PT-Symmetry In Coupled Waveguide Systems

Integrated Photonics Course by Dr. Alina Karabchevsky

Presenter: Fyodor Morozko, Date: December 6, 2020

Basic Concepts of PT-symmetry

Non-Hermitian Hamiltonians with Real Spectrum

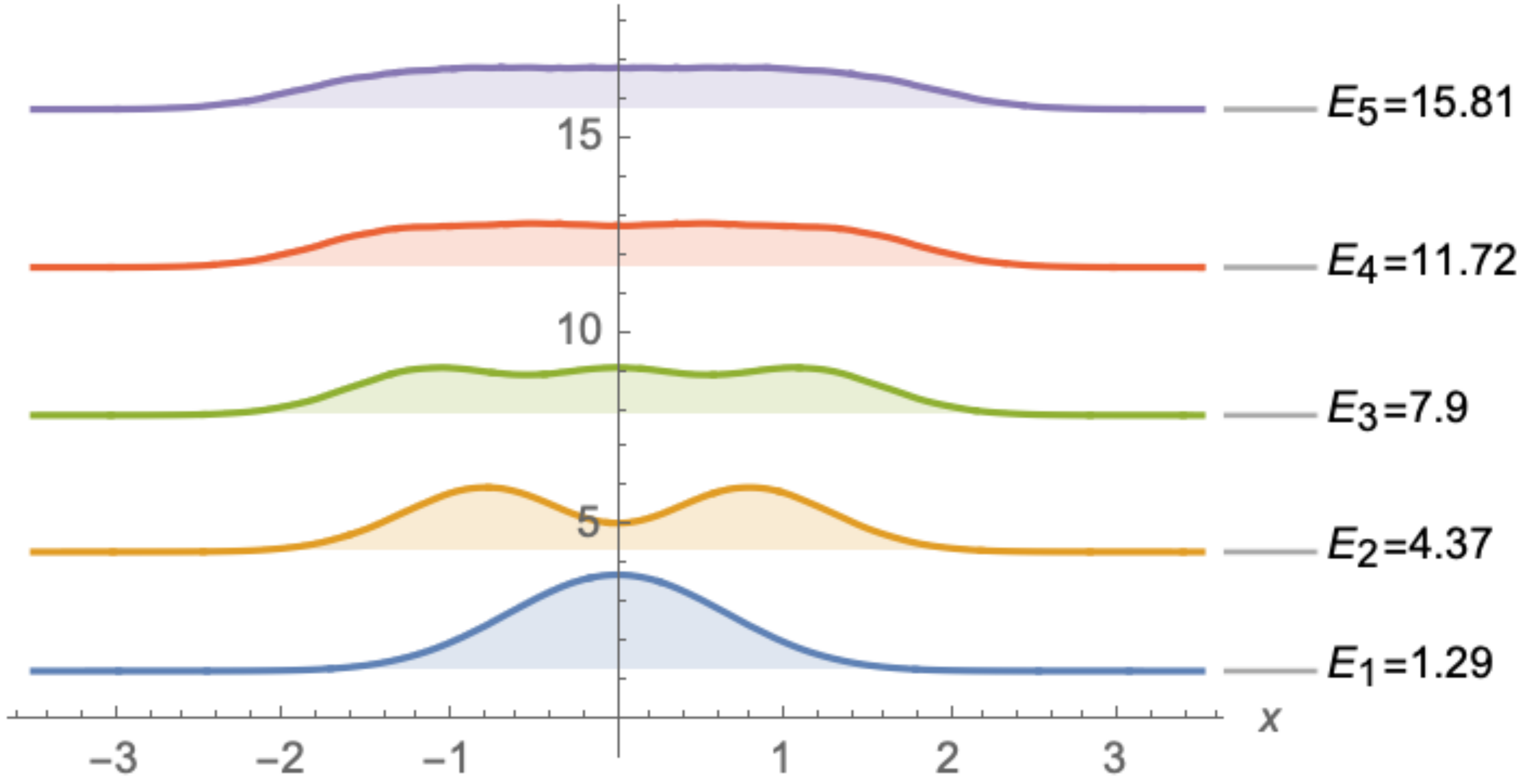
- In 1998, Bender and Böttcher in their seminal work considered the Hamiltonian

$$H = p^2 + x^2 + ix^3,$$

where p and x are momentum and coordinate operators, respectively.

- This Hamiltonian is non-Hermitian since $H^\dagger \neq H$, but appears to have entirely **real** and **positive** spectrum. Such Hamiltonians are referred to as *pseudo-Hermitian.*

- The authors claimed that the reality of the spectrum of H is due to **parity-time** (**PT**) **symmetry** i.e. invariance under simultaneous application of space and time reversal operators.



Eigenfunctions $|\psi_n|^2$ and eigenvalues E_n of the Hamiltonian H.

C. Bender, & S. Böttcher. Real spectra in non-Hermitian Hamiltonians having PT symmetry, *Phys. Rev. Lett.* **80**, 5243–5246 (1998)

Space and Time Reversal Operators

Parity, or space reversal, operator P changes sign of coordinate and momentum:

$$P : x \to -x, p \to -p.$$

Time reversal operator T changes sign of momentum and performs complex conjugation:

$$T : x \to x, p \to -p, i \to -i.$$

So simultaneous application of P and T operators results in coordinate reversal and complex conjugation:

$$PT : x \to -x, p \to p, i \to -i.$$

PT-symmetry Condition

- Generally, PT-symmetry requires that $[H, PT] = 0$.
- PT-symmetry of the Hamiltonian can be also stated in the form

$$H(p, r, t) = H^*(p, -r, -t).$$

- For the Hamiltonians of the form $H = \frac{p^2}{2m} + V(r)$ with complex potential $V(r) = V_R(r) + iV_I(r)$ the above requirement reduces to the condition

$$V(r) = V^*(-r).$$

Properties of Eigenstates and Eigenvalues

- Eigenstates of PT-symmetric Hamiltonians are ***not*** *orthogonal* under conventional (Dirac) inner product:

$$\langle \psi_n | \psi_m \rangle \neq 0$$

for $n \neq m$.

- The eigenstates of PT-symmetric Hamiltonians may be or may not be PT-symmetric.

- If the eigenstates of the PT-symmetric Hamiltonian are PT-invariant, the eigenvalues are real:

$$\psi_n(r) = \psi_n^*(-r) \leftrightarrow E_n = E_n^*.$$

- If the eigenstates of the PT-symmetric Hamiltonian are complex, the eigenstates are *essentially* non-PT-invariant:

$$\psi_n(r) \neq \psi_n^*(-r) \leftrightarrow E_n \neq E_n^*.$$

Phase Transition

- When varying some parameters, the system can go from the phase with PT-symmetric eigenfunctions to the phase with non-PT-symmetric functions.
- When a system passes from PT-symmetric to PT-symmetry-broken phase it experiences **phase transition** related to spontaneous PT symmetry breaking**.**
- A point in parameter space at which phase transition occurs is called **exceptional point** (EP).
- At an EP not only the eigenvalues coincide, but also the eigenvectors.
- The eigenvectors at EP become completely parallel so the Hamiltonian is defective and not diagonalizable.

Example: Two-Level System

- The 2×2 non-Hermitian Hamiltonian

$$\hat{H} = \begin{pmatrix} \omega_1 - i\gamma_1 & \kappa \\ \kappa & \omega_2 - i\gamma_2 \end{pmatrix}$$

describes a two-component system with complex frequencies $\omega_1 - \gamma_1, \omega_2 - \gamma_2$. γ_1, γ_2 are the corresponding gain/loss coefficients, κ is the coupling strength between the components.

- The eigenvalues of $\hat{H}$ are

$$\omega_\pm = \omega_0 - i\chi + \sqrt{\kappa^2 + \Gamma^2},$$

where $\omega_0 = (\omega_1 + \omega_2)/2, \chi = (\gamma_1 + \gamma_2)/2$, and $\Gamma = \delta + i\beta$ with $\delta = (\omega_1 - \omega_2)/2$ and $\beta = (\gamma_1 - \gamma_2)/2$.

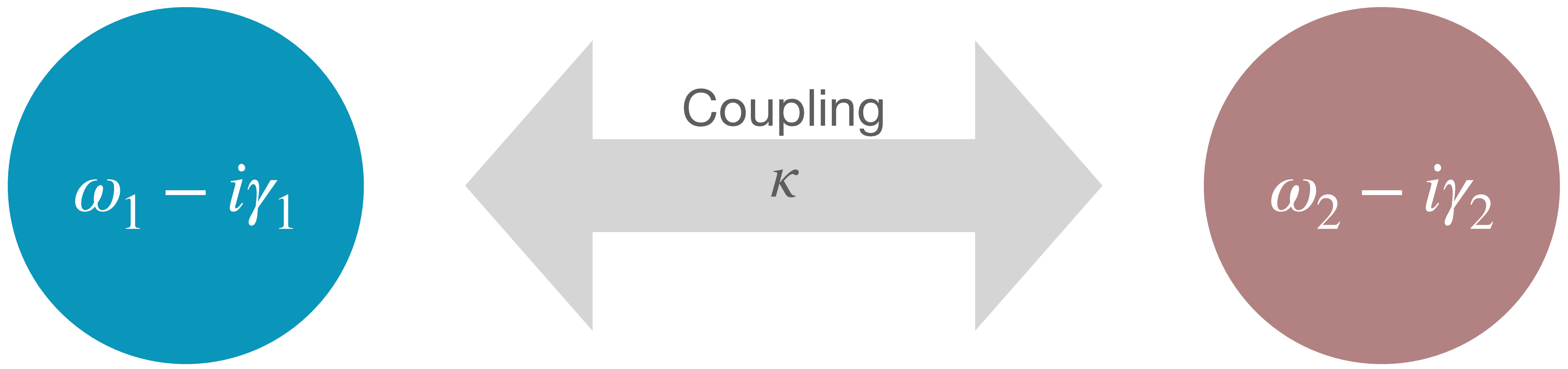
- The necessary (but not sufficient!) conditions for the eigenvalues to be real are

  (i) $\chi = 0 \leftrightarrow \gamma_1 = -\gamma_2,$

  (ii) $\kappa^2 + \Gamma^2 \geq 0 \rightarrow \delta = 0 \leftrightarrow \omega_1 = \omega_2.$

- Under the necessary conditions of $\gamma_1 = -\gamma_2 = \gamma$ and $\omega_1 = \omega_2 = \omega$ the Hamiltonian becomes

$$\hat{H} = \begin{pmatrix} \omega - i\gamma & \kappa \\ \kappa & \omega + i\gamma \end{pmatrix}.$$



Schematics of the two-component system

Ş. Özdemir *et al.* Parity–time symmetry and exceptional points in photonics. *Nature Materials* **18**, 783–798 (2019)

Example: Two-Level System

- Eigenvalues of the Hamiltonian

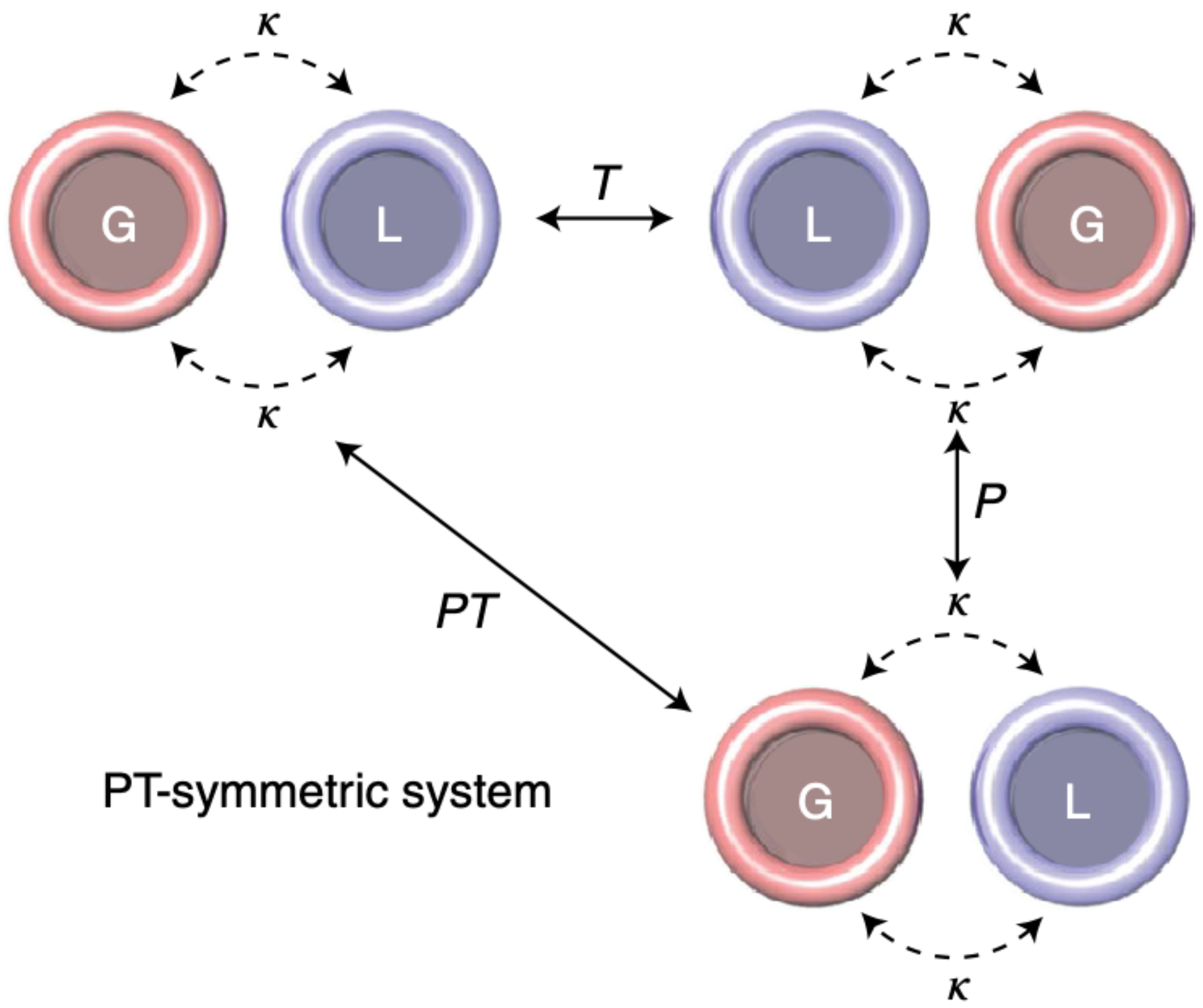
$$\hat{H} = \begin{pmatrix} \omega - i\gamma & \kappa \\ \kappa & \omega + i\gamma \end{pmatrix}$$

are

$$\omega_{\pm} = \omega \pm \sqrt{\kappa^2 - \gamma^2}.$$

- The system is PT-symmetric since it stays invariant after action of $\hat{P}\hat{T}$ operator: $\hat{P}$ exchanges the structures spatially while $\hat{T}$ turns gain into loss and vice versa.

- The eigenvalues are real when $\gamma/\kappa \leq 1$ and are complex when $\gamma/\kappa \geq 1$.



PT-symmetric system of two coupled structures with coupling strength κ, one with loss (L) and one with gain (G) compensating the loss of the other.

Ş. Özdemir *et al.* Parity–time symmetry and exceptional points in photonics. *Nature Materials* **18**, 783–798 (2019)

Example: Two-Level System

- In **PT-symmetric** phase ($\gamma/\kappa \le 1$):

$$\omega_\pm = \omega \pm \kappa \cos\theta,\ |\phi_\pm\rangle = \begin{pmatrix} 1 \\ \pm e^{\pm i\theta} \end{pmatrix}$$

with $\theta = \sin^{-1}(\gamma/\kappa)$.

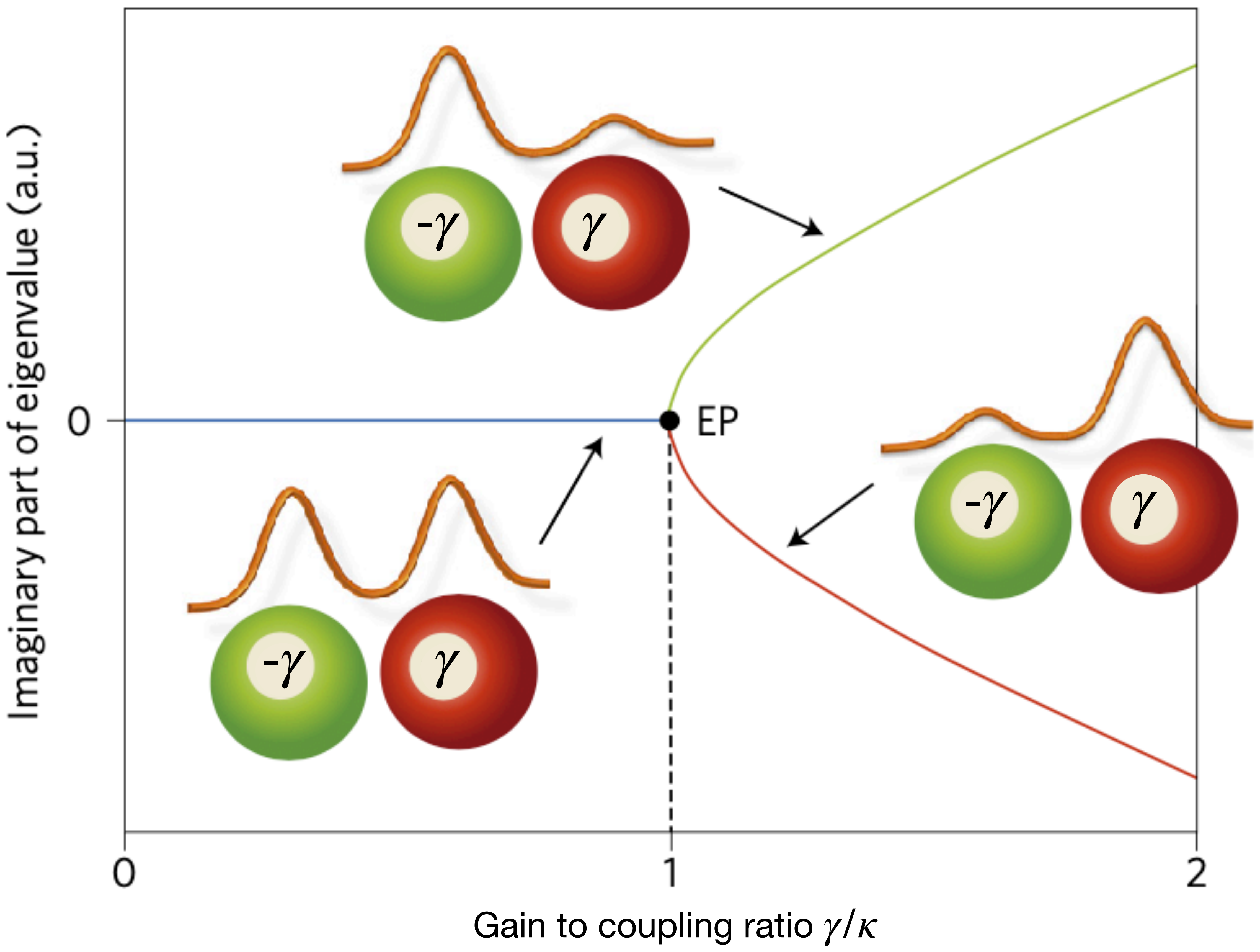
- In **PT-broken** phase $\gamma/\kappa \ge 1$:

$$\omega_\pm = \omega \pm i\kappa \sinh\theta,\ |\phi_\pm\rangle = \begin{pmatrix} 1 \\ ie^{\pm\theta} \end{pmatrix}$$

with $\theta = \cosh^{-1}(\gamma/\kappa)$.

- At the **exceptional point**:

$$\omega_\pm = \omega,\ |\phi_\pm\rangle = \begin{pmatrix} 1 \\ i \end{pmatrix}.$$



Adapted from R. El-Ganainy *et al.* Non-Hermitian physics and PT symmetry. Nature Physics **14**, 11–19 (2018).

PT-symmetry in Optics

Analogy between QM and Optics

- Schrödinger equation:

$$i\hbar\frac{\partial\psi(r,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right)\psi(r,t).$$

- Paraxial diffraction equation for slowly-varying envelope E in scalar approximation:

$$i\frac{\partial E(r)}{\partial z} = \frac{1}{2n_0k_0}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + V(r)\right)E(r),$$

where $V(x,y,z) = k_0^2(\varepsilon(x,y,z) - n_0^2)$, $k_0 = \frac{2\pi}{\lambda}$, n_0 is reference index.

- PT symmetry condition in optics

$$\varepsilon(x,y,z) = \varepsilon^*(-x,-y,-z)$$

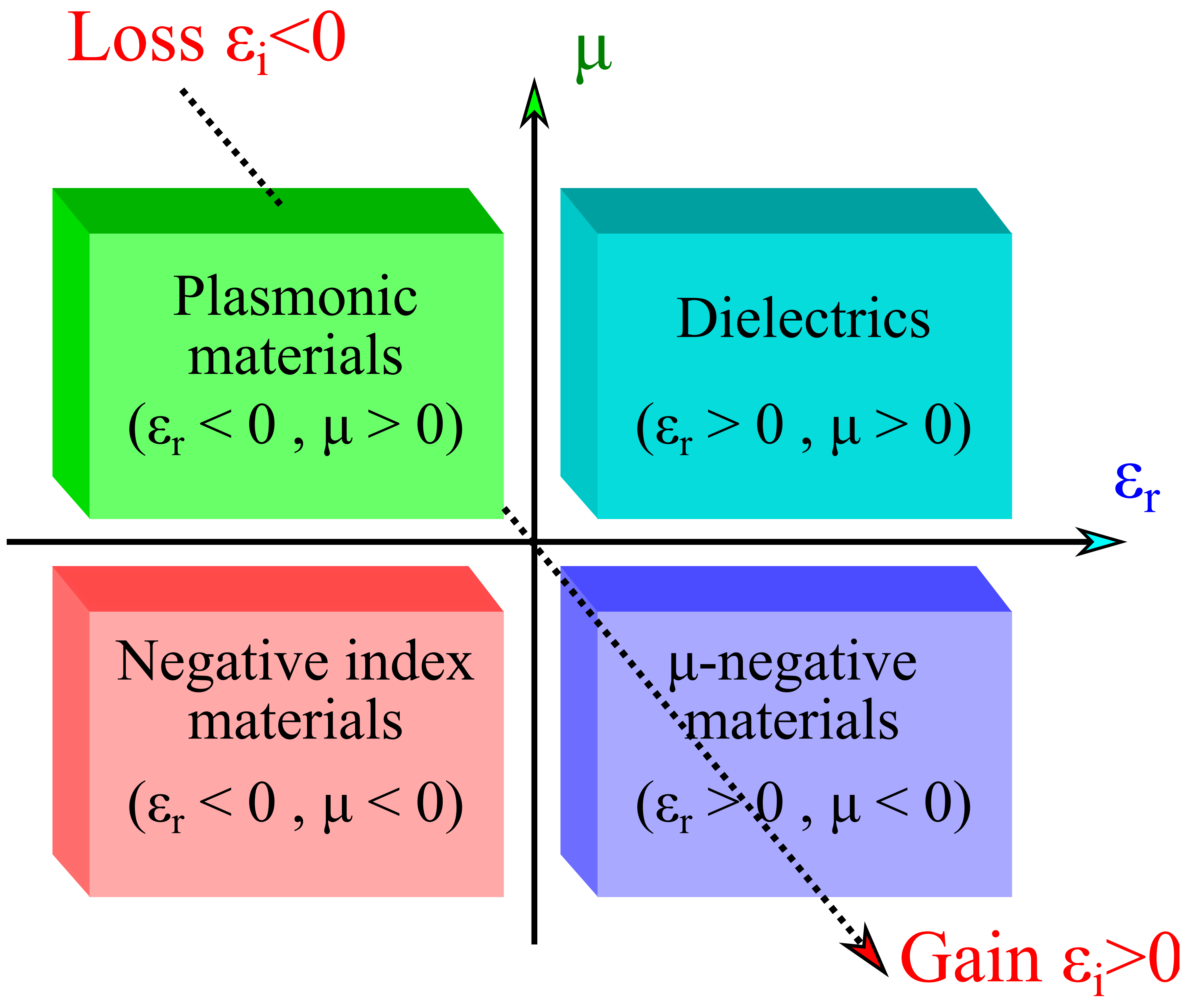
or

$$\varepsilon_r(r) = \varepsilon_r(-r),\ \ \varepsilon_i(r) = -\varepsilon_i(-r)$$

while $\varepsilon(r) = \varepsilon_r(r) + i\varepsilon_i(r)$.

Overview of Optical Materials

- Real part of electric permittivity ε_r and magnetic permeability μ form four quadrants that represent the entire range of the electromagnetic response.
- All four quadrants can be covered by specifically designed materials without taking the imaginary part into consideration.
- The positive/negative sign of the imaginary part of permittivity ε_i represents gain/loss in the materials.
- Therefore, adding non-Hermiticity (i.e. gain and/or loss) provides additional degree of freedom in designing electromagnetic response.



Schematics of materials characterization in terms of loss and gain.
From A. Karabchevsky, A. Novitsky & F. Morozko. Chapter 18: Purcell Effect In PT-Symmetric Waveguides. in *Chirality, Magnetism and Magnetoelectricity: Separate Phenomena and Joint Effects in Metamaterial Structures* (Springer Nature, 2021).

Fundamental Limitations on Realization of PT Symmetry

- Any physical dielectric function must be **causal**. So it satisfies **Kramers–Kronig** relations:

$$\varepsilon_r(\omega, r) = 1 + \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{\varepsilon_i(\omega', r)}{\omega' - \omega}d\omega'. \quad (1)$$

- PT-symmetry condition $\varepsilon_r(r) = \varepsilon_r(-r),\ \varepsilon_i(r) = -\varepsilon_i(-r)$ implies that

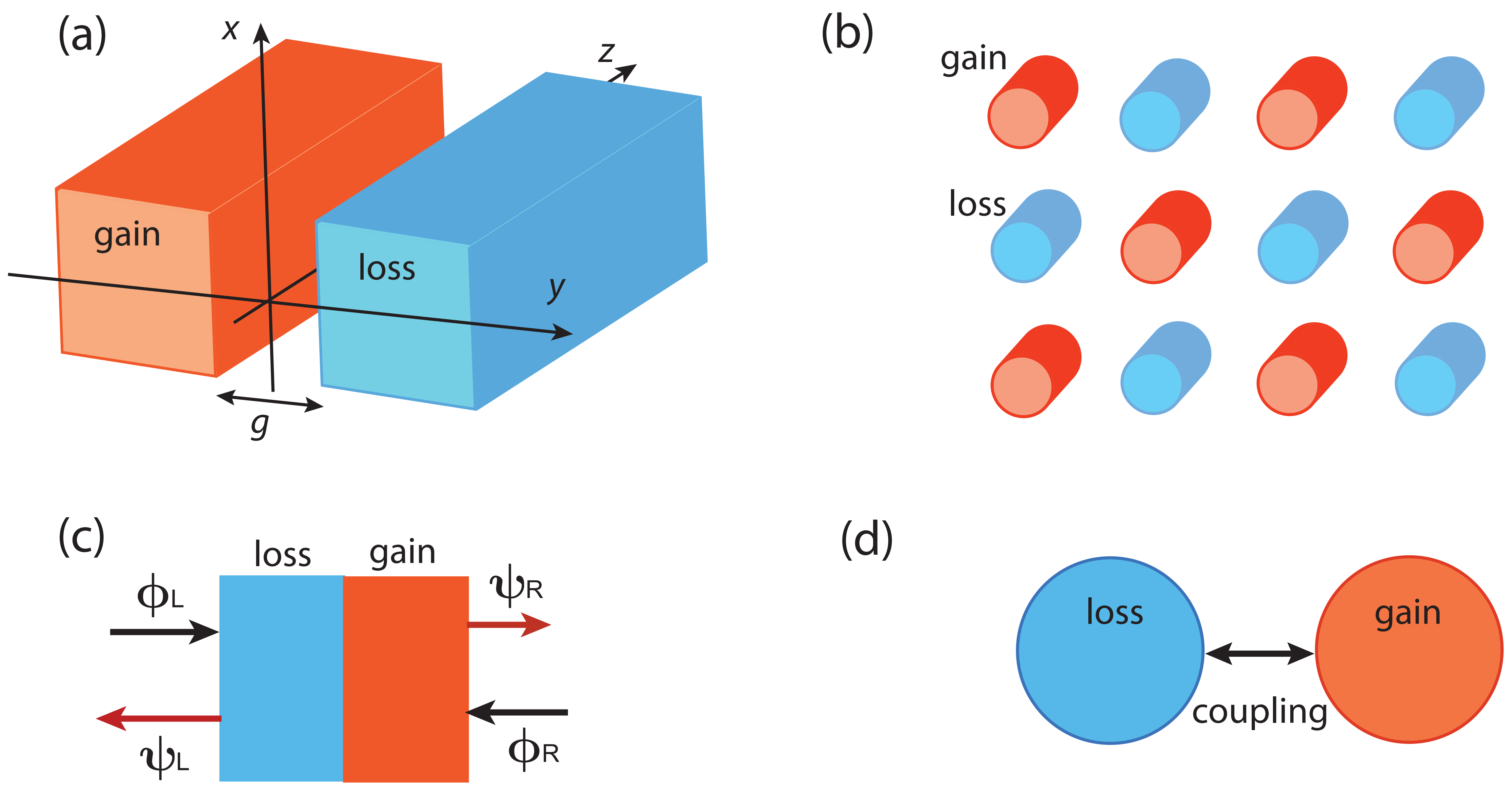
$$\varepsilon_r(\omega, -r) = 1 + \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{\varepsilon_i(\omega', -r)}{\omega' - \omega} = 1 - \frac{1}{\pi}\mathrm{P.V.}\int_{-\infty}^{\infty}\frac{\varepsilon_i(\omega', r)}{\omega' - \omega}d\omega'. \quad (2)$$

- (1) and (2) can be equal to one another for all frequencies only if $\varepsilon_i(\omega, r) = 0$ at all ω.
- In any nontrivial PT-symmetric structure $\varepsilon_i \neq 0$. So the relation *cannot* be satisfied for all frequencies.
- Actually, *PT-symmetry condition can be realized only with a* ***discrete*** *set of frequencies*.

A. Zyablovsky, *et al.* PT-symmetry in optics. *Uspekhi Fizicheskih Nauk* **184**, 1177–1198 (2014).

Overview of PT-Symmetric Photonic Devices

PT-symmetric Structures	Reported Effects
Coupled waveguides	• Loss induced transparency • Asymmetric power oscillations
Photonic lattices	• Beam splitting • Band merging • Secondary emissions
Multilayer systems	• Unidirectional invisibility • Coherent perfect absorption and lasing
Microresonators	• Non-reciprocal light transmission • Ultra sensitive sensing • Suppression of spontaneous relaxation rate



Schematics of state-of-art PT-symmetric structures: (a) coupled waveguides, (b) photonic lattices, (d) multilayer systems, (d) microresonators From A. Karabchevsky, A. Novitsky & F. Morozko. Chapter 18: Purcell Effect In PT-Symmetric Waveguides. in *Chirality, Magnetism and Magnetoelectricity: Separate Phenomena and Joint Effects in Metamaterial Structures* (Springer Nature, 2021).

Coupled Waveguide Systems

Coupled Waveguide Systems

- The field in the coupled waveguide system can be described in terms of coupled modes

$$E(x, y, z) = g(z)E_g(x, y) + l(z)E_l(x, y),$$

where E_g and E_l are modes of isolated gain and loss channels, g and l are complex amplitudes.

- The evolution of the amplitudes g and l is governed by the system of coupled equations:

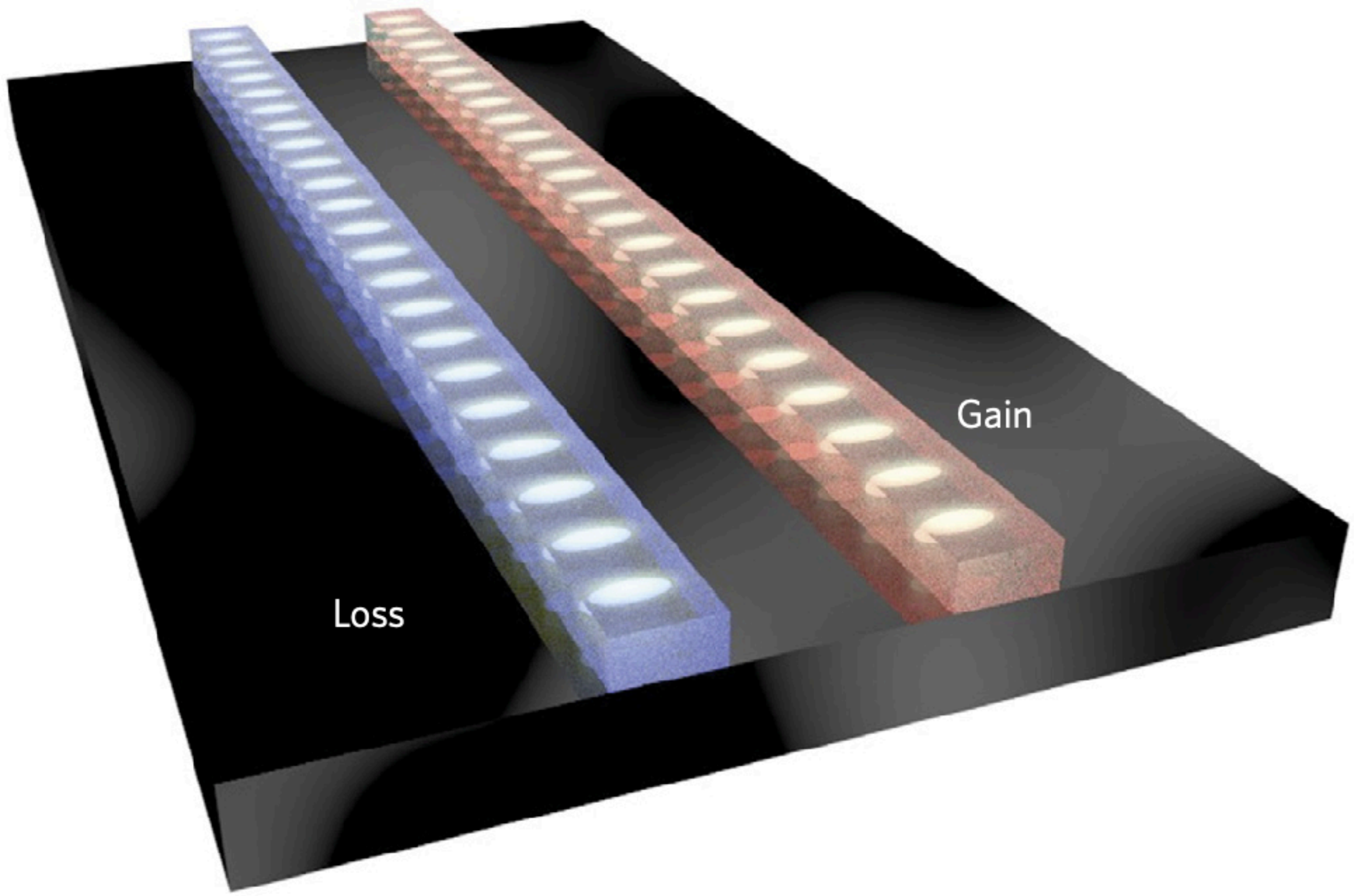
$$i\frac{d}{dz}\begin{pmatrix} g \\ l \end{pmatrix} = \begin{pmatrix} \beta + \delta + i\alpha/2 & \kappa \\ \kappa & \beta + \delta - i\alpha/2 \end{pmatrix}\begin{pmatrix} g \\ l \end{pmatrix},$$

where β is a propagation constant, κ is a coupling coefficient, δ is a correction to the propagation constant, α is an effective gain (or loss).

- We can assign to this system the Hamiltonian

$$\hat{H} = \begin{pmatrix} \beta + \delta - i\alpha/2 & \kappa \\ \kappa & \beta + \delta - i\alpha/2 \end{pmatrix},$$

which is the considered Hamiltonian of a two-level PT-symmetric system.



From R. El-Ganainy *et al.* Non-Hermitian physics and PT symmetry. *Nature Physics* **14,** 11–19 (2018).

Eigenmodes and Propagation Constants

- In **PT-symmetric** phase $\alpha/2\kappa \leq 1$:

$$\beta_{1,2} = \mathrm{Re}(\beta + \delta) \pm \kappa \cos\theta,$$

$$E_{1,2}(x,y) = E_g(x,y) \pm e^{\pm i\theta} E_l(x,y)$$

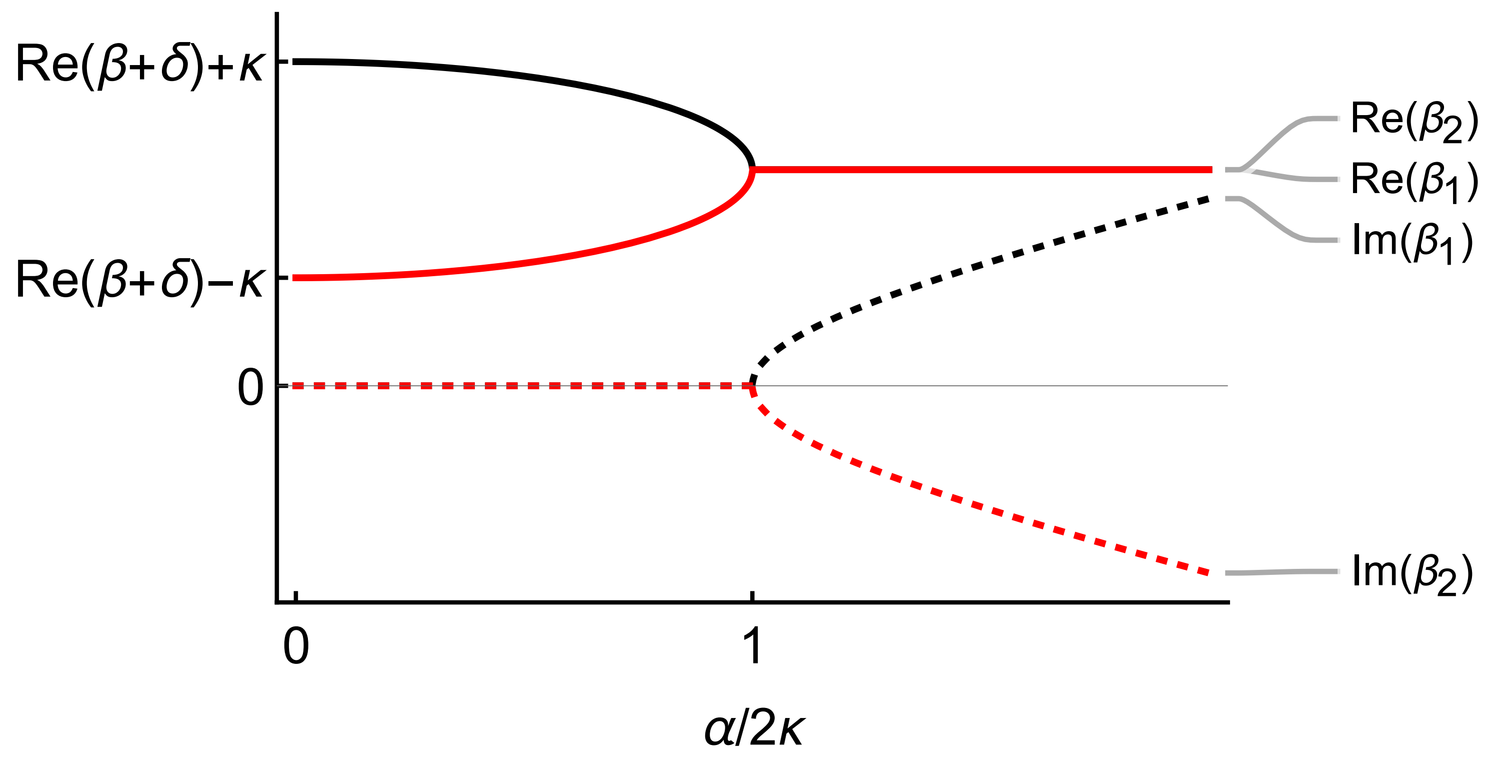
with $\theta = \sin^{-1}(\alpha/2\kappa)$.

- In **PT-broken** phase $\alpha/2\kappa \geq 1$:

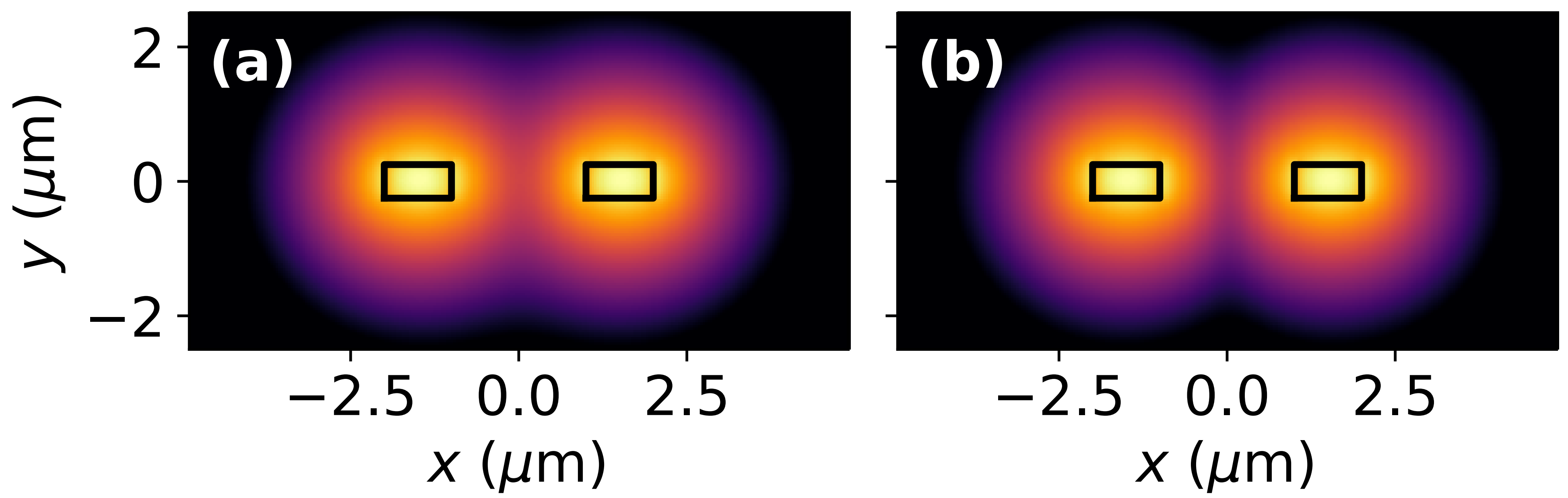
$$\beta_{1,2} = \mathrm{Re}(\beta + \delta) \pm i\kappa \sinh\theta,$$

$$E_{1,2}(x,y) = E_g(x,y) + ie^{\mp\theta} E_l(x,y)$$

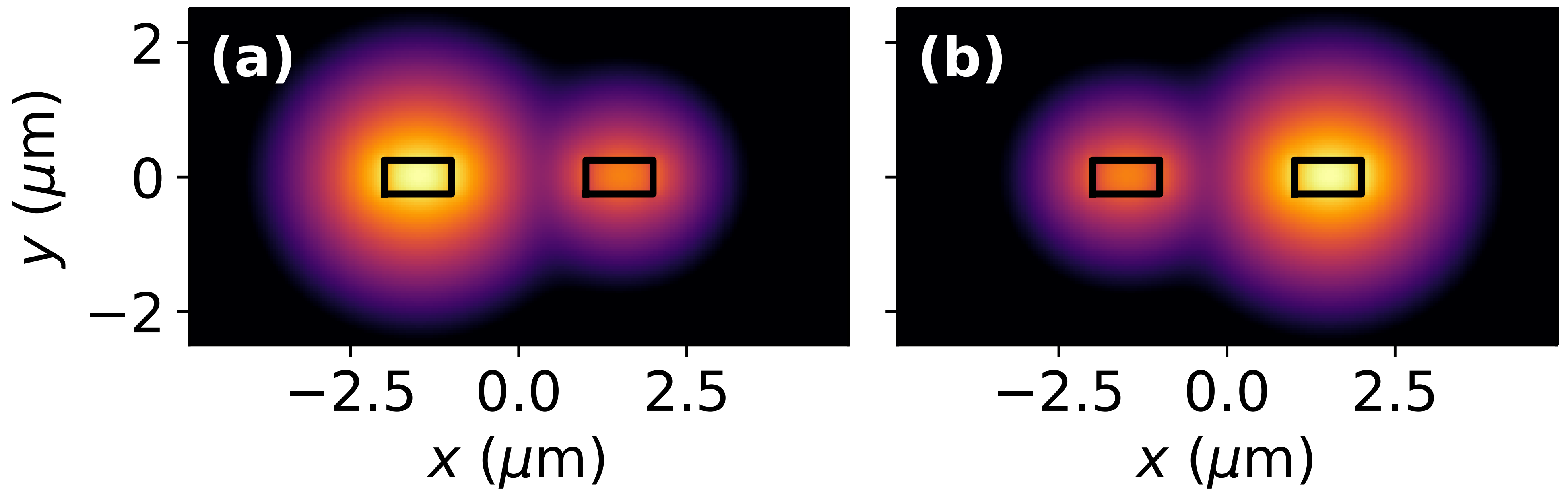
with $\theta = \cosh^{-1}(\alpha/2\kappa)$.



Propagation constants of the supermodes



Numerically calculated mode intensity profiles in **PT-symmetric** phase: (a) “Even” mode, (b) “Odd” mode



Numerically calculated mode intensity profiles in **PT-broken** phase: (a) Gain mode, (b) Loss mode

A. Karabchevsky, A. Novitsky & F. Morozko. Chapter 18: Purcell Effect In PT-Symmetric Waveguides in *Chirality, Magnetism and Magnetoelectricity: Separate Phenomena and Joint Effects in Metamaterial Structures* (Springer Nature, 2021)

PT-symmetry in Passive Systems

- PT-symmetry breaking can also occur in entirely passive dual systems where one channel exhibits loss while the other is lossless:

$$i\frac{d}{dz}\begin{pmatrix} g \\ l \end{pmatrix} = \begin{pmatrix} \beta & \kappa \\ \kappa & \beta - i\alpha \end{pmatrix}\begin{pmatrix} g \\ l \end{pmatrix}.$$

- In such a configuration PT symmetry can be reestablished through the gauge transformation $\begin{pmatrix} g \\ l \end{pmatrix} = e^{-\alpha z/2}\begin{pmatrix} \tilde{g} \\ \tilde{l} \end{pmatrix}$:

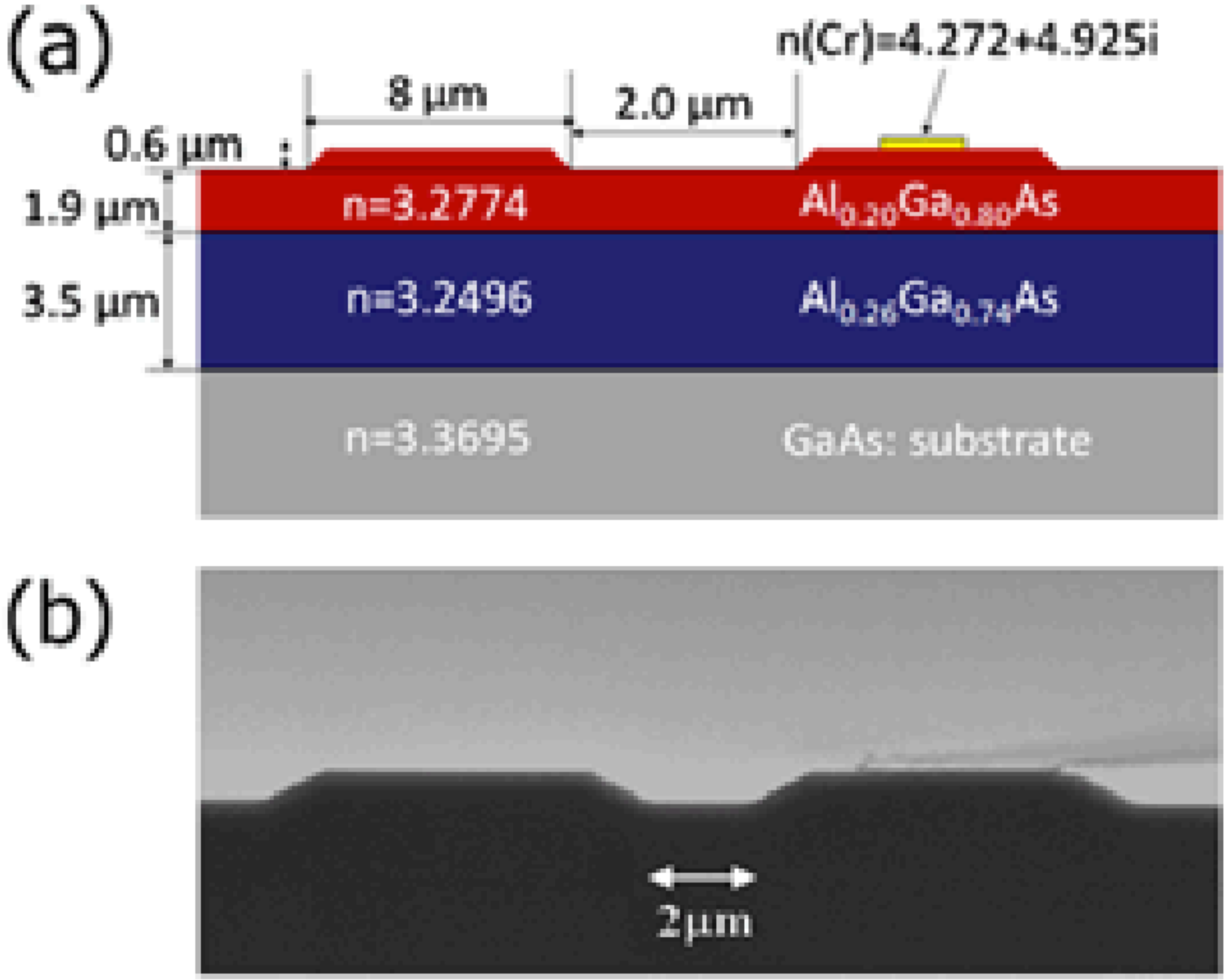
$$i\frac{d}{dz}\begin{pmatrix} \tilde{g} \\ \tilde{l} \end{pmatrix} = \begin{pmatrix} \beta + i\alpha/2 & \kappa \\ \kappa & \beta - i\alpha/2 \end{pmatrix}\begin{pmatrix} \tilde{g} \\ \tilde{l} \end{pmatrix}.$$

- The new “effective” Hamiltonian

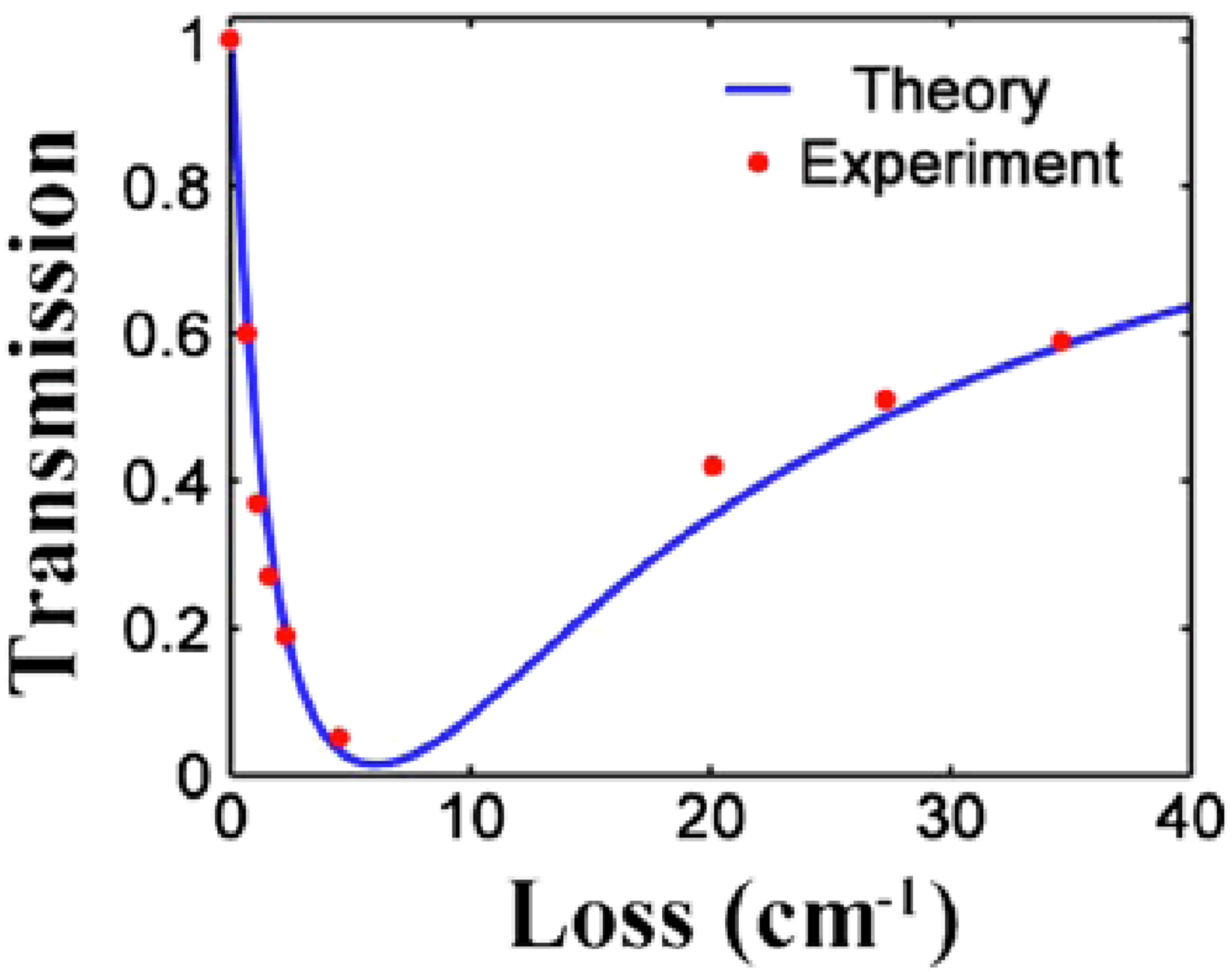
$$\tilde{H} = \begin{pmatrix} \beta + i\alpha/2 & \kappa \\ \kappa & \beta - i\alpha/2 \end{pmatrix}$$

is PT-symmetric.

PT Symmetry Breaking in a Passive Coupled Waveguide System



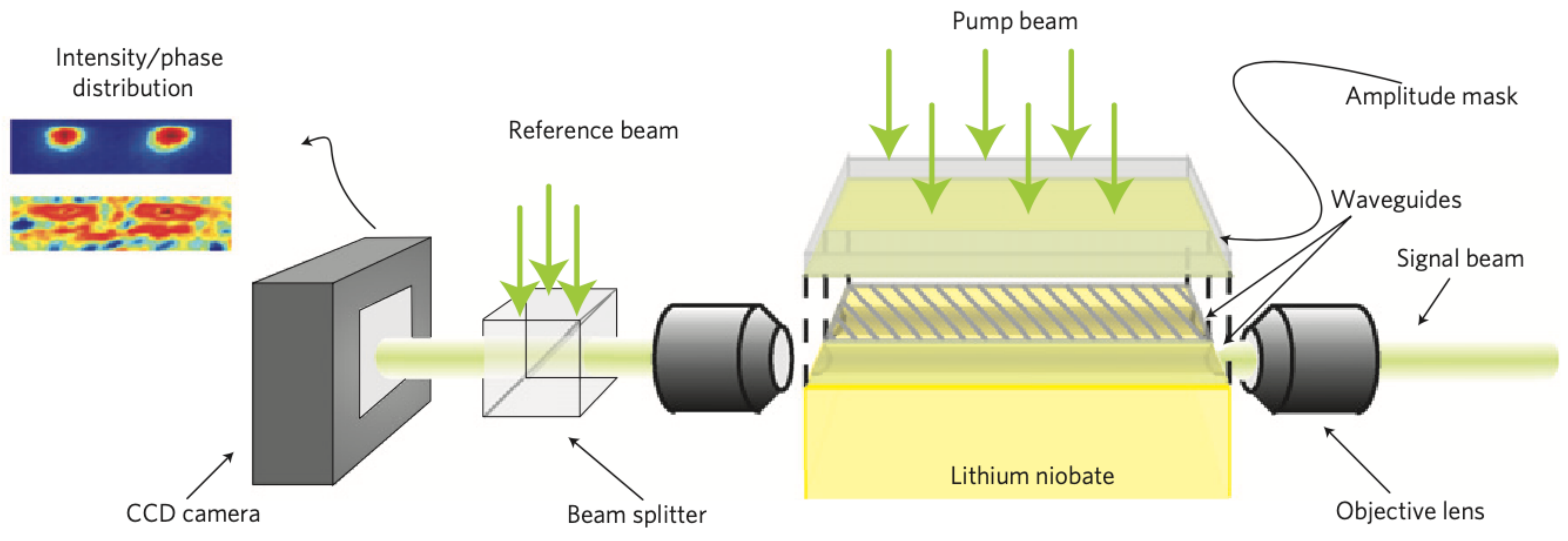
(a) Design details and complex refractive index distribution.
(b) Scanning electron microscopy picture of the finalized passive PT device with the Cr stripe shown on the right.



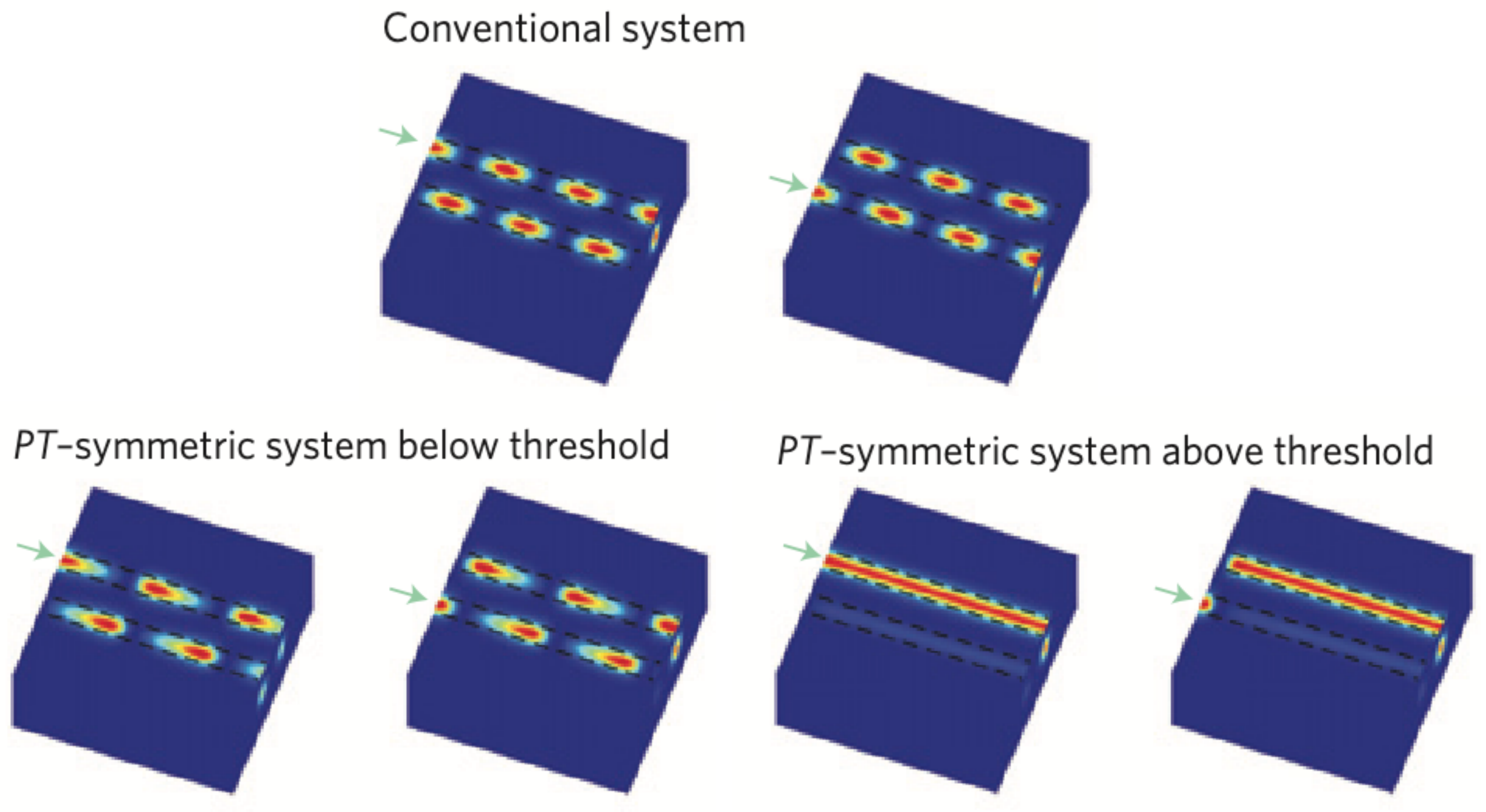
Loss-induced transparency due to spontaneous passive PT symmetry breaking. Output transmission of a passive PT complex system as the loss in the lossy waveguide arm is increased. The transmission attains a minimum at 6 cm^{-1}. When *more loss* is added the *transmission grows*.

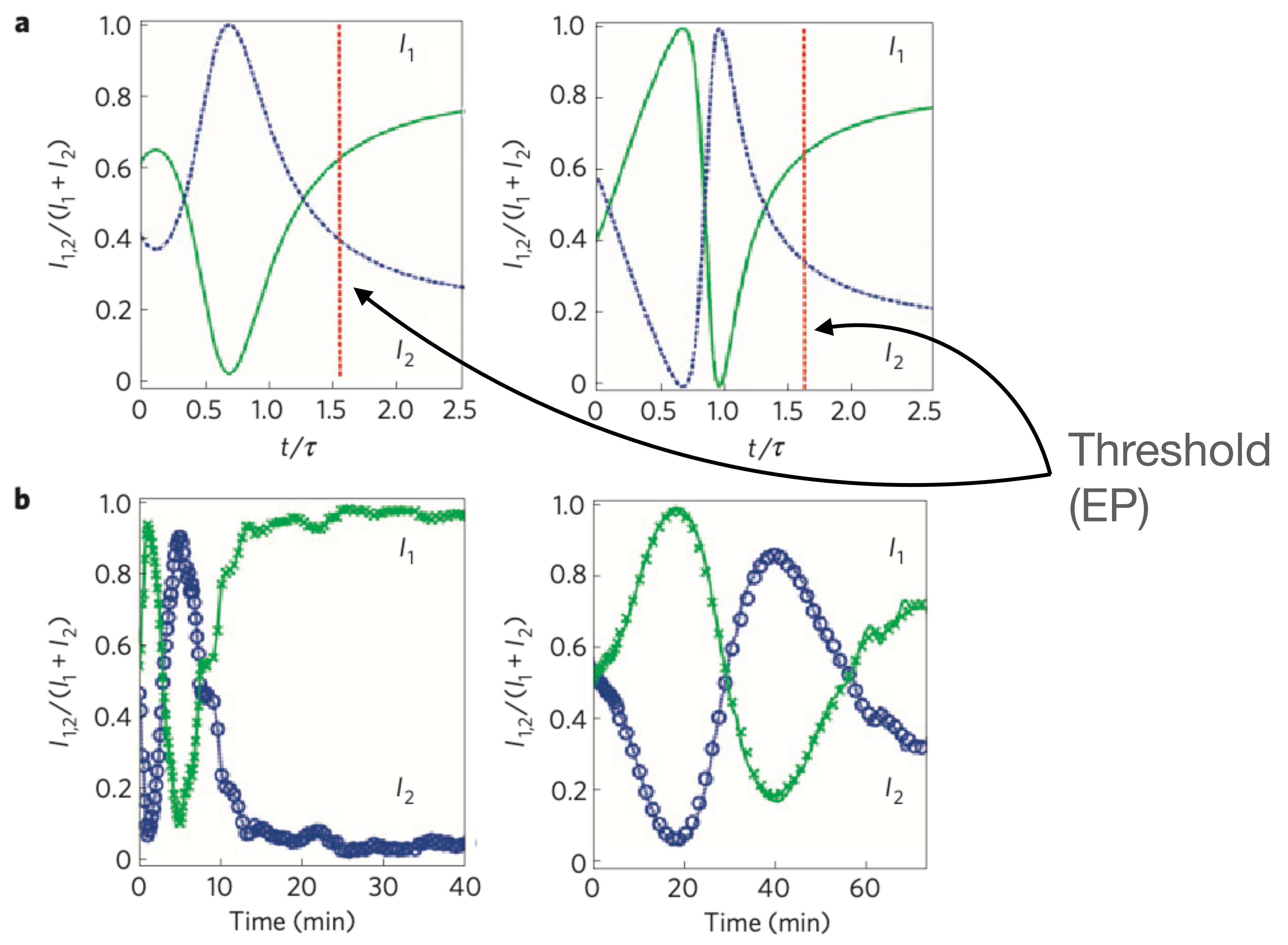
A. Guo *et al.* Observation of PT symmetry breaking in complex optical potentials. *Phys. Rev. Lett.* **103,** 093902 (2009)

Observation of PT Symmetry in a Balanced Gain/Loss System



Experimental set-up





Computed and experimentally measured response of a PT-symmetric coupled system.
a Numerical solution of the coupled equations (1) describing the PT-symmetric system. The left (right) panel shows the situation when light is coupled into channel 1 (2).
b Experimentally measured (normalized) intensities at the output facet during the gain build-up as a function of time.

C. Rüter *et al.* Observation of parity–time symmetry in optics, *Nature Physics* **6**, 192–195 (2010)

Photonic Lattices

Beam Dynamics in PT-symmetric Photonic Lattices

- Makris *et al.* considered beam diffraction in a PT-symmetric photonic lattice with the complex potential

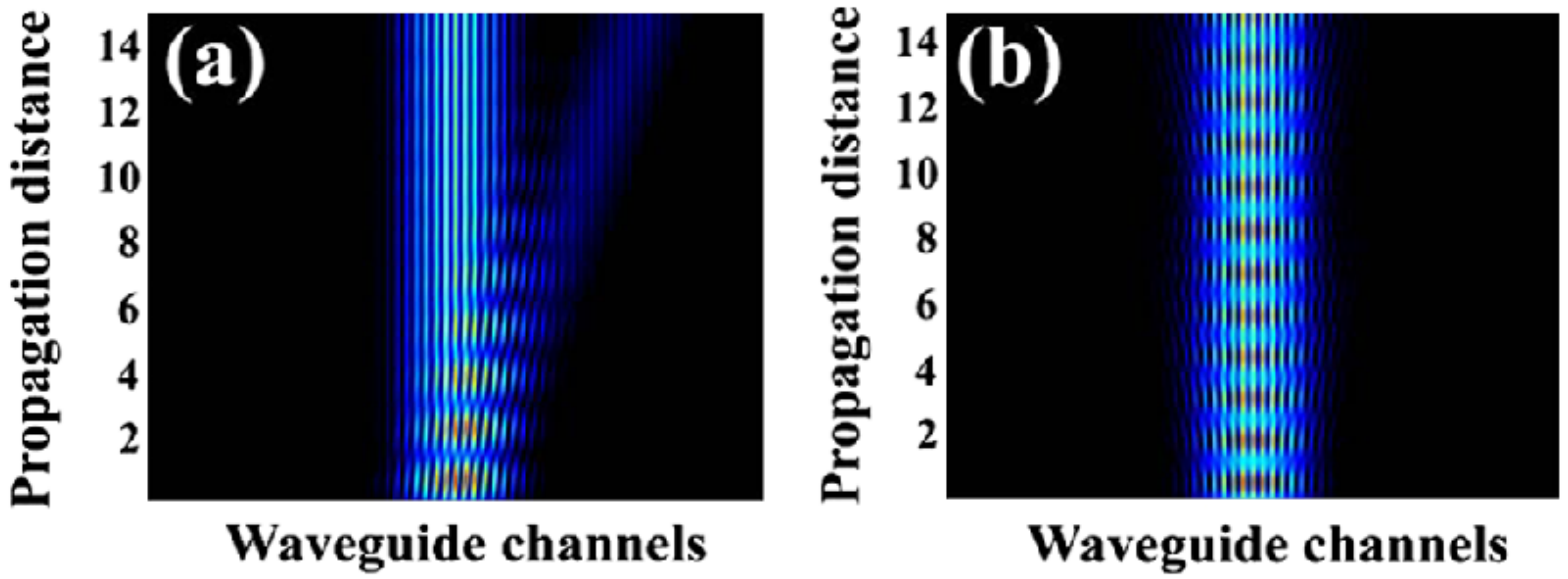
$$V(x) = 4(\cos^2(x) + iV_0 \sin(2x)).$$

- They looked for the solutions to the paraxial diffraction equation

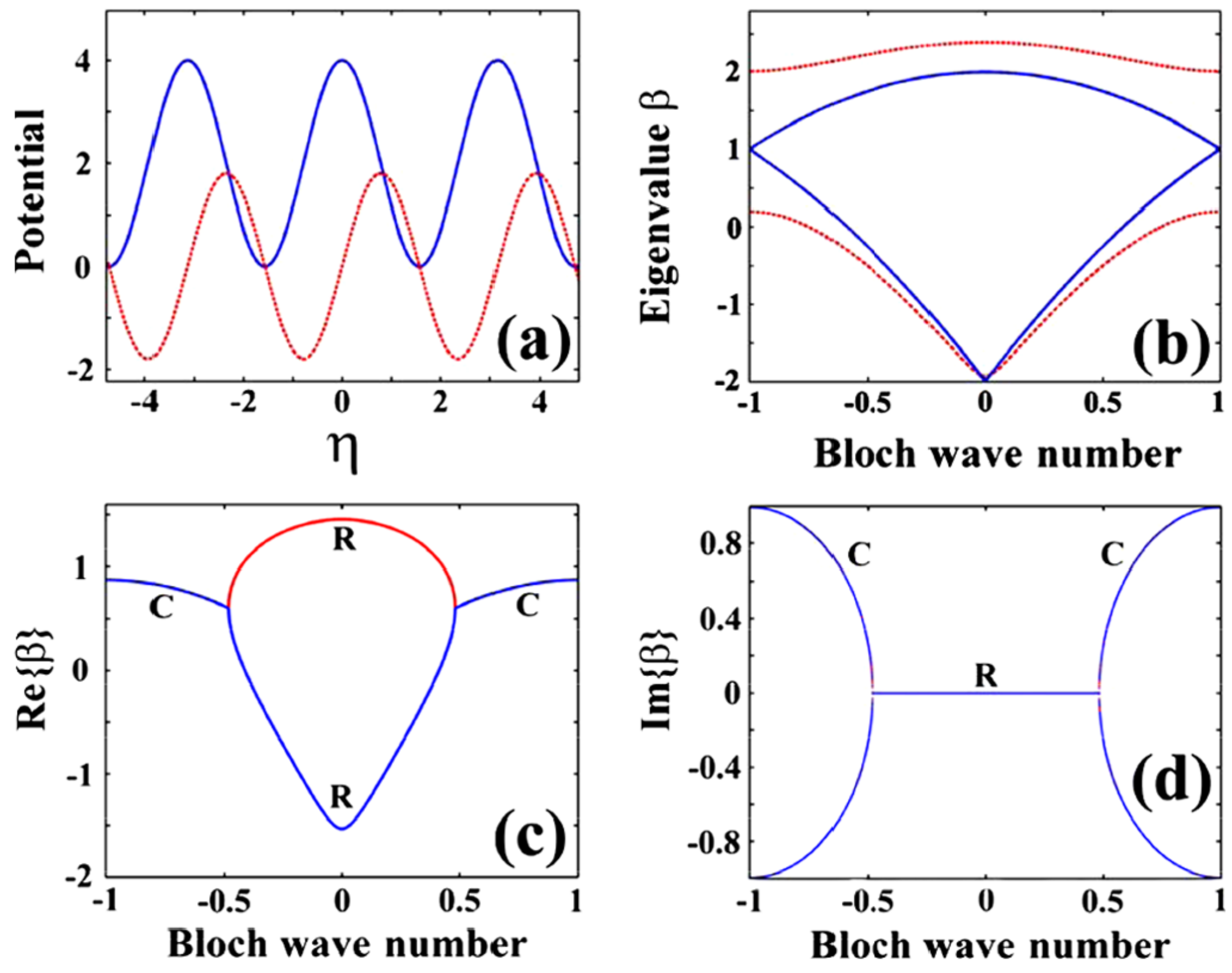
$$i\frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial x^2} + V(x)E = 0$$

of the form $E(x,z) = \phi_{kn}(x)\exp(i\beta_{kn}z)$, where $\phi_{kn}(x)$ is the n-band Floquet-Bloch mode at Bloch momentum k and β_{kn} is the associated eigenvalue (propagation constant).

- For $V_0 < 0.5$ the band structure is entirely real while for $V_0 > 0.5$ it becomes complex.
- In the PT array the beam splits in two and double refraction occurs. This peculiar effect can be attributed to the mode nonorthogonality.



Intensity evolution of a broad optical beam under normal incidence when (a) $V_0 = 0.49$, (b) $V_0 = 0$.



(a) Real (blue) and imaginary (red) parts of the PT-symmetric periodic potential. (b) Corresponding band structure for $V_0 = 0.2$ (blue) and $V_0 = 0.5$ (red). (c), (d) Real and imaginary part of the double-valued band for $V_0 = 0.7$, respectively, resulting from the merging of the two first bands.

K. G. Makris *et al.* Beam dynamics in PT-symmetric optical lattices, *Phys. Rev. Lett.* **100,** 103904 (2008)

Multilayer Systems

Unidirectional Invisibility

- Z. Lin *et al. have* investigated a Bragg grating with PT-symmetric refractive index distribution

$$n(z) = n_0 + n_1 \cos(2\beta z) + i n_2 \sin(2\beta z), -L/2 \le z \le L/2,$$
$$n = n_0 \text{ elsewhere.}$$

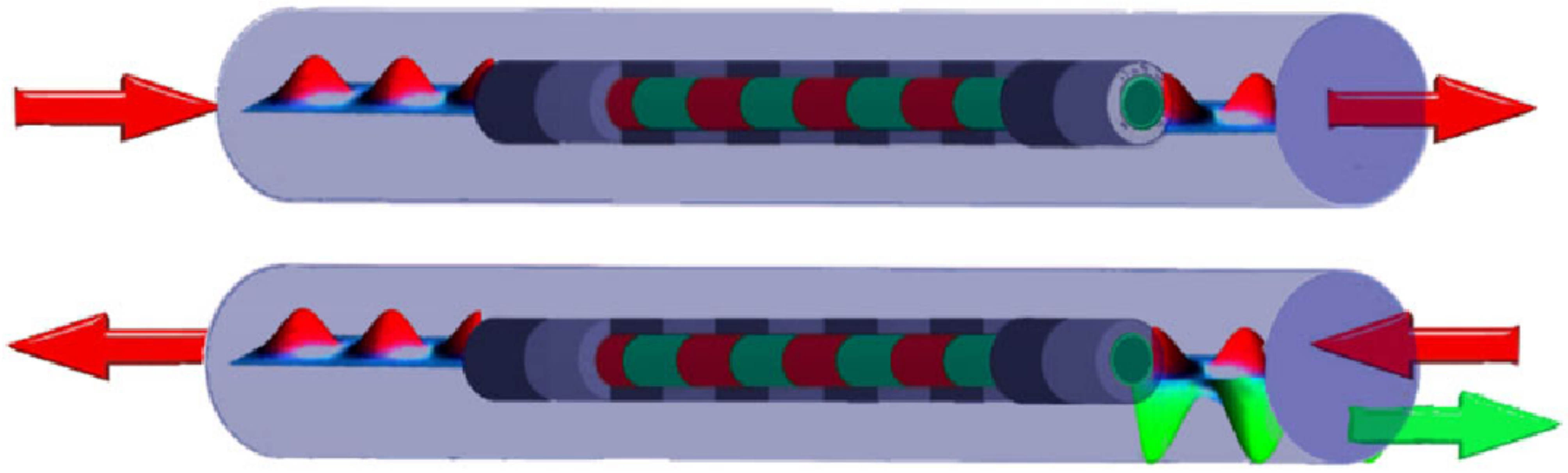
- Such a grating shows symmetric *transmission* from left to right and vice versa:

$$T = \frac{|\lambda|^2}{|\lambda|^2 \cos^2(\lambda L) + \delta^2 |\sin(\lambda L)|^2}.$$

- But, surprisingly, *reflection* at different sides when $n_2 \neq 0$ is *different*:

$$R_L = \frac{(n_1 - n_2)^2 k^2 / 4n_0^2}{\delta^2 + |\lambda \cot(\lambda L)|^2}, \quad R_R = \frac{(n_1 + n_2)^2 k^2 / 4n_0^2}{\delta^2 + |\lambda \cot(\lambda L)|^2}.$$

$\lambda = \sqrt{\delta^2 - k^2(n_1^2 - n_2^2)/4n_0^2}$, $\delta = \beta - k$ is the detuning,
$\beta = \pi/\Lambda$ is the grating wavenumber.



Unidirectional invisibility of a PT-symmetric Bragg scatterer.

- The wave entering from the left (upper figure) does not recognize the existence of the periodic structure and goes through the sample entirely unaffected.
- A wave entering the same grating from the right (lower figure), experiences enhanced reflection.

Z. Lin *et al.* Unidirectional invisibility induced by PT-Symmetric periodic structures. *Phys. Rev. Lett.* **106**, 213901 (2011)

Unidirectional Invisibility

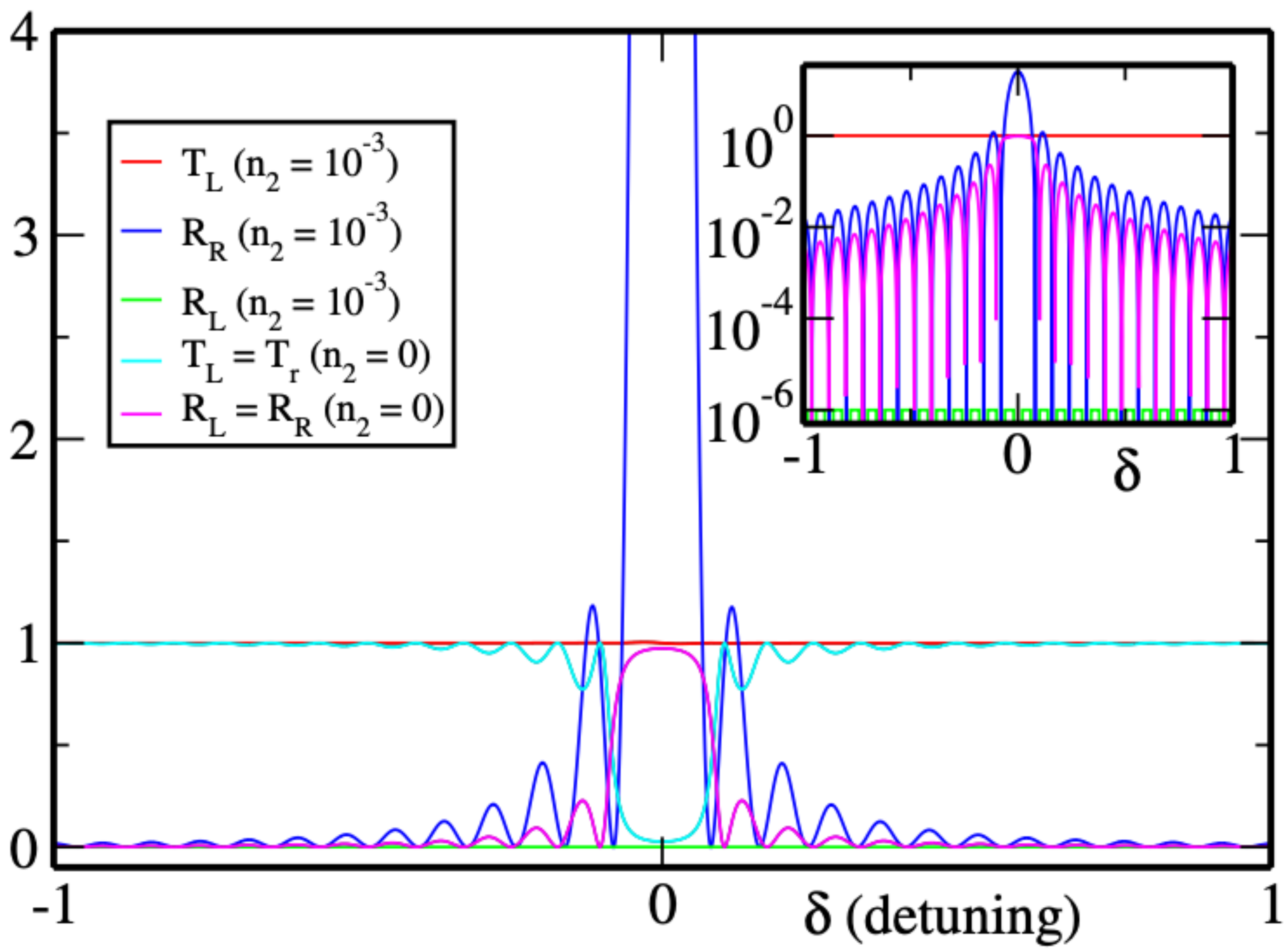
$$R_L = \frac{(n_1 - n_2)^2 k^2 / 4n_0^2}{\delta^2 + |\lambda \cot(\lambda L)|^2},$$

$$R_R = \frac{(n_1 + n_2)^2 k^2 / 4n_0^2}{\delta^2 + |\lambda \cot(\lambda L)|^2}.$$

The reflectivity asymmetry becomes most pronounced when $n_1 = n_2$:

$$R_L = 0\,,$$

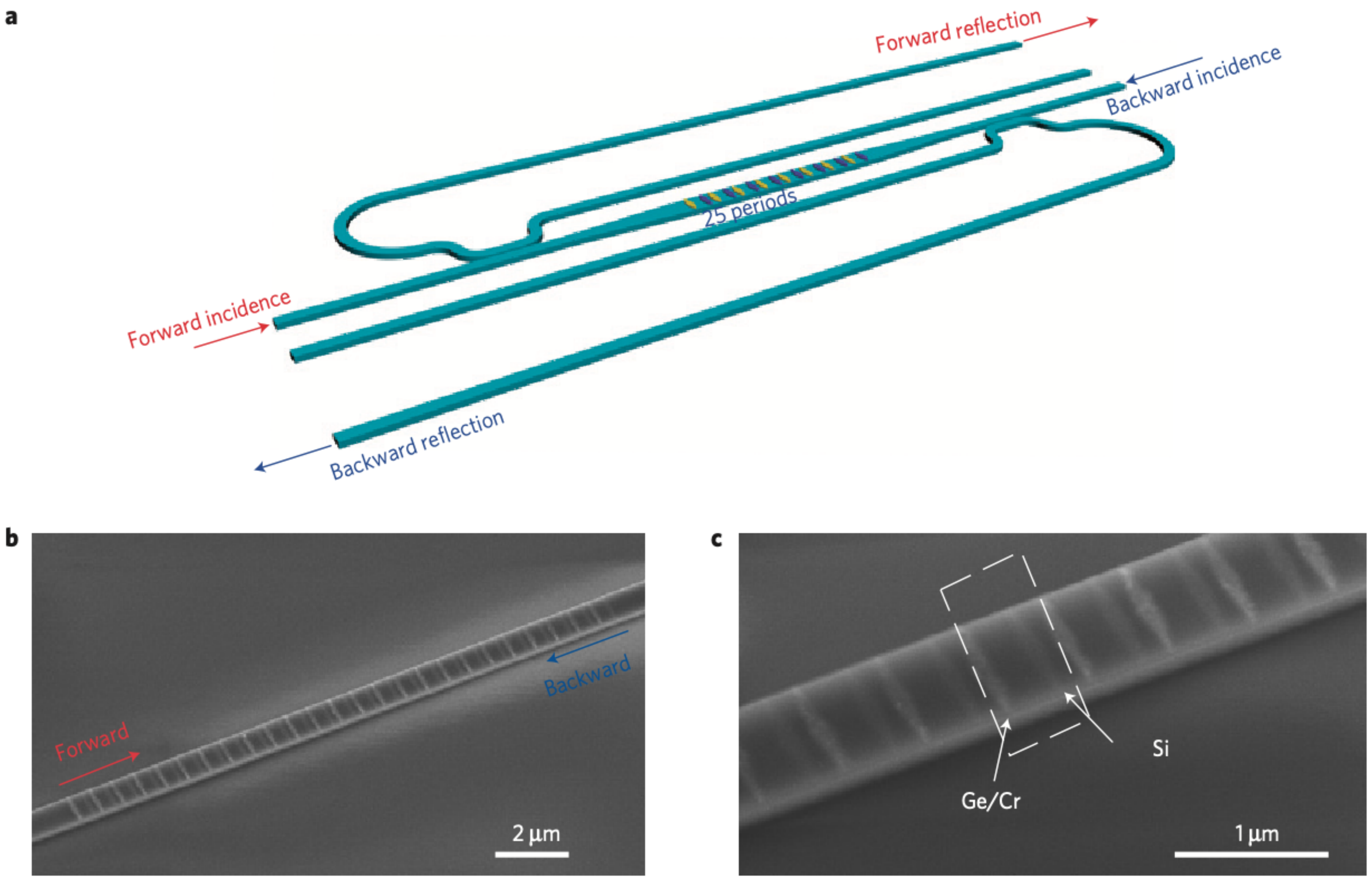
$$R_R = L^2 \left(k \frac{n_1}{n_0} \right)^2 \left(\frac{\sin(L\delta)}{L\delta} \right)^2 \xrightarrow{\delta \to 0} L^2 \left(k \frac{n_1}{n_0} \right)^2.$$



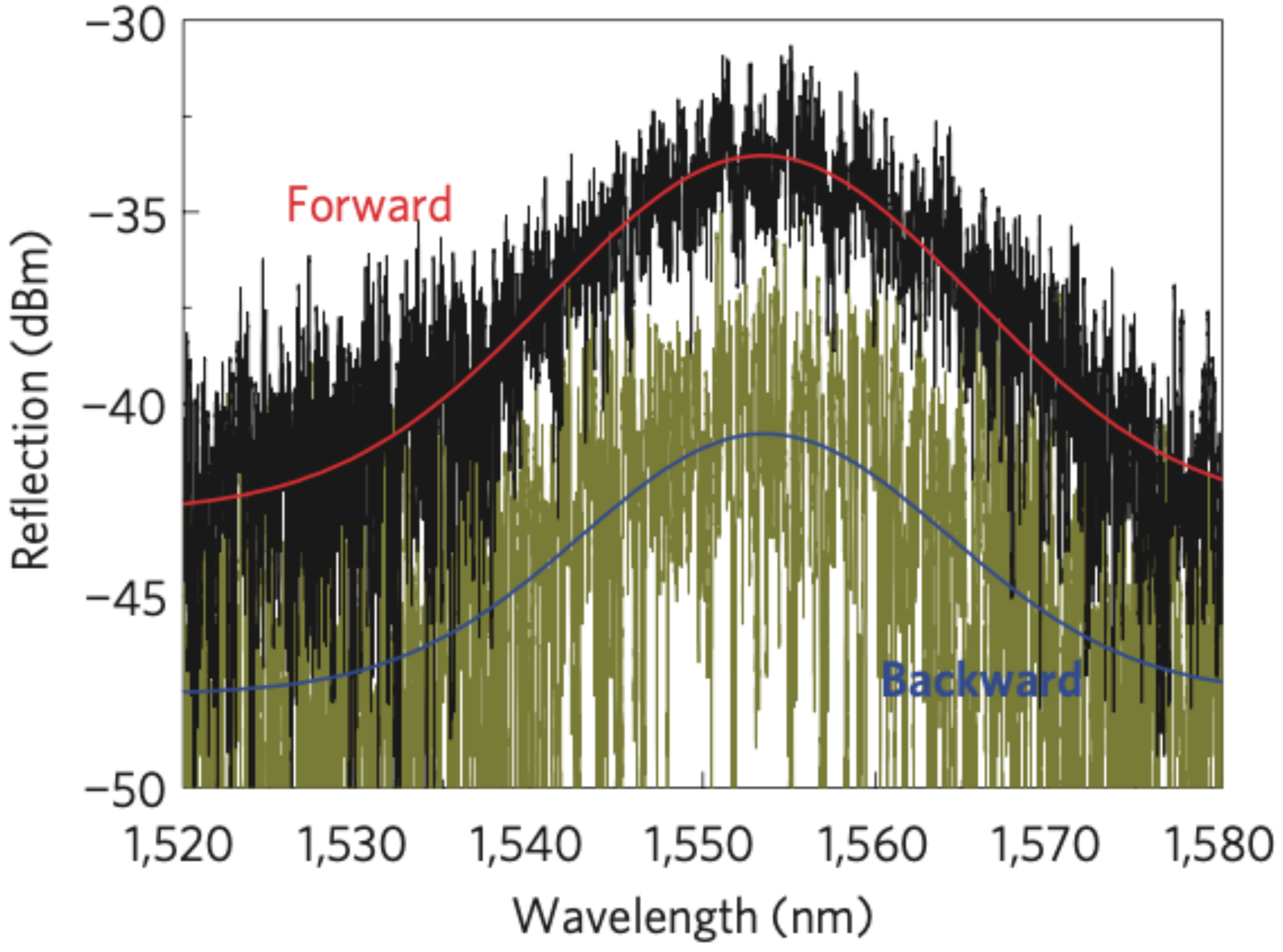
Transmission and reflection coefficients of the PT-symmetric Bragg grating vs detuning parameter

Z. Lin *et al.* Unidirectional invisibility induced by PT-Symmetric periodic structures. *Phys. Rev. Lett.* **106**, 213901 (2011)

Experimental Demonstration of Unidirectional Invisibility



Experimental setup. The real and imaginary parts of the complex potential are realized by 51nm Si and 14nm Ge / 24nm Cr.

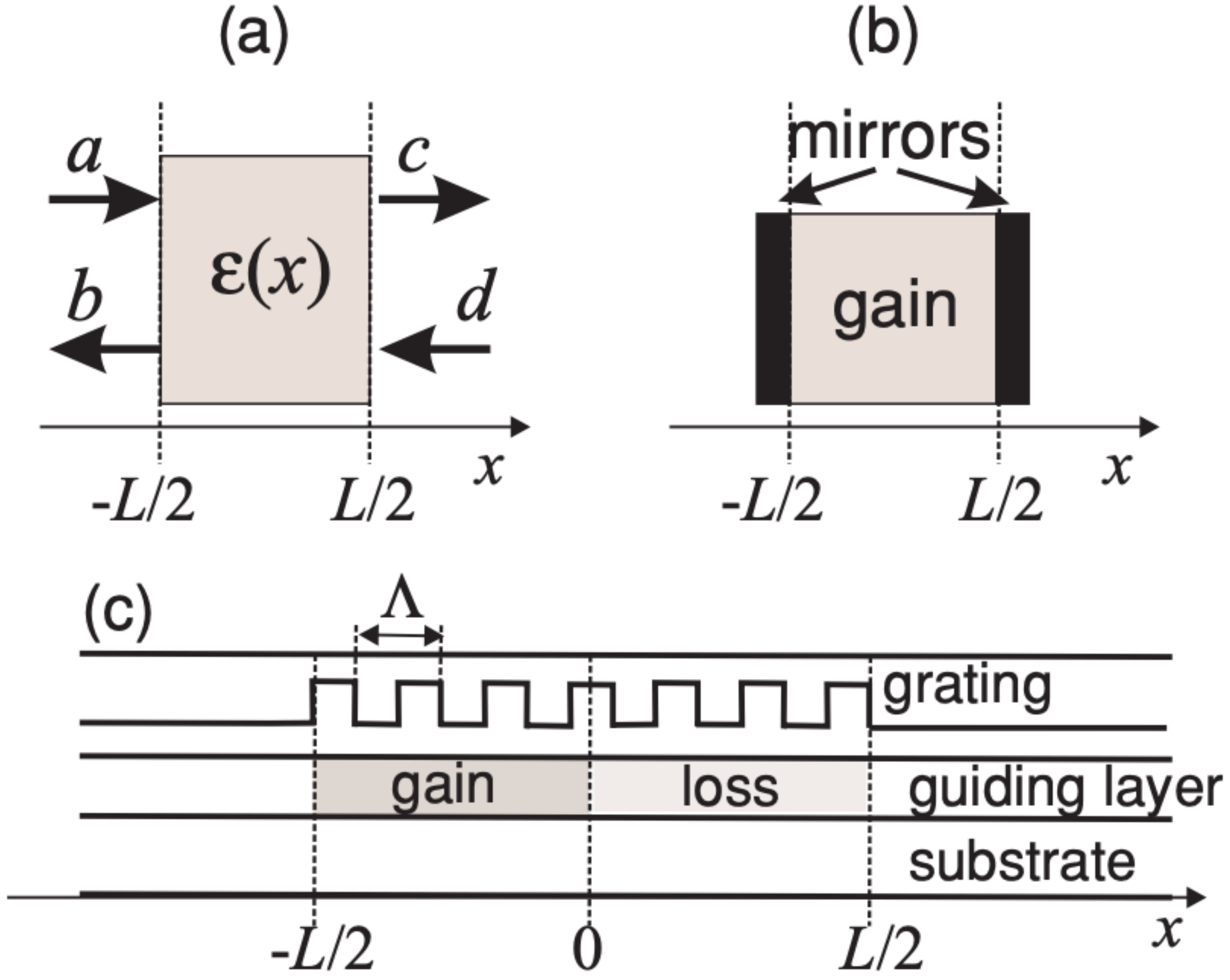


Measured reflection spectra of the device through the waveguide coupler for both directions over a broad band of telecom wavelengths from 1.520 to 1.580 nm. It is found that the *forward reflectivity is about* ***7.5 dB*** *higher than the backward*.

L. Feng *et al.* Experimental demonstration of a unidirectional reflectionless parity-time metamaterial at optical frequencies. *Nature Materials* **12**, 108–113 (2013)

Coherent Perfect Absorption Lasing

- Coherent perfect absorber (CPA) is the time-reversed counterpart of a laser in which gain medium is substituted by loss medium.
- Longhi introduced the idea of PT-symmetric CPA laser by unifying CPA with a laser.
- It was shown by analysis of the scattering matrix that such a system operating above PT phase transition can operate as CPA and laser *simultaneously.*



(a) Schematic of wave scattering in a one-dimensional optical structure with complex dielectric constant $\varepsilon(x)$.
(b) Schematic of a laser oscillator comprising two equal lossless mirrors filled with a gain medium.
(c) Distributed-feedback structure, consisting of a uniform index grating with two homogeneous and symmetric gain and loss regions, that realizes a PT CPA laser.

S. Longhi. PT-symmetric laser absorber. *Phys. Rev. A* **82**, 031801 (2010)

Microresonators

Optical Microcavity Sensor

- Let us consider first a system at Hermitian degeneracy (diabolic point, or DP):

$$H_0^{\mathrm{DP}} = \begin{pmatrix} \omega_0 & 0 \\ 0 & \omega_0 \end{pmatrix}.$$

- Adding a non-Hermitian perturbation ϵH_1 where $H_1 = \begin{pmatrix} \omega_1 & A_1 \\ B_1 & \omega_1 \end{pmatrix}$ leads to eigenvalues of the perturbed Hamiltonian $H = H_0^{\mathrm{DP}} + \epsilon H_1$ that are split by

$$\Delta\omega_{\mathrm{DP}} = 2\epsilon\sqrt{A_1 B_1}.$$

- When applying the same perturbation ϵH_1 to a Hamiltonian that is located at an EP, where

$$H_0^{\mathrm{EP}} = \begin{pmatrix} \omega_0 & A_0 \\ 0 & \omega_0 \end{pmatrix}$$

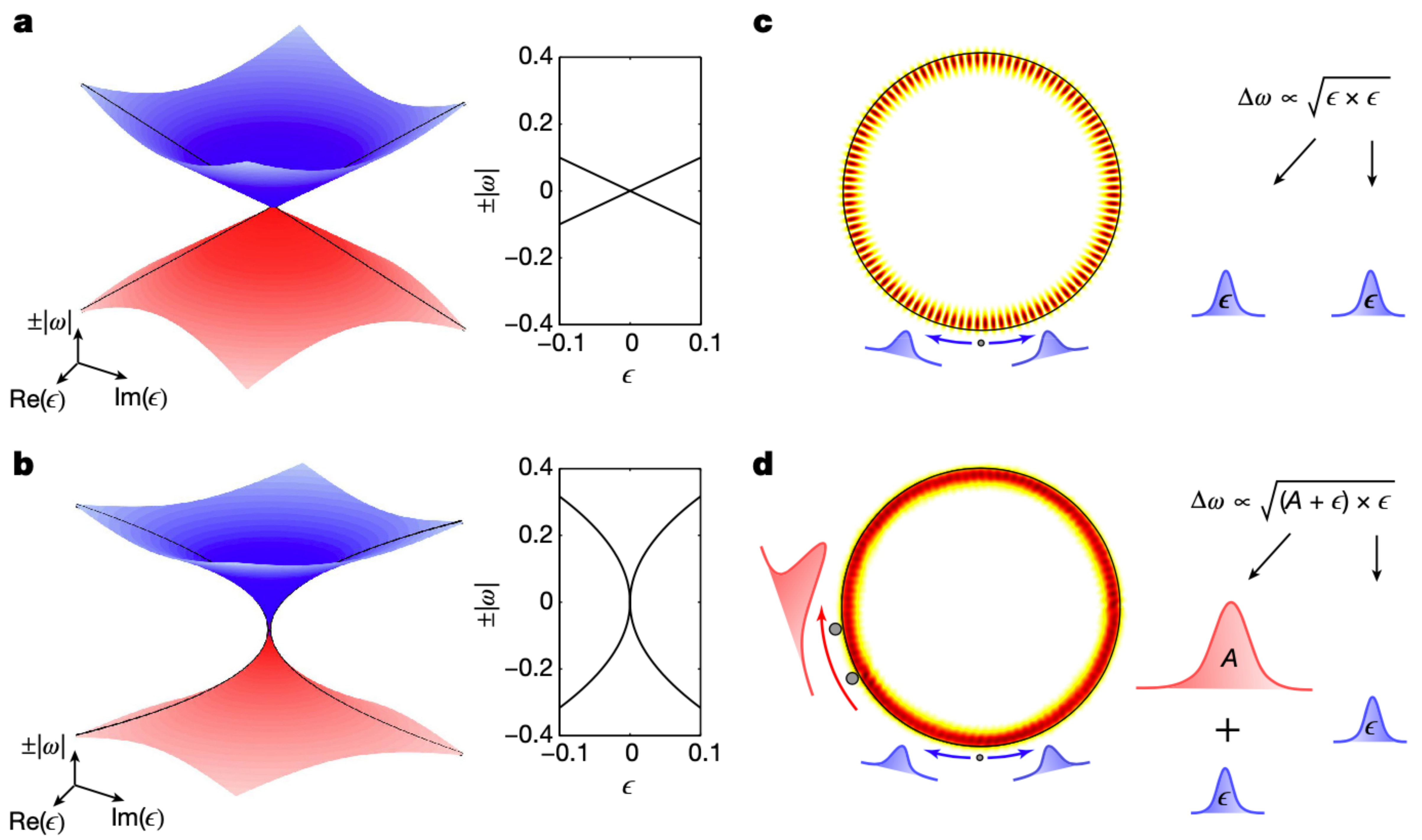
we obtain an eigenvalue splitting of

$$\Delta\omega_{\mathrm{EP}} = 2\sqrt{\epsilon}\sqrt{A_0 B_1 + \epsilon A_1 B_1}.$$

- For weak perturbations ($\epsilon \ll 1$ or $|A_0| \ll |A_1|$) the splitting at the EP is always enhanced compared to the DP:

$$\Delta\omega_{\mathrm{EP}} = \Delta\omega_{\mathrm{DP}}\sqrt{\epsilon + A_0/A_1}/\sqrt{\epsilon}.$$

- The situation described here is typically encountered in particle sensors based on microcavities.



A comparison of sensors operating at an exceptional point and a diabolic point. Topology of the surface that characterizes the complex frequency ω (the energy level) of diabolic- (a) and exceptional- point (b) sensors. (c) Visualization of the electric-field distribution in a diabolic-point sensor. (d) Visualization of the electric-field distribution in an exceptional-point sensor.

W. Chen *et al*. Exceptional points enhance sensing in an optical microcavity, *Nature* **548**, 192–196 (2017)

Other PT-Symmetry-Related Phenomena

- Encircling EPs;
- Single-mode operation in PT-symmetric lasers;
- Enhancement or suppression of the spontaneous relaxation rate near EPs;
- Non-Reciprocal light transmission;
- PT optomechanics.

Ş. Özdemir *et al.* Parity–time symmetry and exceptional points in photonics. *Nature Materials* **18**, 783–798 (2019)

Perspectives and Future Outlook

- Non-Hermitian topological photonics
- PT symmetry in nonlinear optics
- Metamaterials and plasmonics
- PT symmetry in quantum domain

Ş. Özdemir *et al.* Parity–time symmetry and exceptional points in photonics. *Nature Materials* **18**, 783–798 (2019)