Toroidal Dipole Associated Resonant Forward Scattering of Light by Silicon Nanoparticles

P. D. Terekhov[1, 2], **K. V. Baryshnikova**[1], **A. S. Shalin**[1], **A. Karabchevsky**[2], **and A. B. Evlyukhin**[1, 3]

[1]ITMO University, Russia
[2]Electrooptical Engineering Unit and Ilse Katz Institute for Nanoscale Science & Technology
Ben-Gurion University, Beer-Sheva 84105, Israel
[3]Laser Zentrum Hannover e.V., Germany

Abstract— In this work we investigate in terms of Cartesian multipoles the Kerker-type effect in high-index dielectric nanoparticles for which the third order multipoles give a considerable contribution to the light scattering process. We show that the Kerker-type effect can be associated with the resonant excitation of toroidal dipole moment (third order multipole) and, namely, with the interference of the scattered waves generated by electric, magnetic and toroidal dipole moments of high-index dielectric nanoparticles. From our theoretical results reveal, that the interplay between these moments with dominating contribution of toroidal dipole moment can provide strong suppression of the backward light scattering and, simultaneously, resonant forward light scattering.

1. INTRODUCTION

Resonant optical responses in high-index nanostructures attract noticeable attention because of their practical applications [1–3]. Recently such resonances were experimentally demonstrated for silicon nanospheres [1, 4]. Dielectric nanoparticles with optical resonant responses are promising building blocks for metasurfaces, which can provide effective ways to manipulate the light [2, 5]. Spectral positions of multipole resonances of dielectric nanoparticles are determined by their size and aspect ratio (height to base edge of parallelepiped) [6, 7]. This opportunity can be used for the realization of the spectral overlap of electric and magnetic dipole resonances resulting in strong suppression of the backward light scattering due to the Kerker effect [8], when the electric and magnetic dipole magnitudes and phases are equal to each other. Directional scattering is very advantageous for nanophotonics and photovoltaics applications, for instance nanoantennas [9–11], lensing [12], nonlinearities [13], emission [14] and metasurfaces [15]. The resonant Kerker-type effect was demonstrated for silicon nanoparticles with low aspect ratio (height to diameter) in [16]. Dielectric Huygens' surfaces based on such nanoparticles were created and investigated [17]. We have demonstrated resonant forward scattering of light with toroidal dipole contributions for silicon nanocylinders with large aspect ratio in [18]. Here we expand this work to show in detail similar effect for silicon nanoparallelepipeds with large aspect ratio. In contradiction to recently researched destructive interference between toroidal and electric dipole moment (*"anapole"*-mode) [19, 20], here we consider so-called *"super-dipole"* interference regime.

2. SYSTEM AND METHODS

We consider plane wave scattering by silicon nanoparallelepipeds with different aspect ratios (Fig. 1). Base edges of the particles are fixed and equal to 100 nm. Dielectric permittivity of silicon is taken from [21].

Our theoretical approach is based on the multipole decomposition method presented in [22]. Concisely, regular electric dipole moment of a scatterer is calculated as

$$\mathbf{p} = \int \mathbf{P}\left(\mathbf{r}'\right) d\mathbf{r}', \tag{1}$$

where $\mathbf{P}(\mathbf{r}')$ is the polarization induced in the scatterer by incident light wave, $\mathbf{r}'$ is the radius-vector of a volume element inside the scatterer. The toroidal dipole moment, having the same radiation pattern, is determined as:

$$\mathbf{T} = \frac{i\omega}{10} \int \left\{ 2\mathbf{r}'^2 \mathbf{P}\left(\mathbf{r}'\right) - \left(\mathbf{r}' \cdot \mathbf{P}\left(\mathbf{r}'\right)\right) \mathbf{r}' \right\} d\mathbf{r}'. \tag{2}$$

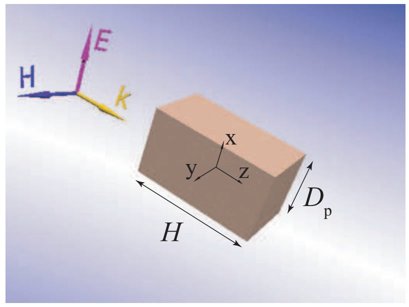

Figure 1: Parallelepipedal particle, irradiated by a plane wave, which propagates along the particle's axis. **k** is the wave vector, **E** is the electric field, **H** is the maghetic field, D_p and **H** are the base edge and the height of the parallelepiped respectively.

The expressions for the other multipole moments can be found in [22]. When considering multipole moments up to the electric octupole moment, the scattering cross-section can be presented as:

$$\begin{aligned}\sigma_{\text{sca}} \simeq\ & \frac{k_0^4}{6\pi\varepsilon_0^2|\mathbf{E}_{inc}|^2}\left|\mathbf{p}+\frac{ik_0\varepsilon_d}{c}\mathbf{T}\right|^2+\frac{k_0^4\varepsilon_d\mu_0}{6\pi\varepsilon_0|\mathbf{E}_{inc}|^2}|\mathbf{m}|^2\\ &+\frac{k_0^6\varepsilon_d}{720\pi\varepsilon_0^2|\mathbf{E}_{inc}|^2}\sum|Q_{\alpha\beta}|^2+\frac{k_0^6\varepsilon_d^2\mu_0}{80\pi\varepsilon_0|\mathbf{E}_{inc}|^2}\sum|M_{\alpha\beta}|^2\\ &+\frac{k_0^8\varepsilon_d^2}{1890\pi\varepsilon_0^2|\mathbf{E}_{inc}|^2}\sum|O_{\alpha\beta\gamma}|^2. \end{aligned} \tag{3}$$

where k_0 is the free-space wave number; ε_0 is the vacuum permittivity; ε_d is the relative dielectric permittivity of a surrounding medium (here we consider $\varepsilon_d = 1$); μ_0 is the vacuum magnetic permeability; c is the light speed in the vacuum; $\mathbf{E}_{inc}$ is the electric field amplitude of the incident light wave; **m** is the magnetic dipole moment (MD) of a particle; a term $\mathbf{p} + i\frac{k_0\varepsilon_d}{c}\mathbf{T}$ including the interference of electric dipole (ED) and toroidal dipole (TD) moments can be treated as total electric dipole moment (TED); Q, M and O are the electric quadrupole moment tensor (EQ), the magnetic quadrupole moment tensor (MQ) and the tensor of electric octupole moment (OCT) in irreducible representations, respectively. Note that these tensors are symmetric and traceless [22]. Total scattering cross-section is obtained through the integration of the Poynting vector over a closed surface in the far-field zone and the normalization to the incident field intensity [22]. Total electric fields and corresponding induced polarization in scatterers are calculated numerically using COMSOL Multiphysics. Using the calculated polarization, the multipole moments and their contributions into scattering cross-sections are obtained by a numerical integration.

Previously, Kerker-type effect was considered without the investigation of TD moment contribution [16, 23]. Here we note that in nanoparallelepipeds with signficant TD contribution Kerker-type effect can be modulated by the toroidal dipole far-field radiation which is similar to the electric dipole one. Multipole analysis based on the Eq. (3) clarifies that scattering resonance peak at the wavelength of 587 nm corresponds to the overlap of the resonant contributions of several different multipoles (Fig. 2). This spectrum area combines the total electric dipole TED, the magnetic dipole MD, and the magnetic quadrupole MQ resonant contributions. It is important to note that the basic contributions into this resonant peak of the nanoparallelepiped with high aspect ratio ($H = 300$ nm, Fig. 2) come from the total electric and magnetic dipole moments. This is achieved because the TD contribution into the TED moment raises with the increasing of nanoparticle height. As it is shown in [18], TED contribution to the scattering cross-section is negligible for the particles with aspect ratio ≈ 1. Another fact to note is the much smaller resonant MQ contribution with respect to the contribution of resonant TED and MD. This is substantial for the realization of a resonant Kerker-type effect in large-aspect-ratio nanoparticles due to the TED and MD resonances overlap.

For the illustration of the backscattering attenuation in Fig. 3 we show two different radiation patterns for the nanoparallelepiped with height $H = 300$ nm and base edge $D_p = 100$ nm. The forward scattering corresponds to the angle 90 degrees in Fig. 3, and the backward scattering corresponds to the angle 270 degrees in Fig. 3. In the Fig. 3(a) we present a radiation pattern for the spectrum point corresponding to the scattering maximum wavelength ($\lambda = 587$ nm). Here one can observe strong backscattering attenuation. As it is shown with black dashed line in Fig. 2,

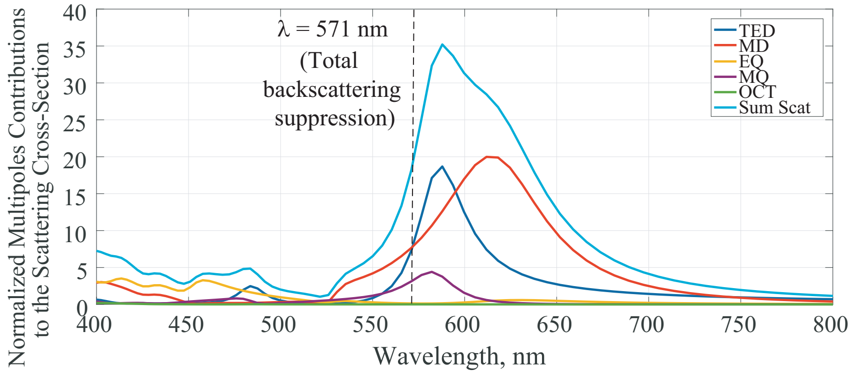

Figure 2: Multipole contributions to the scattering cross-section spectrum for parallelepipedal nanoparticle, height $H = 300$ nm. Hereinafter the values are normalized to geometric cross-section of the parallelepiped. Black dashed line corresponds to the wavelength of total backscattering suppression (see text below).

at the wavelength $\lambda = 571$ nm magnitudes of MD and TED contributions to the scattering cross-sections are equal to each over. Together with phase equality [18] such parameters correspond to the Kerker-type conditions realization. As it shown in Fig. 3(b), the backscattering suppression here is very strong.

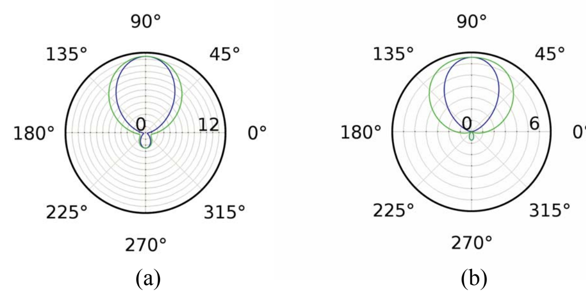

Figure 3: Scattered field patterns for the silicon parallelepipedal nanoparticle with $H = 300$ nm, calculated at the wavelength (a) 587 nm, (b) 571 nm. The different colors correspond to the scattered field patterns calculated in the mutually perpendicular planes. The angle of 90° (270°) corresponds to the forward (backward) direction.

3. CONCLUSION

In conclusion, in this work using the multipole decomposition method we have shown, that a constructive interference between toroidal and electric dipole moments of the parallelepiped can be realized in the optical frequency range. Total electric dipole moment with dominant contribution of the toroidal dipole is resonantly excited in such nanoparallelepipeds with large aspect ratio and so-called super-dipole mode can exist. We found that, due to the interference between electromagnetic fields generated by the total electric dipole and magnetic dipole moments of the nanoparallelepipeds with relatively high aspect ratio, the backward scattering suppression effect can be realized.

REFERENCES

1. Kuznetsov, A. I., A. E. Miroshnichenko, Y. H. Fu, J. B. Zhang, and B. Luk'yanchuk, "Magnetic light," *Scientific Reports*, Vol. 2, No. 492, 2012.
2. Jahani, S. and Z. Jacob, "All-dielectric metamaterials," *Nature Nanotechnology*, Vol. 11, No. 1, 23–36, 2016.
3. Baryshnikova, K. V., A. B. Evlyukhin, and A. S. Shalin, "Magnetic hot-spots in hollow silicon cylinders," *Journal of Physics: Conference Series*, Vol. 741, 012156, IOP Publishing, 2016.
4. Evlyukhin, A. B., S. M. Novikov, U. Zywietz, R. L. Eriksen, C. Reinhardt, S. I. Bozhevolnyi, and B. N. Chichkov, "Demonstration of magnetic dipole resonances of dielectric nanospheres in the visible region," *Nano Letters*, Vol. 12, No. 7, 3749–3755, 2012.

5. Kuznetsov, A. I., A. E. Miroshnichenko, M. L. Brongersma, Y. S. Kivshar, and B. Lukyanchuk, "Optically resonant dielectric nanostructures," *Science*, Vol. 354, No. 6314, 2472, 2016.
6. Evlyukhin, A. B., C. Reinhardt, and B. N. Chichkov, "Multipole light scattering by nonspherical nanoparticles in the discrete dipole approximation," *Physical Review B*, Vol. 84, No. 23, 235429, 2011.
7. Markovich, D., K. Baryshnikova, A. Shalin, A. Samusev, A. Krasnok, P. Belov, and P. Ginzburg, "Enhancement of artificial magnetism via resonant bianisotropy," *Scientific Reports*, 6, 2016.
8. Kerker, M., D.-S. Wang, and C. L. Giles, "Electromagnetic scattering by magnetic spheres," *JOSA*, Vol. 73, No. 6, 765–767, 1983.
9. Novotny, L. and N. Van Hulst, "Antennas for light," *Nature Photonics*, Vol. 5, No. 2, 83–90, 2011.
10. Krasnok, A. E., C. R. Simovski, P. A. Belov, and Y. S. Kivshar, "Superdirective dielectric nanoantennas," *Nanoscale*, Vol. 6, No. 13, 7354–7361, 2014.
11. Karabchevsky, A., A. Mosayyebi, and A. V. Kavokin, "Tuning the chemiluminescence of a luminol ow using plasmonic nanoparticles," *Light: Science & Applications*, Vol. 5, No. 11, e16164, 2016.
12. Khorasaninejad, M., W. T. Chen, J. Oh, and F. Capasso, "Superdispersive off-axis meta-lenses for compact high resolution spectroscopy," *Nano Letters*, Vol. 16, No. 6, 3732–3737, 2016.
13. Shcherbakov, M. R., D. N. Neshev, B. Hopkins, A. S. Shorokhov, I. Staude, E. V. Melik-Gaykazyan, M. Decker, A. A. Ezhov, A. E. Miroshnichenko, I. Brener, et al., "Enhanced third-harmonic generation in silicon nanoparticles driven by magnetic response," *Nano Letters*, Vol. 14, No. 11, 6488–6492, 2014.
14. Rusak, E., I. Staude, M. Decker, J. Sautter, A. E. Miroshnichenko, D. A. Powell, D. N. Neshev, and Y. S. Kivshar, "Hybrid nanoantennas for directional emission enhancement," *Applied Physics Letters*, Vol. 105, No. 22, 221109, 2014.
15. Arbabi, A., Y. Horie, M. Bagheri, and A. Faraon, "Dielectric metasurfaces for complete control of phase and polarization with subwavelength spatial resolution and high transmission," *Nature Nanotechnology*, 2015.
16. Staude, I., A. E. Miroshnichenko, M. Decker, N. T. Fofang, S. Liu, E. Gonzales, J. Dominguez, T. S. Luk, D. N. Neshev, I. Brener, et al., "Tailoring directional scattering through magnetic and electric resonances in subwavelength silicon nanodisks," *ACS Nano*, Vol. 7, No. 9, 7824–7832, 2013.
17. Decker, M., I. Staude, M. Falkner, J. Dominguez, D. N. Neshev, I. Brener, T. Pertsch, and Y. S. Kivshar, "High-efficiency dielectric huygens surfaces," *Advanced Optical Materials*, Vol. 3, No. 6, 813–820, 2015.
18. Terekhov, P. D., K. V. Baryshnikova, A. S. Shalin, A. Karabchevsky, and A. B. Evlyukhin, "Resonant forward scattering of light by high refractive-index dielectric nanoparticles with toroidal dipole contribution," *Optics Letters*, Vol. 42, No. 4, 835–838, 2017.
19. Miroshnichenko, A. E., A. B. Evlyukhin, Y. F. Yu, R. M. Bakker, A. Chipouline, A. I. Kuznetsov, B. Lukyanchuk, B. N. Chichkov, and Y. S. Kivshar, "Nonradiating anapole modes in dielectric nanoparticles," *Nature Communications*, 6, 2015.
20. Terekhov, P. D., K. V. Baryshnikova, A. S. Shalin, A. B. Evlyukhin, and I. A. Khromova, "Nonradiating anapole modes of dielectric particles in terahertz range," *Days on Diffraction (DD)*, 406–409, IEEE, 2016.
21. Palik, E. D., *Handbook of Optical Constants of Solids*, Vol. 3, Academic Press, 1998.
22. Evlyukhin, A. B., T. Fischer, C. Reinhardt, and B. N. Chichkov, "Optical theorem and multipole scattering of light by arbitrary shaped nanoparticles," *Physical Review B*, Vol. 94, No. 20, 205434, 2016.
23. Campione, S., L. I. Basilio, L. K. Warne, and M. B. Sinclair, "Tailoring dielectric resonator geometries for directional scattering and huygens metasurfaces," *Optics Express*, Vol. 23, No. 3, 2293–2307, 2015.