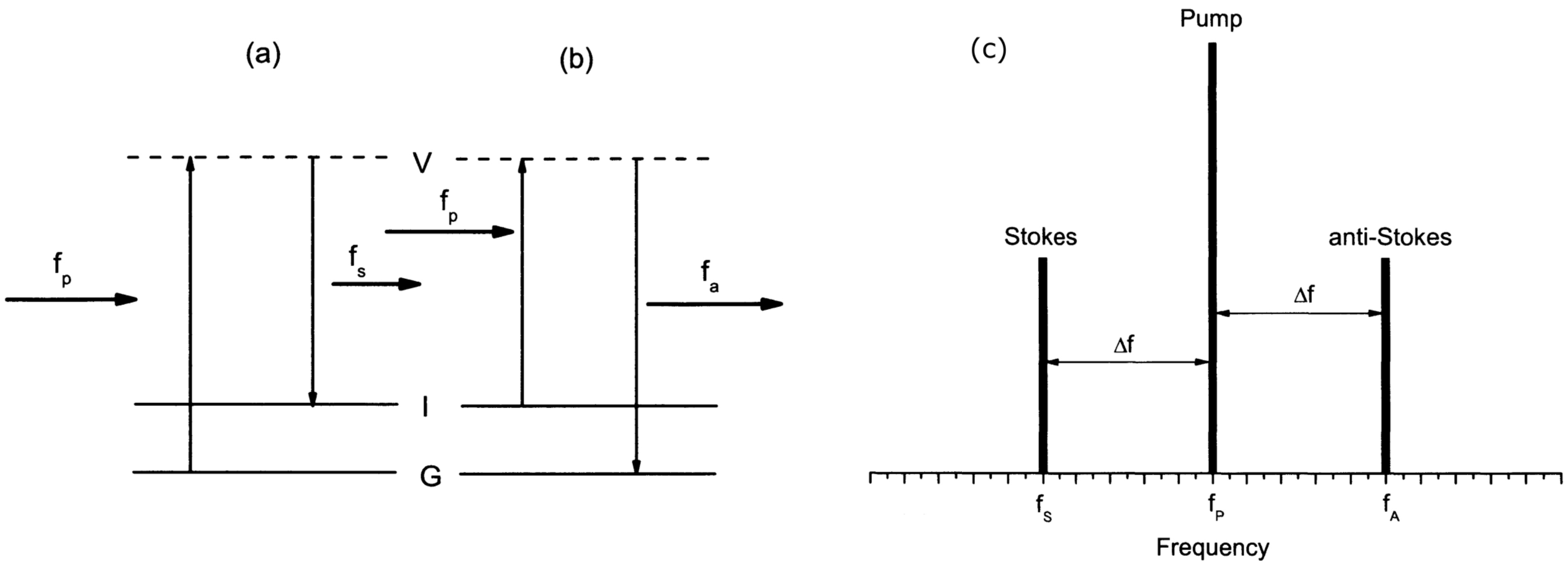


Figure 29: Energy diagram for the description of the Raman Scattering. (a) Generation of the Stokes. (b) Generation of the anti-Stokes wave. (c) Distribution of the distinct frequencies for the Raman process [1].

RAMAN SCATTERING SPECTRA

Raman scattering can be used for spectroscopy of vibration that are IR-inactive. The Figure below shows a frequency shift due to Raman scattering.

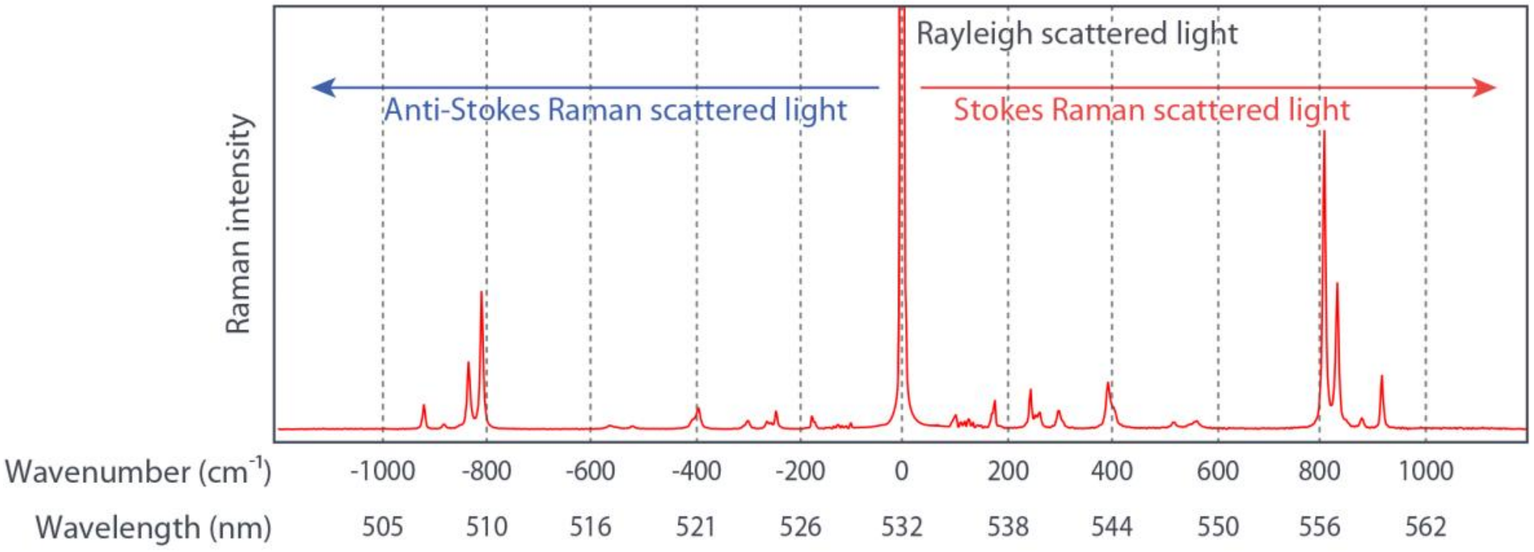


Figure 30: Raman spectrum of ethanol obtained by 532 nm excitation wavelength.

RAMAN SCATTERING

The Raman coefficient for the pump is slightly different from that for the Stokes wave because the pump is frequency-shifted. The Raman gain coefficient in fibers scales, to first order, with the inverse pump wavelength. Therefore, the Raman coefficient of the pump is related to that of the probe as:

$$g_{\mathrm{p}} = \frac{\omega_{\mathrm{p}}}{\omega_{\mathrm{s}}} g_{\mathrm{s}}$$

The intensity is connected to the power via the effective area of the waveguide. The Raman gain coefficient depends on the effective area of the fiber, as follows:

$$g_{\mathrm{R}} = g_{\mathrm{s}} \cdot A_{\mathrm{eff}}$$

where g_{R} as the Raman gain.

RAMAN SCATTERING IN OPTICAL FIBERS

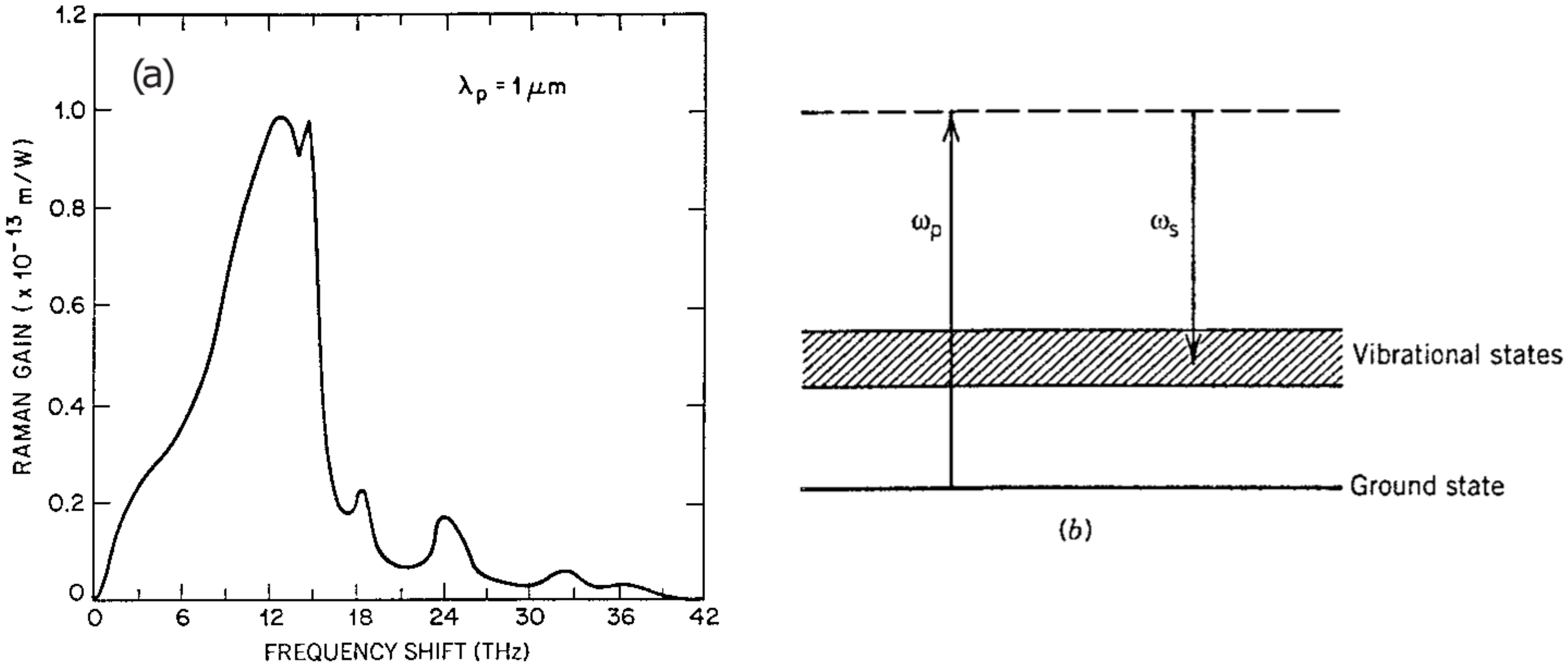


Figure 31: (a) Raman gain spectrum of fused silica at $\lambda_p = 1\ \mu m$ and (b) energy levels participating in the SRS process [3].

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RAMAN SCATTERING IN OPTICAL FIBERS

The spectrum of the Raman gain depends on the decay time associated with the excited vibrational state. In the case of a molecular gas or liquid, the decay time is relatively long (~1 ns), resulting in a Raman-gain bandwidth of ~1 GHz. In the case for optical fibers, the bandwidth exceeds 10 THz. Figure 31 shows the Raman-gain spectrum of silica fibers.

The broadband and multipeak nature of the spectrum is due to the amorphous nature of glass. More specifically, vibrational energy levels of silica molecules merge together to form a band. As a result, the Stokes frequency ω_s can differ from the pump frequency ω_p over a wide range. The maximum gain occurs when the Raman shift $f_R = f_p - f_s$ is about 13 THz. Another major peak occurs near 15 THz while minor peaks persist for values of f_R as large as 35 THz. The peak value of the Raman gain g_R is about $1\cdot10^{-13}$ m/W at a wavelength of 1 μm. This value scales linearly with f_p (or inversely with the pump wavelength λ_p), resulting in $g_R \approx t \cdot 10^{-13}$ m/W at 1.55 μm.

RAMAN SCATTERING

At the same time, the relative polarization between pump and Stokes wave has, of course, an influence on the efficiency of the Raman scattering as well. The differential equation system, describing the intensities of both waves under the influence of Raman scattering during its propagation through the medium is then represented by:

$$\frac{dI_\mathrm{s}}{dz} = \frac{g_\mathrm{R}}{K_\mathrm{s}} I_\mathrm{p} I_\mathrm{s} - \alpha_\mathrm{s} I_\mathrm{s} \tag{64}$$

$$\frac{dI_\mathrm{p}}{dz} = -\frac{\omega_\mathrm{p}}{\omega_\mathrm{s}} \frac{g_\mathrm{R}}{K_\mathrm{s}} I_\mathrm{p} I_\mathrm{s} - \alpha_\mathrm{p} I_\mathrm{p} \tag{65}$$

where K_s is a factor that includes the relative polarization between the pump wave and the Stokes wave. The Raman gain has its maximum if $K_\mathrm{s} = 1$, the case where both waves have an identical polarization. Standard single mode fibers show birefringence and hence, the polarization state of the waves is arbitrary. In this case, the polarization factor is $K_\mathrm{s} = 2$.

STIMULATED RAMAN SCATTERING

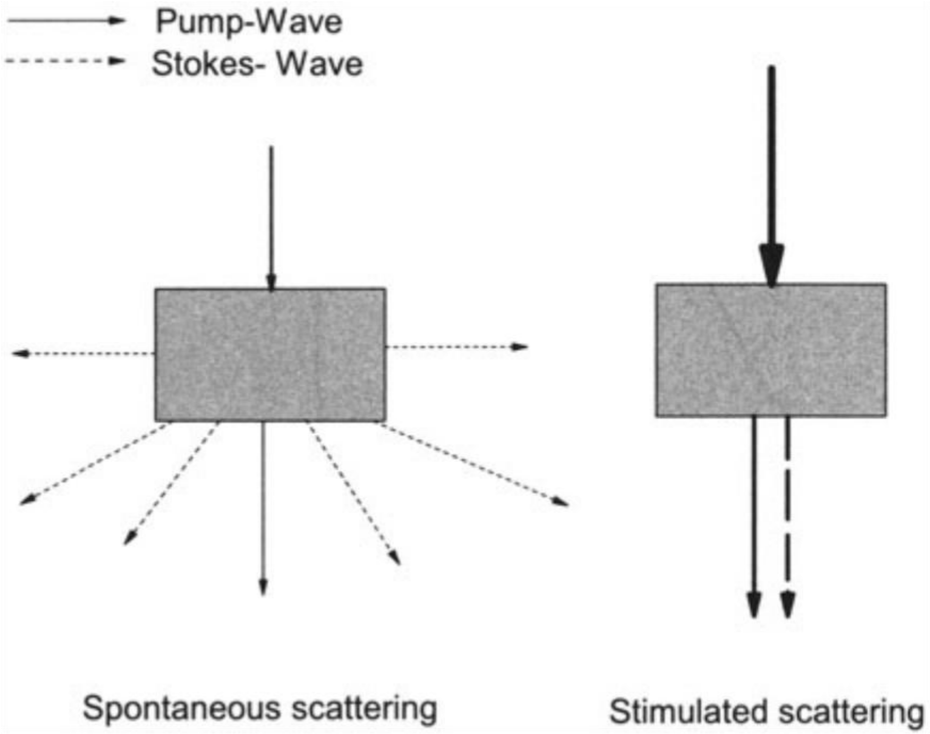


Figure 32: Basic differences between (a) spontaneous and (b) stimulated scattering [1].

STIMULATED RAMAN SCATTERING

The Raman scattering process becomes stimulated if the pump power exceeds a threshold value. The differences can be explained by the model of harmonic oscillator:

- The pump wave hits a dipole oscillating with its resonance frequency, an additional Stokes wave will be generated by the dipole. The dipole emits this Stokes wave with a radiation pattern typical for a dipole that shows a $\sin\theta$ dependence.
- If the intensity of the pump wave is higher than a particular threshold, then the wave scattered at the first dipole is relatively intense. The Stokes wave of the following dipole superimposes coherently in the forward direction with the Stokes wave of the first dipole.

THRESHOLD OF RAMAN SCATTERING

The intensities of the pump and Stokes waves under the influence of Raman scattering in an optical fiber are described by the differential equation system (64) and (65). If, in a first approximation, it is assumed that the intensity depletion of the pump due to the Raman interaction is small, the first term on the right side of (65) can be neglected.

$$I_{\mathrm{p}}(z) = I_{\mathrm{p}}(0) \exp\left(-\alpha_{\mathrm{p}} z\right)$$

Using Equation (64) we obtained

$$I_{\mathrm{s}}(z) = I_{\mathrm{s}}(0) \exp\left[\frac{g_{\mathrm{R}}}{K_{\mathrm{s}}} I_{\mathrm{p}}(0) L_{\mathrm{eff}} - \alpha_{\mathrm{s}} z\right]$$

where the effective interaction length is

$$L_{\mathrm{eff}} = 1 - \exp\left(-\alpha_{\mathrm{p}} z\right)/\alpha_{\mathrm{s}}$$

THRESHOLD OF RAMAN SCATTERING

The threshold for stimulated scattering is defined as the input intensity value of the pump wave for which the Stokes wave shows a growth in the fiber I_{pG}. The amplification due to the Raman process must exceed the attenuation loss of the Stokes wave.

$$\frac{g_{\mathrm{R}}}{K_{\mathrm{s}}} I_{\mathrm{pG}} L_{\mathrm{eff}} \gg \alpha_{\mathrm{s}} z$$

where $I_{\mathrm{pG}} = P_{0\mathrm{G}}/A_{\mathrm{eff}}$. The threshold of stimulated Raman scattering (SRS) is

$$P_{0\mathrm{G}} \gg \frac{\alpha_{\mathrm{s}} A_{\mathrm{eff}} K_{\mathrm{s}}}{g_{\mathrm{R}}}$$

THRESHOLD OF RAMAN SCATTERING

$$\frac{g_R}{K_s} I_{pG} L_{eff} \gg \alpha_s z$$

- The intensity of the Stokes wave increases very strongly from 0 to 5 km in this range the inequality is valid.
- If the waves propagate further, the intensity of the pump decreases. The growth rate of the Stokes wave is smaller and comes to a maximum after a propagation distance of 14 km. After this the attenuation (the right side) exceeds the amplification and the Stokes wave will decrease.

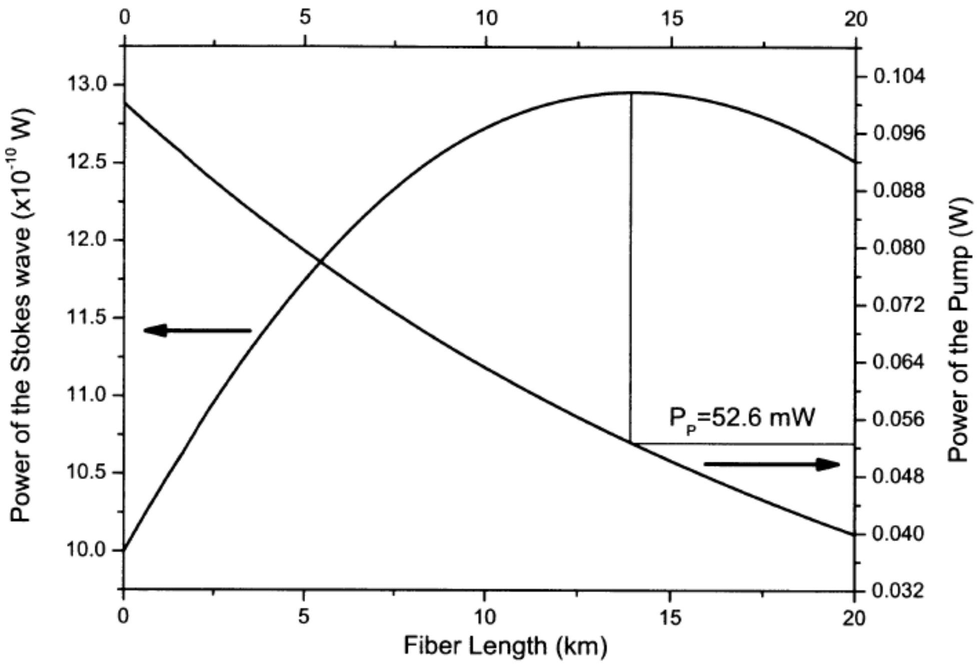


Figure 33: Computed power of a pump wave and a Stokes wave along a fiber [1].

THRESHOLD OF RAMAN SCATTERING

The forward threshold for Raman scattering in optical fibers is defined as the input pump power at which the output powers for pump and Stokes wave are equal. It is estimated as:

$$P_{\mathrm{th}} = 16 \frac{K_{\mathrm{s}} A_{\mathrm{eff}}}{g_{\mathrm{R}} L_{\mathrm{eff}}} \tag{66}$$

The Equation is an approximation and is only valid under the conditions that the power transfer of the pump to the Stokes wave due to the Raman process is negligible, the effective area for pump- and Stokes wavelength are equal, the Raman gain can be approximated by a Lorentz function, and the initial Raman signal is generated by spontaneous scattering only, i.e., no wave with the Raman frequency shift is launched into the fiber.

THRESHOLD OF RAMAN SCATTERING

- The threshold for the stimulated Raman scattering is $P_{\mathrm{th}} \approx 1\,\mathrm{W}$ in polarization-maintaining fibers ($K_s = 1$) with an effective area of $A_{\mathrm{eff}} = 80\ \mu\mathrm{m}^2$, a Raman gain of $g_R \approx 7 \cdot 10^{-14}\ \mathrm{m/W}$, and an effective length of $L_{\mathrm{eff}} \approx 22\,\mathrm{km}$. Whereas, in standard single mode fibers, due to the arbitrary distribution of the polarization states of pump and Stokes wave ($K_s = 2$), it has a threshold of $P_{\mathrm{th}} \approx 2\,\mathrm{W}$.
- If the intensity of the pump wave is higher than the threshold, the power of the Stokes wave at the end of the fiber is greater than the output power of the pump.
- Figure below shows the behavior for a pump intensity of $I_p = 1.9 \cdot 10^{10}\ \mathrm{W/m^2}$ and an input power of $P_p = 1.5\,\mathrm{W}$ for a polarization-maintaining fiber ($g_R \approx 7 \cdot 10^{-14}\ \mathrm{m/W}$, $A_{\mathrm{eff}} = 80\ \mu\mathrm{m}^2$, $\alpha = 0.2\ \mathrm{dB/km}$). The figure shows clearly the intensity loss of the pump due to the power transfer to the Stokes wave.

RAMAN SCATTERING

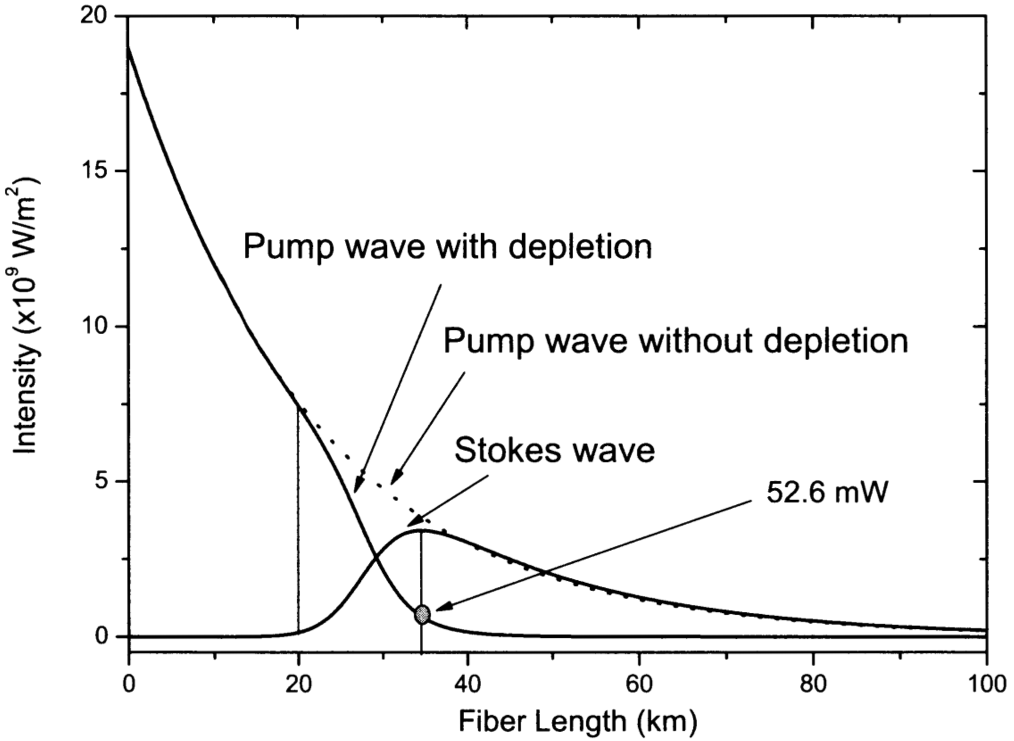
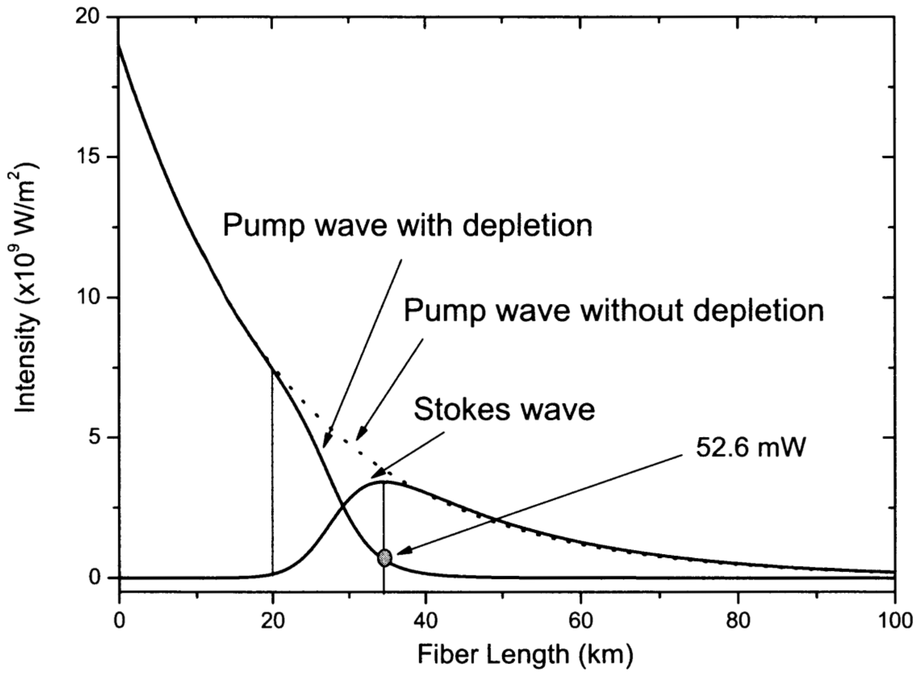


Figure 34: Pump and Stokes waves in an optical fiber whose pump depletion with the parameters above was taken into consideration and an input pump power above the threshold far stimulated scattering [1].

RAMAN SCATTERING

- After a propagation distance of 20 km, the Stokes wave increases very strongly and the depletion of the pump can be no longer neglected.
- At a distance of 29 km, the intensity of the Stokes exceeds that of the pump.
- After a distance of 34.6 km, the pump intensity is no longer strong enough to amplify the Stokes wave further.
- For longer distance, the attenuation in the fiber is stronger than the power transfer between pump and Stokes wave.



STIMULATED RAMAN SCATTERING

- If $A_{\text{eff}} = 50\ \mu\text{m}^2$ and $\alpha = 0.2\ \text{dB/km}$ as the representative values, P_{th} is about 570 mW near 1.55 μm. It is important to emphasize that Eq. (66) provides an order-of-magnitude estimate only as many approximations are made in its derivation. As channel powers in optical communication systems are typically below 10 mW, SRS is not a limiting factor for single-channel lightwave systems. However, it affects the performance of WDM systems considerably.
- Both SRS and SBS can be used to advantage while designing optical communication systems because they can amplify an optical signal by transferring energy to it from a pump beam whose wavelength is suitably chosen. SRS is especially useful because of its extremely large bandwidth. Indeed, the Raman gain is used routinely for compensating fiber losses in modern lightwave systems.

RAMAN AMPLIFIER

- Using Raman amplification, the whole transmission bandwidth of an optical fiber can be exploited. For instance, studies proposed the realization of a U-band (1625 – 1675 nm) amplifier with the Raman effect.
- In an optical fiber, multiple Raman processes can be carried out simultaneously. This means that a broadband amplification is possible if many pump lasers are combined. With such a concept, a Raman amplifier with a gain bandwidth of more than 100 nm was demonstrated.
- A particular advantage of the Raman amplification is the distributed amplification inherent in the process.

RAMAN AMPLIFIER

Hence, a Raman amplifier can be pumped in forward, backward, or both directions. The basic set up of a Raman amplifier pumped backwards is shown in the Figure below.

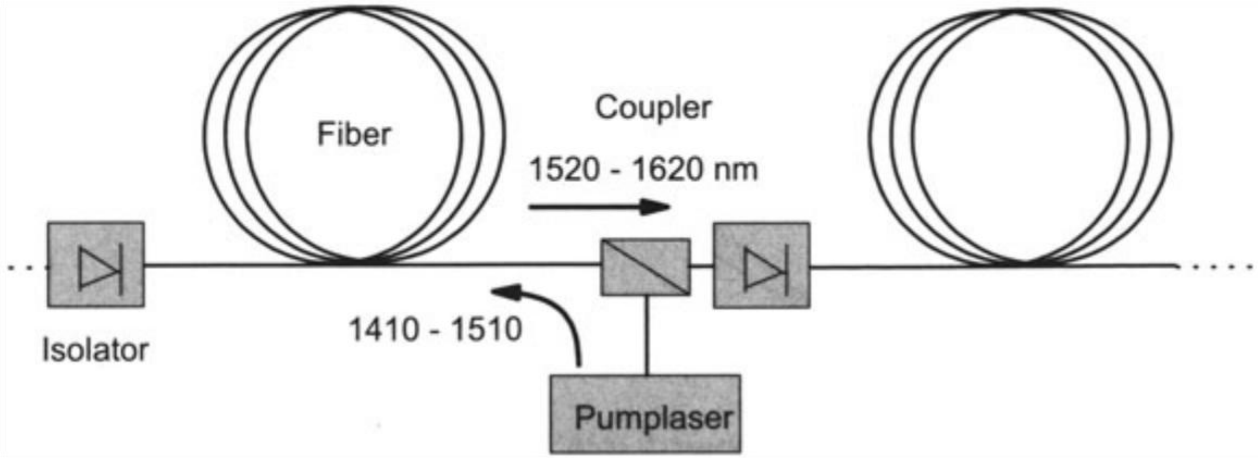


Figure 35: Schematic set-up of a backward-pumped Raman amplifier. [1]

RAMAN LASER

- A Raman laser is, in principle, a Raman amplifier in a resonator cavity. Due to the cavity, the threshold of stimulated scattering is decreased.
- The Raman scattering in the fiber generates new waves with different frequencies. Only the beam, or the wavelength, that hits the tuning mirror perpendicularly forms a Fabry-Perot resonator together with the first dichroic mirror.
- For compact laser devices, the mirrors can be replaced by a periodic alteration of the refractive index in the core of the fiber (fiber Bragg grating).

RAMAN LASER

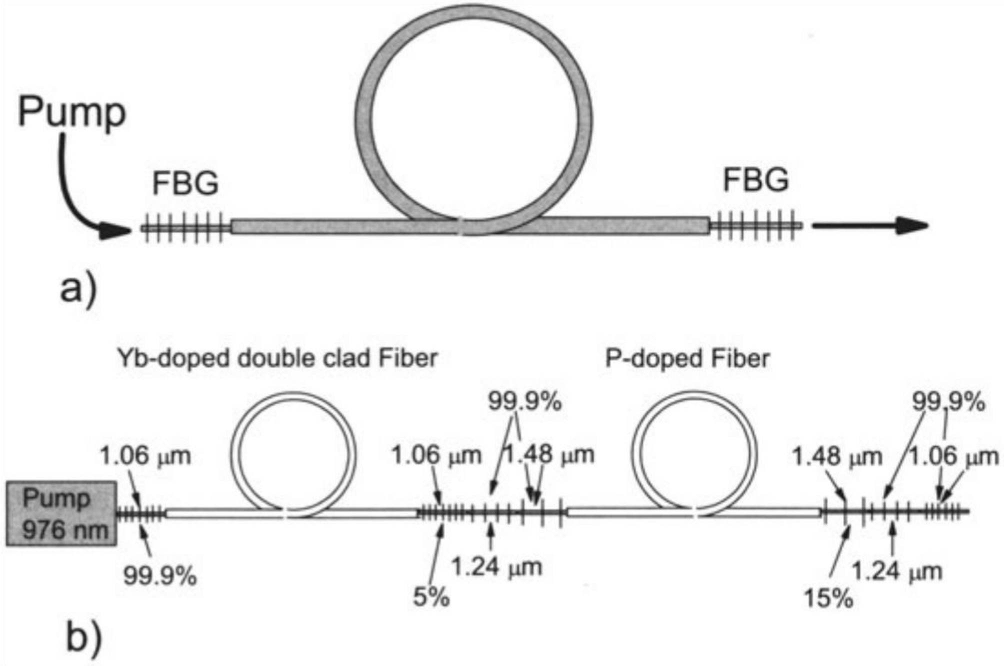


Figure 36: Fiber Raman laser with a resonator composed of fiber Bragg gratings (FBG = fiber Bragg grating, Yb = Ytterbium, P = Phosphosilicate). (a) Basic set up. (b) Diode-pumped fiber laser [1].

BRILLOUIN SCATTERING

- In the 1920s the French physicist Leon Brillouin investigated the scattering of light at acoustic waves.
- Brillouin scattering is cause by interaction between light and material. Different from Raman scattering no vibrations are involved. Density fluctuations of the medium are involved that can be seen as acoustic waves or phonons. The density fluctuations can be caused by acoustic wave.
- SBS decreases the SNR and increases BER (Bit-Error-Rate).
- When SBS exceeded the threshold, the signal can not be increased and all excess power is scattered back.

BRILLOUIN SCATTERING

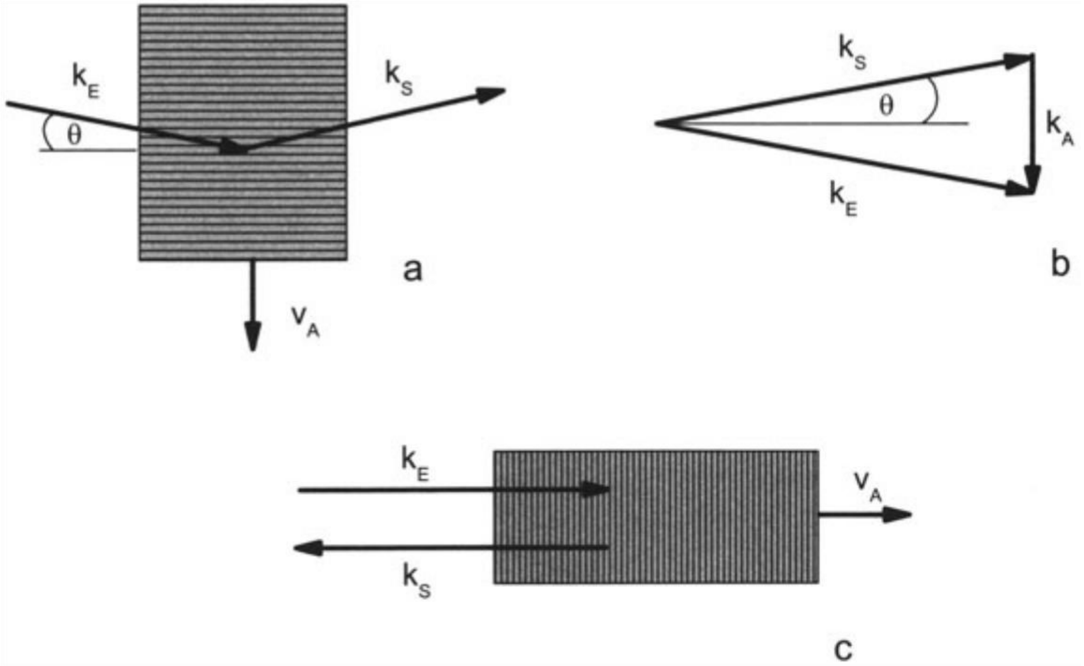


Figure 37: Scattering at a density modulation of an optical medium and corresponding vector diagram if the wave moves under an angle (a,b) or in the opposite direction to the density modulation (c) [1].

BRILLOUIN SCATTERING

Brillouin scattering is the result of the deviation of an optical wave on a density modulation in the material, caused by an acoustic wave with the sound velocity v_{A}.

The scattering process requires that energy as well as moment um are conserved during the interaction. The momentum conservation requires that the wave vectors satisfy:

$$k_{\mathrm{S}} = k_{\mathrm{E}} \pm k_{\mathrm{A}} \qquad f_{\mathrm{S}} = f_{\mathrm{E}} \pm f_{\mathrm{A}}$$

As expected from the Doppler effect, the frequency of the scattered wave decreases if the density modulation moves away and it increases if the density modulation comes nearer.

BRILLOUIN SCATTERING

In our case $k_S = k_E \pm k_A$. Assume that the absolute values of the wave vectors for the incident and the scattered wave are approximately equal ($|k_S| \approx |k_E|$),we obtain

$$|k_A| = 2|k_E| \sin\theta$$

Using $|k_A| = \omega/v_A$ and $|k_E| = 2\pi n/\lambda_E$, where v_A is the acoustic velocity, the acoustic frequency is given as:

$$f_A = \frac{2v_A n}{\lambda_E} \sin\theta \quad (67)$$

- In the forward direction, no frequency shift occurs because $\theta = 0$ and $f_A = 0$.
- If the incident and the scattered waves propagate in opposite directions, $\theta = 0$ and the frequency shift is at a maximum.

BRILLOUIN SCATTERING

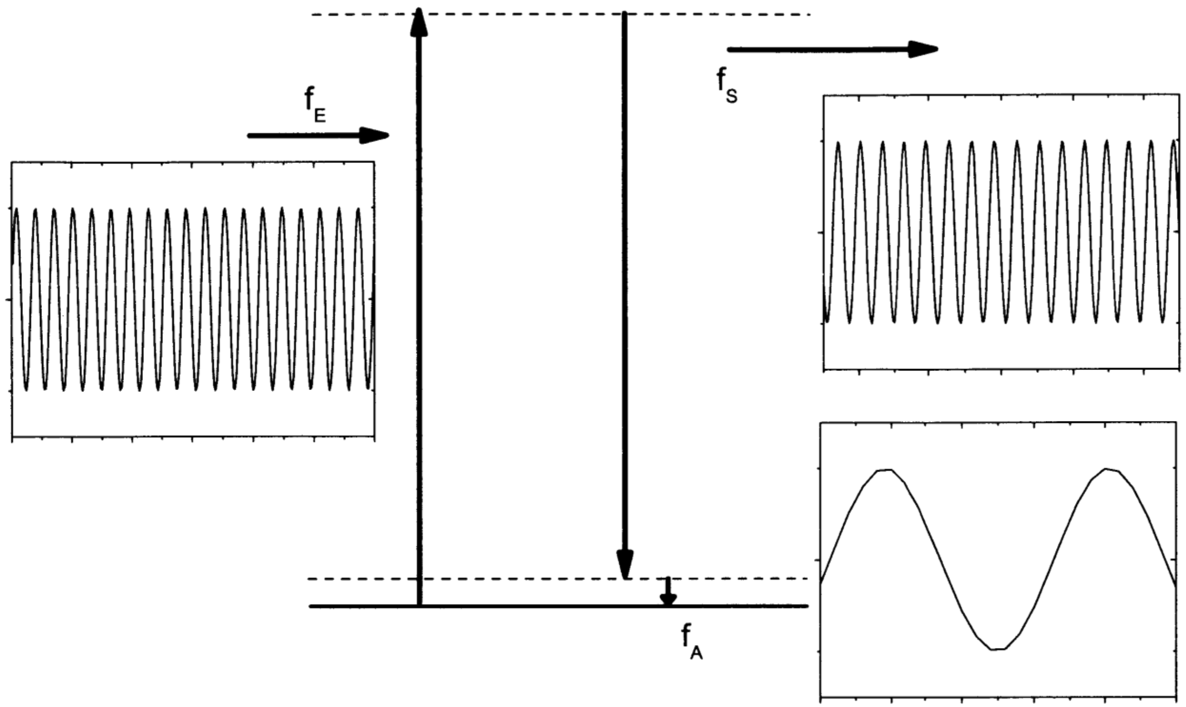


Figure 38: Quantum mechanical model of Brillouin scattering. One photon of the incident wave is annihilated and creates simultaneously a photon of the scattered wave and a phonon [1].

STIMULATED BRILLOUIN SCATTERING

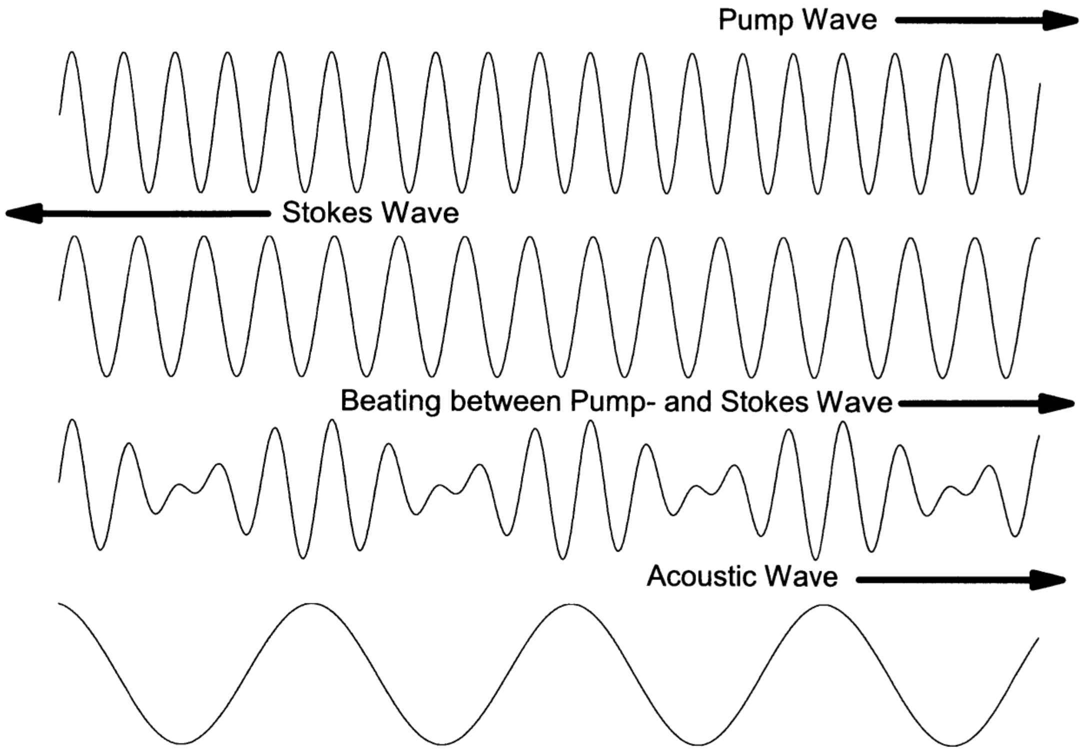


Figure 39: Generation of an acoustic wave due to the superposition between the pump wave, propagating in forward direction, and the backscattered Stokes wave [1].

STIMULATED BRILLOUIN SCATTERING

- If the intensity of the pump wave is so high that - as in the case of Raman scattering - the power transfer to the Stokes wave (generated at an arbitrary point in the medium) is higher than the attenuation it will experience, a stimulated process can occur. In this case the superposition between the **pump wave** (propagating in the forward direction) and the **backscattered Stokes wave** will cause a fading with a frequency that corresponds to the acoustic wave.
- Therefore, the fading leads to an amplification of the acoustic wave. A stronger acoustic wave causes a stronger Stokes wave which results in a stronger intensity modulation, and so on. Since the pump by itself is responsible for an amplification of the effect, the process is called stimulated.

STIMULATED BRILLOUIN SCATTERING check

Stimulated Brillouin scattering is the interaction between the pump wave, the generated Stokes wave and the acoustic wave in the fiber.

As a result, the beating term acts as source that increases the amplitude of the sound wave, which in turn increases the amplitude of the scattered wave, resulting in a positive feedback loop. SBS has its origin in this positive feedback, which ultimately can transfer all power from the pump to the scattered wave. The feedback process is governed by the following set of two coupled equations:

$$\frac{dI_{\mathrm{p}}}{dz} = -g_{\mathrm{B}} I_{\mathrm{p}} I_{\mathrm{s}} - \alpha I_{\mathrm{p}} \qquad (69)$$

$$\frac{dI_{\mathrm{s}}}{dz} = -g_{\mathrm{B}} I_{\mathrm{p}} I_{\mathrm{s}} + \alpha I_{\mathrm{s}} \qquad (70)$$

where I_{p} and I_{s} are the intensities of the pump and Stokes fields, g_{B} is the SBS gain, and α is the fiber losses.

STIMULATED BRILLOUIN SCATTERING

For small intensities, the pump intensity depends only on the fiber attenuation:

$$I_{\mathrm{p}}(z) = I_{\mathrm{p}}(0)\exp(-\alpha z) \tag{71}$$

The pump intensity at distance L is

$$I_{\mathrm{p}}(z) = I_{\mathrm{p}}(0)\int_0^L \exp(-\alpha z)\ dz = \frac{I_{\mathrm{p}}(0)}{\alpha}\left(1 - e^{-\alpha L}\right) = I_{\mathrm{p}}(0)L_{\mathrm{eff}} \tag{72}$$

where L_{eff} is the effective interaction length.

$$\frac{dI_{\mathrm{s}}}{dz} = \left[-g_{\mathrm{B}}I_{\mathrm{p}}(z) + \alpha\right]I_{\mathrm{s}} \tag{73}$$

The intensity of stokes wave in distance L is

$$I_{\mathrm{s}}(L) = I_{\mathrm{s}}(0)\exp\left(-\frac{g_{\mathrm{B}}P_0L_{\mathrm{eff}}}{A_{\mathrm{eff}} - \alpha L}\right) \tag{74}$$

STIMULATED BRILLOUIN SCATTERING

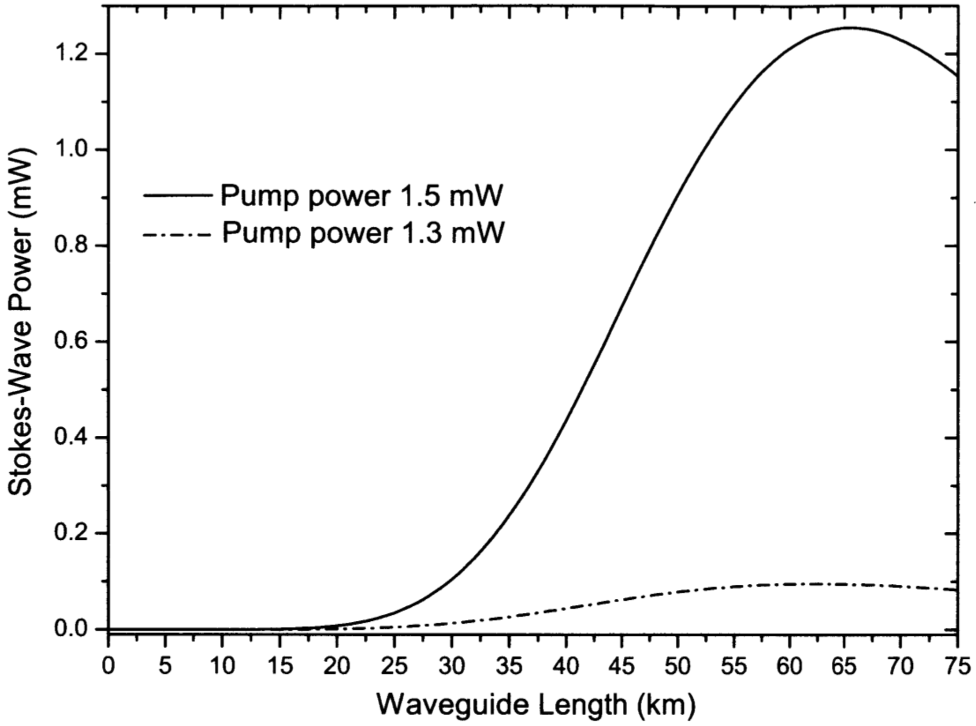


Figure 40: Power of the Stokes wave at the input of a polarization-maintaining fiber against the fiber length for two different pump powers [1].

THE BRILLOUIN GAIN

The Brillouin gain coefficient of a fiber is determined by three important parameters: the frequency shift between the pump and Stoke waves (f_A), the peak Brillouin gain ($g_{\mathrm{B}_\mathrm{max}}$) and the linewidth of the distribution (Δf_A).

The Brillouin gain coefficient has a very narrow bandwidth and its maximum determines the frequency shift or the frequency of the acoustic wave. The distribution is approximated by a Lorentzian function:

$$g_B(f) = \frac{g_{\mathrm{B}_\mathrm{max}}}{1 + [(f - f_\mathrm{A})/(\Delta f_\mathrm{A}/2)]^2} \tag{75}$$

For pulses with a temporal duration much longer than the phonon lifetime, the maximum of the Brillouin gain $g_{\mathrm{B}_\mathrm{max}}$ is

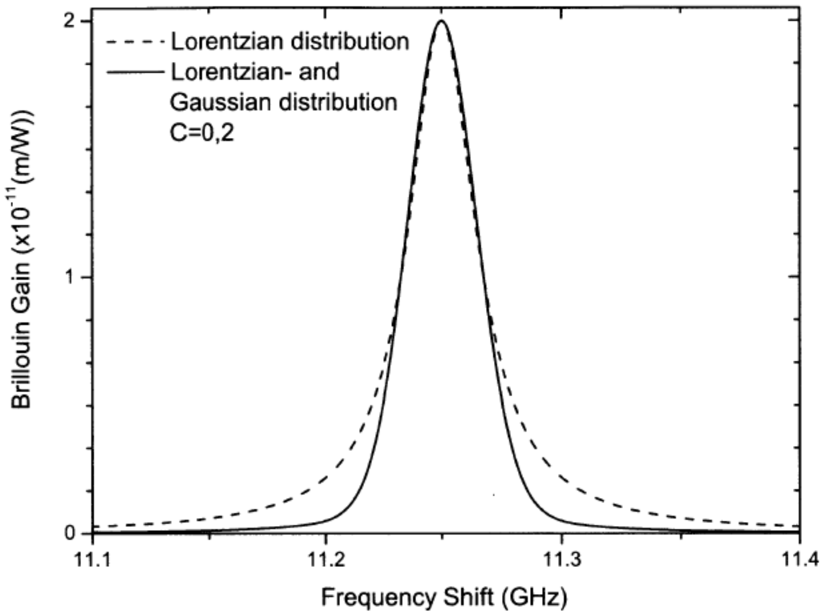
$$g_{\mathrm{B}_\mathrm{max}} = \frac{4\pi n^8 p^2}{c{\lambda_\mathrm{P}}^3 \rho f_\mathrm{A} \Delta f_\mathrm{A}}$$

where p is the elasto-optic constant and ρ is the material density.

THE BRILLOUIN GAIN

If the pump pulses are temporally short, their spectrum will be correspondingly large and the gain curve merges into a Gaussian distribution:

$$g_B(f) = \left\{ \frac{C}{1 + [(f - f_\mathrm{A})/(\Delta f_\mathrm{A}/2)]^2} + (1 - C)\exp\left[-\ln(2)\frac{(f - f_\mathrm{A})^2}{(\Delta f_\mathrm{A}/2)^2} \right] \right\} g_{\mathrm{B}_\mathrm{max}} \qquad (76)$$



THE BRILLOUIN GAIN

check

Figure below shows the Brillouin gain spectra at $\lambda_p = 1.525\ \mu\text{m}$ for three different kinds of single-mode silica fibers. Both the Brillouin shift υ_B and the gain bandwidth $\Delta\upsilon_B$ can vary from fiber to fiber because of the guided nature of light and the presence of dopants in the fiber core. The fiber labeled (a) in Fig. 40 has a core of nearly pure silica (germania concentration of about 0.3% per mole). The measured Brillouin shift $\upsilon_B = 11.25\ \text{GHz}$ is in agreement with Eq. (68). The Brillouin shift is reduced for fibers (b) and (c) of a higher germania concentration in the fiber core.

BRILLOUIN-GAIN SPECTRA

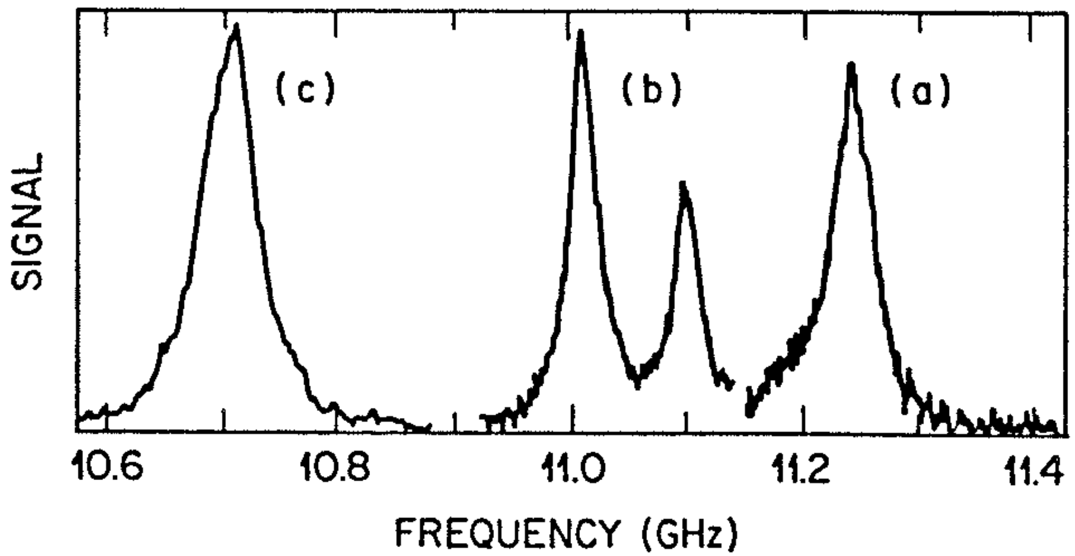


Figure 41: Brillouin-gain spectra measured using a 1.525 μm pump for three fibers with different germania doping: (a) silica-core fiber, (b) depressed-cladding fiber and (c) dispersion-shifted fiber. Vertical scale is arbitrary [3].

STIMULATED BRILLOUIN SCATTERING check

The doublepeak structure for fiber (b) results from inhomogeneous distribution of germania within the core. The gain bandwidth is larger than that expected for bulk silica ($\Delta \upsilon_B \approx 17$ MHz at $\lambda_p = 1.525$ μm). A part of the increase is due to the guided nature of acoustic modes in optical fibers. However, most of the increase in bandwidth can be attributed to variations in the core diameter along the fiber length. Because such variations are specific to each fiber, the SBS gain bandwidth is generally different for different fibers and can exceed 100 MHz; typical values are ~50 MHz for λ_p near 1.55 μm.

The peak value of the Brillouin gain in Eq. (75) occurs for $\Omega = \Omega_B$ and depends on various material parameters such as the density and the elasto-optic coefficient. For silica fibers $g_B \approx 5 \cdot 10^{-11}$ m/W. The threshold power level for SBS can be estimated by solving Eqs. (69) and (73) and finding at what value of I_p, I_s grows from noise to a significant level.

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STIMULATED BRILLOUIN SCATTERING check

The threshold power $P_{th} = I_p A_{eff}$, where A_{eff} is the effective core area, satisfies the condition:

$$g_B P_{th} L_{eff}/A_{eff} \approx 21 \quad (77)$$

where L_{eff} is the effective interaction length defined as

$$L_{eff} = \frac{1 - \exp(-\alpha L)}{\alpha} \quad (78)$$

and α represents fiber losses. For optical communication systems L_{eff} can be approximated by $1/\alpha$ as $\alpha L \gg 1$ in practice. Using $A_{eff} = \pi w^2$, where w is the spot size, P_{th} can be as low as 1 mW depending on the values of w and α. Once the power launched into an optical fiber exceeds the threshold level, most of the light is reflected backward through SBS. Clearly, SBS limits the launched power to a few milliwatts because of its low threshold.

STIMULATED BRILLOUIN SCATTERING

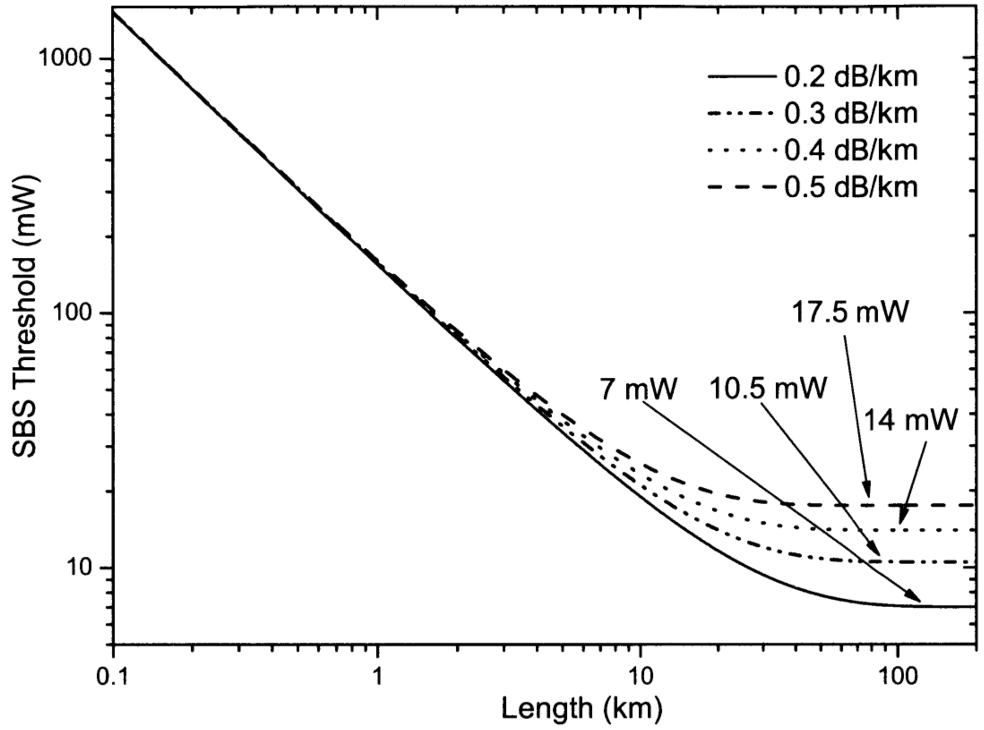

Figure 42: Threshold of stimulated Brillouin scattering versus fiber length in a standard single mode fiber with different attenuation ($K_B = 2$, $g_B = 2 \cdot 10^{-11}$ m/W, $A_{\text{eff}} = 80\ \mu\text{m}^2$) [1].

STIMULATED BRILLOUIN SCATTERING check

The preceding estimate of P_{th} applies to a narrowband CW beam as it neglects the temporal and spectral characteristics of the incident light. In a lightwave system, the signal is in the form of a bit stream.

For a single short pulse whose width is much smaller than the phonon lifetime, no SBS is expected to occur. However, for a highspeed bit stream, pulses come at such a fast rate that successive pulses build up the acoustic wave, similar to the case of a CWbeam, although the SBS threshold increases. The exact value of the average threshold power depends on the modulation format (RZ versus NRZ) and is typically ~5 mW. It can be increased to 10 mW or more by increasing the bandwidth of the optical carrier to >200 MHz through phase modulation. SBS does not produce interchannel crosstalk in WDM systems because the 10-GHz frequency shift is much smaller than typical channel spacing.

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