

DISCRETE FOURIER TRANSFORM (DFT): LONG SEQUENCES, MATRIX NOTATION

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Reading: Chapter 4 by B. Porat

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DFT PROPERTY: SYMMETRY

- Let assume signal $x[n]$ as real

Proof that $X[k] = X^*[N - k]$

Proof:

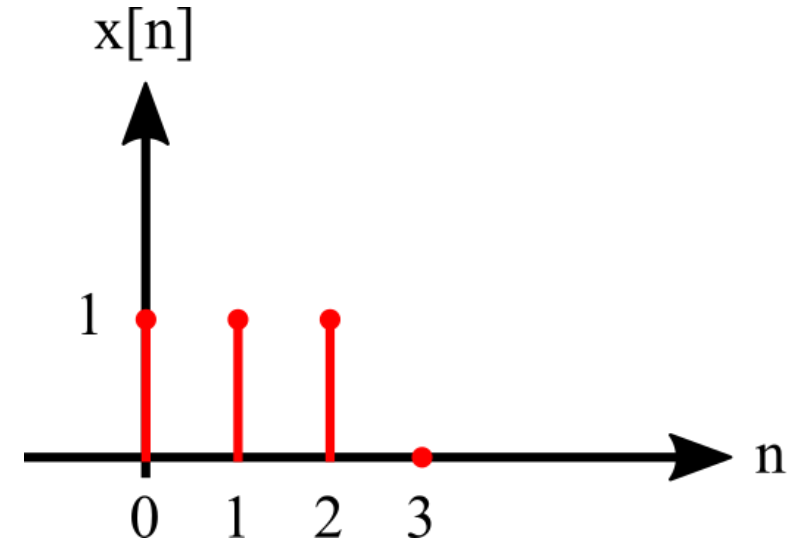
$$\begin{aligned} X^*[N - k] &= \sum_{n=0}^{N-1} \left[x[n] e^{-j(N-k)\frac{2\pi}{N}n} \right]^* \\ &= \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{N}n} e^{jN\frac{2\pi}{N}n} = 1 \\ &= X[k] \end{aligned}$$

Meaning:

- 1) One can calculate only samples $k = 0 \dots N/2$ (N is even) or $k = 0 \dots (N - 1)/2$ (N is odd)
- 2) If N is **even** then $X[N/2]$ is real.

EXAMPLE

- Let $x[n]$ a signal composed on ones of Length $N = 4$
- This is discrete square wave



- 1) Calculate the DFT $X[k]$. Note $X[k]$ is complex
- 2) In Personal Computer (PC) there will be two vectors X_R and jX_I as two separated vectors/arrays. What is the inverse transform (IDFT) of each?

EXAMPLE: SOLUTION – DFT CALCULATION

1) From the definition of DFT:

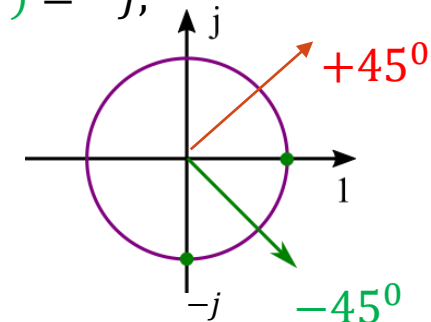
$$X[k] = \sum_{n=0}^3 x[n]e^{-j\frac{2\pi}{4}nk} = \sum_{n=0}^2 e^{-j\frac{2\pi}{4}nk} = \frac{1 - e^{-j\frac{2\pi}{4}k3}}{1 - e^{-j\frac{2\pi}{4}k}} = \frac{1 - e^{-jk\frac{3\pi}{2}}}{1 - e^{-j\frac{\pi}{2}k}}$$

$$X[0] = 3, \text{ sum of } x[n] \longrightarrow X[k=0] = \sum_{n=0}^{N-1} (W_N^{kn}) = N \cdot \delta[k \bmod N]$$

$$X[1] = \frac{1 - j^{\frac{3\pi}{2}}}{1 - (-j)^{\frac{\pi}{2}}} \overset{\text{graphically } -45^\circ}{\Rightarrow} \frac{-45^\circ}{+45^\circ} \overset{\text{graphically } -45^\circ - (+45^\circ)}{\Rightarrow} -j$$

$$X[2] = \frac{1 - (-1)^{-3\pi}}{1 - (-1)^{-\pi}} = 1$$

$$X[3] = X^*[N - 3] = X^*[4 - 3] = -j^* = j$$



To save in Personal Computer (PC) there will be two vectors X_R and jX_I $X^*[1]$ as two separate vectors/arrays:

$$X_R[k] = \{3, 0, 1, 0\}$$

$$jX_I[k] = j\{0, -1, 0, 1\}$$

EXAMPLE: SOLUTION – IDFT CALCULATION

2) Now we will apply the inverse transform on X_R and jX_I

$$X[k] = X_R[k] + jX_I[k] \rightarrow X_R[k] = \{3,0,1,0\} \quad jX_I[k] = \{0, -j, 0, j\} = j\{0, -1, 0, 1\}$$

Saved in PC this way

We now calculate inverse transforms for $X_R[k]$ and for $X_I[k]$:

$$0 \leq n \leq 3 \quad x_1[n] = \frac{1}{4} \sum_{k=0}^3 X_R[k] e^{j\frac{2\pi}{4}nk} =$$

$$= \frac{1}{4} [3 + e^{j\frac{2\pi}{4}n \cdot 2}] = \frac{1}{4} [3 + e^{j\pi n}] = \begin{cases} 1, & n=0 \\ \frac{1}{2}, & n=1 \\ 1, & n=2 \\ \frac{1}{2}, & n=3 \end{cases}$$

$k=0$
 $k=2$
 $(-1)^n$
 $n=0$
 $n=3$

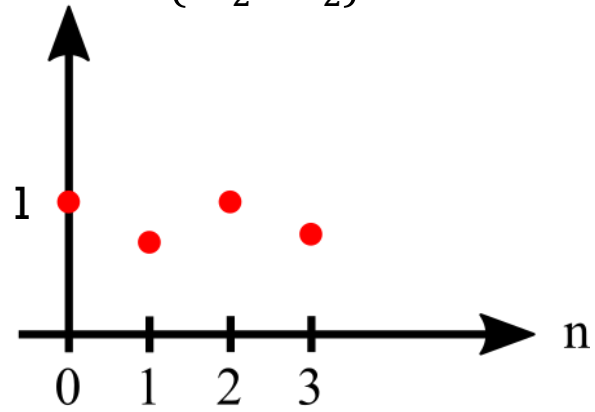
$$x_2[n] = \frac{1}{4} \sum_{k=0}^3 jX_I[k] e^{j\frac{2\pi}{4}nk} =$$

$$= \frac{1}{4} [-je^{j\frac{2\pi}{4}n} + je^{j\frac{2\pi}{4}n \cdot 3}] = \frac{1}{4} [-je^{j\frac{\pi}{2}n} + je^{j\frac{3\pi}{2}n}] = 0$$

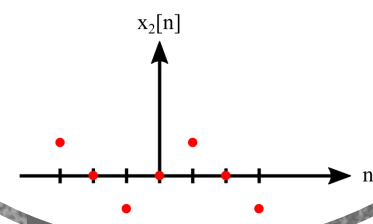
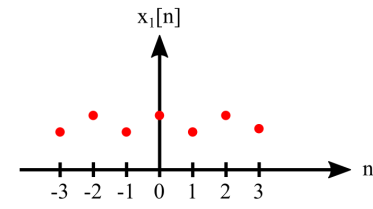
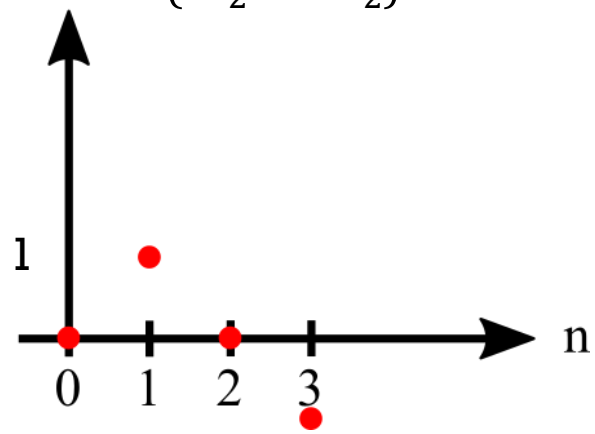
We now insert n and summarize:

$$= \begin{cases} 0, & n=0 \quad \text{H.W.} \\ \frac{1}{4} [(-j) \cdot (j) + (j) \cdot (-j)] = \frac{1}{2}, & n=1 \\ \frac{1}{4} [(-j) \cdot (-1) + (j) \cdot (-1)] = 0, & n=2 \\ \frac{1}{4} [(-j) \cdot (-j) + (j) \cdot (j)] = -\frac{1}{2}, & n=3 \end{cases}$$

$$x_1[n] = \left\{1, \frac{1}{2}, 1, \frac{1}{2}\right\}$$



$$x_2[n] = \left\{0, \frac{1}{2}, 0, -\frac{1}{2}\right\}$$

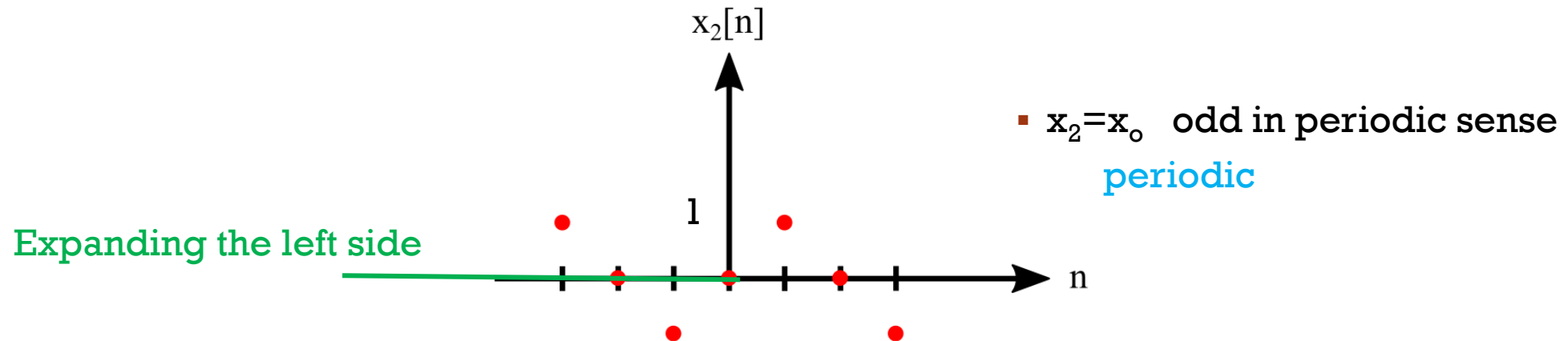
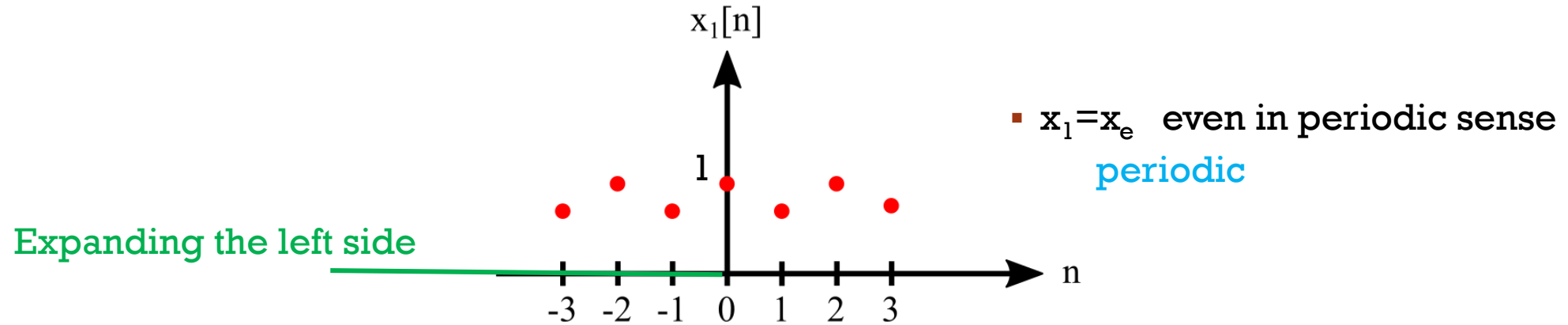


VALIDATION

- $x_1 + x_2 = x$

Even or Odd is a property measured in FT as compared to 0

SIMILAR PROPERTY: FT VS. DFT



SURVEY: DFT TRANSFORM



▪ EasyPolls:

Signal $x[n]$ is unlimited in time.

$X[k]=\text{DFT}\{x[n],n=0,\dots,N-1\}$. Can $x[n]$ be reconstructed from $X[k]$ for all n ?

- Yes, for all $x[n]$
- Yes, for some $x[n]$
- No

results

vote

תשובה :

אם זרקנו מידע
שאנחנו לא יודעים
אזי לא ניתן לשחזרו
– לכן תשובה ראשונה
לא נכונה

אם זרקנו מידע
שאנחנו יודעים אזי
ניתן לשחזרו חלקית

DFT OF LONG SEQUENCES

What is the meaning of DFT when the sequence is longer than N ?

DFT OF LONG SEQUENCES

- Let assume that $x[n]$ is long (length $L > N$). We want to calculate DFT of length N, so we will **sample in frequency at N points**.

Then, there are two options:

Case 1: $x[n]$ is long and periodic

Case 2: $x[n]$ is long and non periodic

DFT OF LONG SEQUENCES: CASE I

Case 1: $x[n]$ is long and periodic:

Assuming periodic signal $\tilde{x}[n]$ with period N while $x[n]$ is one period of $\tilde{x}[n]$.

$$\text{DFT}\{x[n]\} = X[k].$$

Calculate IDFT \rightarrow Solution:

$$\frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} = x[n] \quad 0 \leq n \leq N-1$$

Note: $e^{j\frac{2\pi}{N}kn}$ is periodic with period N in n , therefore $= \tilde{x}[n] \quad n \in \mathbb{Z}$

From here, pair DFT $x[n], X[k]$ are periodic sums with period N - this relation is named **DTFS** (**D**iscrete-**T**ime **F**ourier **S**eries).

THE DFT AND DTFS

Discrete-Time Fourier Series טור פורייה לסדרה

- They are almost the same computation
- Different only in the normalization

DTFS

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

DFT

$$x[n] = \frac{1}{N} \sum_{r=0}^{N-1} X[r] e^{j\frac{2\pi}{N}rn}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

WHY ARE THE NORMALIZATIONS DIFFERENT?

- The DFT is a transform
 - It is sampled from the Fourier of a non-periodic function
 - It calculates the integral over x
 - The DFT will obey Parseval's theorem

$$x[n] = T_s x(nT_s)$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{N-1} T_s x(nT_s) e^{-jnk\omega_0 T} \approx \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$$

DTFS VS. CTFS

- The DTFS is a series
 - The underlying function is periodic
 - The weight of a single coefficient is calculated as an average over one period
 - Parseval: sum of all the coefficients = average over time

Discrete-Time Fourier Series

DTFS

$$x[n] = \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn}$$

$$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn}$$

Continuous-Time Fourier Series טור פורייה

CTFS=FS(Fourier Series)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\omega_0 t} dt$$

EXAMPLE OF DTFS

- We want to find the values of the coefficients
- They turn out to disappear for all values except the base frequency
- Note the coefficients are
 - Symmetric in amplitude
 - Anti-symmetric in phase

$$x[n] = \sum_{r=-10}^9 D_r e^{j0.1\pi rn}$$

$$D_1 = \frac{1}{2j} = \frac{1}{2} e^{-j\frac{\pi}{2}}$$

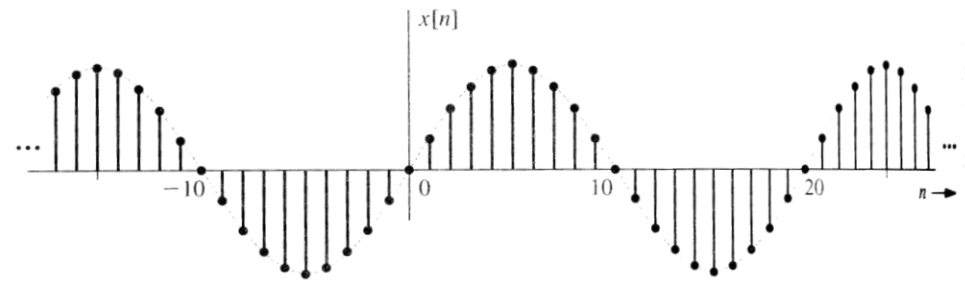
$$D_{-1} = -\frac{1}{2j} = \frac{1}{2} e^{j\frac{\pi}{2}}$$

$$|D_1| = |D_{-1}| = \frac{1}{2}$$

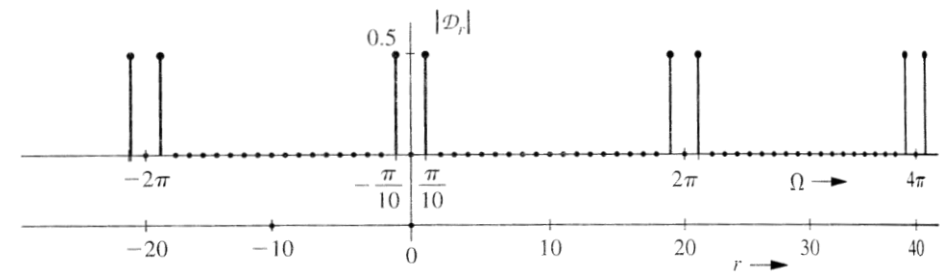
$$\angle D_1 = -\frac{\pi}{2}; \quad \angle D_{-1} = \frac{\pi}{2}$$

FREQUENCY SPECTRUM FOR THE EXAMPLE

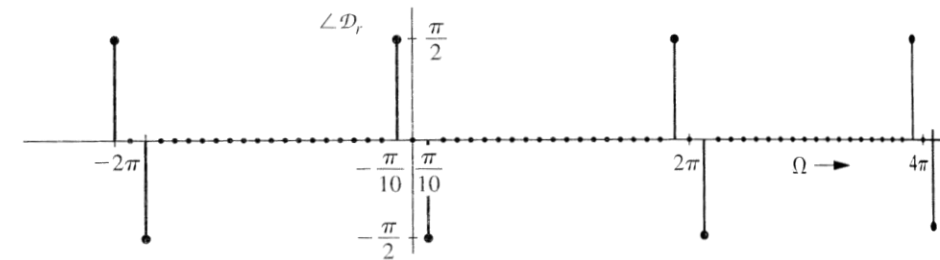
- The sine is a single frequency
- It is composed of two exponentials
 - One positive frequency
 - One negative frequency
- The spectrum is periodic and discrete



(a)



(b)



(c)

DFT OF LONG SEQUENCES: CASE II

We assume that $x[n]$ is long of length $L > N$.

Case 2: $x[n]$ is long and non periodic:

- In this case, we can calculate DFT only on elements $0 \leq n \leq N - 1$
- And so, based on the definition we will calculate the N points DTFT:

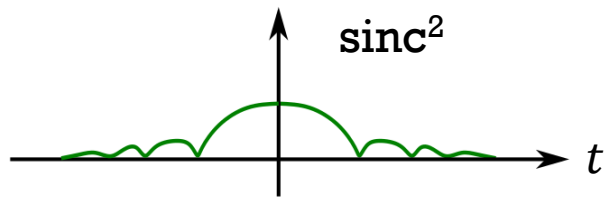
$$\begin{aligned}
 \hat{x}[n] &= \frac{1}{N} \sum_{k=0}^{N-1} X[e^{j\frac{2\pi}{N}k}] e^{j\frac{2\pi}{N}kn} = \frac{1}{N} \sum_{k=0}^{N-1} \sum_{m=-\infty}^{\infty} x[m] e^{-j\frac{2\pi}{N}km} e^{j\frac{2\pi}{N}kn} = \\
 &= \sum_{m=-\infty}^{\infty} x[m] \frac{1}{N} \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}k[n-m]} = \sum_{m=-\infty}^{\infty} x[m] \underbrace{\delta[(n-m) \bmod N]}_{\text{Impulse train}} \xrightarrow{\text{Get rid of modulo via } \delta} \\
 &= \sum_{m=-\infty}^{\infty} x[m] \sum_{l=-\infty}^{\infty} \delta[n-m+lN] = \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x[m] \delta[n+lN-m] = \\
 &= \sum_{l=-\infty}^{\infty} x[n+lN] \quad \text{sifting}
 \end{aligned}$$

Sampled DTFT (points to the first sum)
 Impulse train (points to the delta function)
 Get rid of modulo via δ (points to the transition)

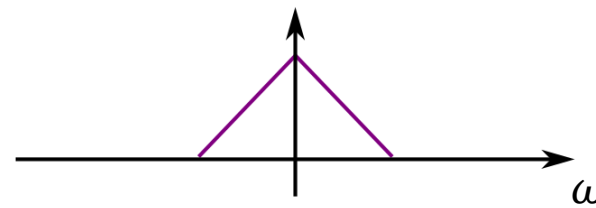
This reminds replications – sampling in frequency-> replication in time when $x[n]$ of length N , the replications will not overlap

EXAMPLE: GRAPHICAL DESCRIPTION

time
 $x(t)$



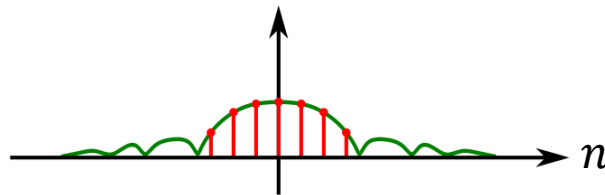
frequency
 $X(j\omega)$



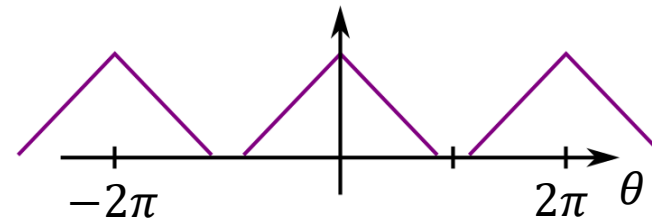
Triangular window
FT

Sampled signal

$x[n]$



$X(e^{j\theta})$

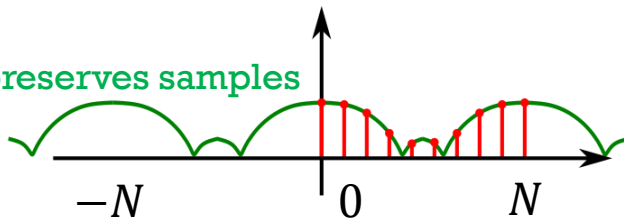


Influence of sampling in time
DTFT

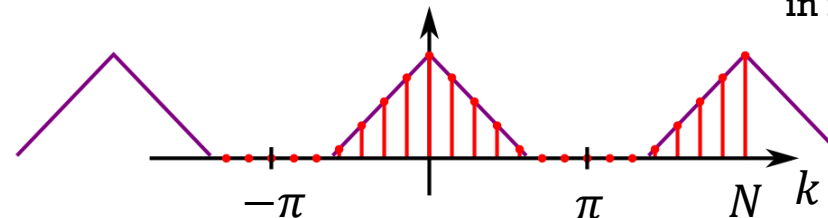
Periodic,
Replications

*this option preserves samples
in frequency

$\hat{x}[n]$



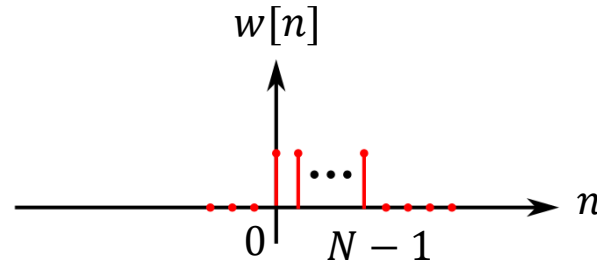
$X[k]$



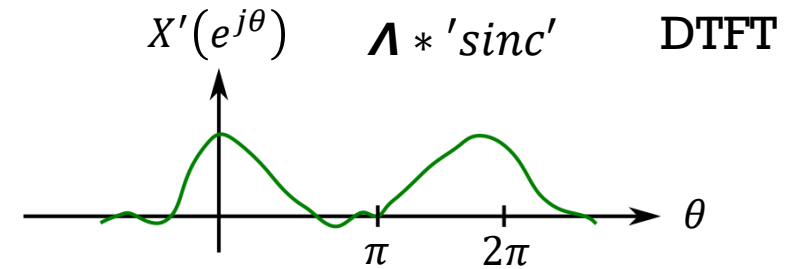
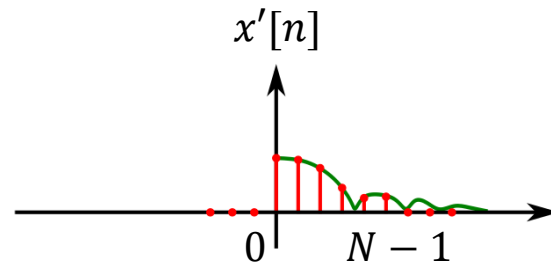
Sampled frequency
Influence of N-points sampling
in frequency
DFT

EXAMPLE: GRAPHICAL DESCRIPTION – WE MULTIPLY $X[N]$ BY A WINDOW

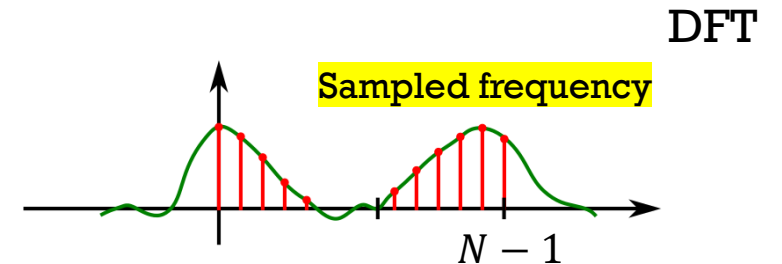
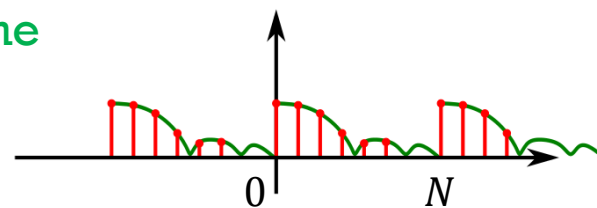
- In this case the replications in time will overlap.
- To avoid the replications, we will apply on $x[n]$ window in time



$$x'[n] = x[n] \cdot w[n]$$

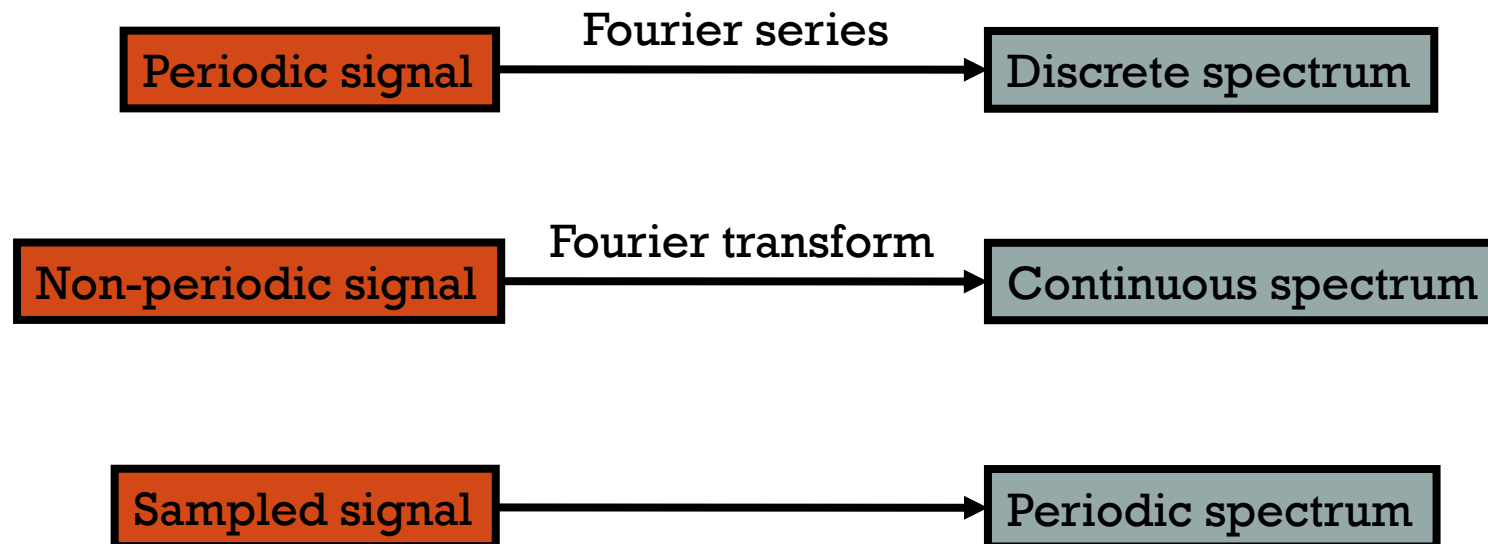


This option preserves the (limited) information in time



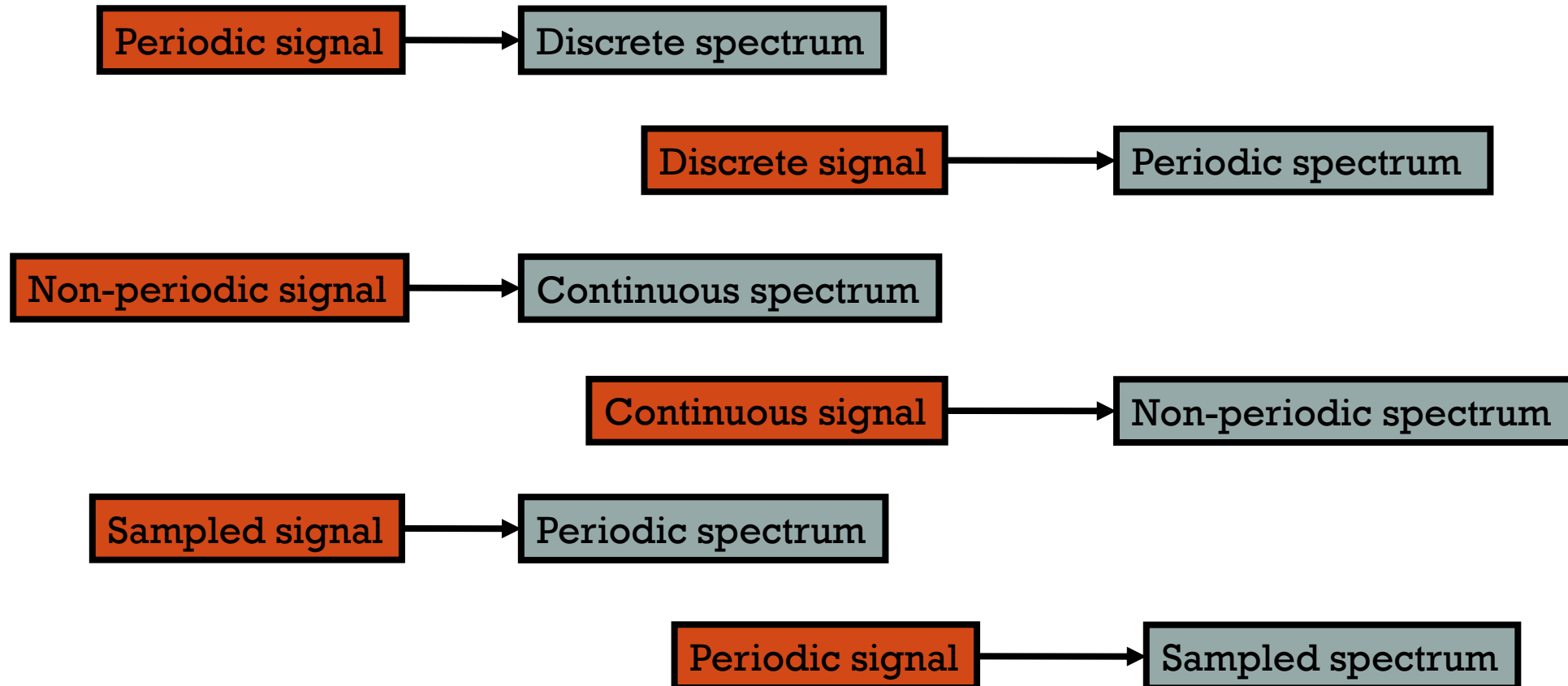
FOURIER TRANSFORM RELATIONS

- There is a relationship between periodicity of a function and the continuous nature of its transform.



FOURIER TRANSFORM AND FOURIER SERIES

- By duality, we get more relationships





SURVEY: SAMPLING IN FREQUENCY

- EasyPolls:

$x=[1,0,1,1]$, $X=\text{DTFT}\{x\}$ sampled with $N=3$,
 $y=\text{IDFT}\{X,3\}=?$

[1,0,1]

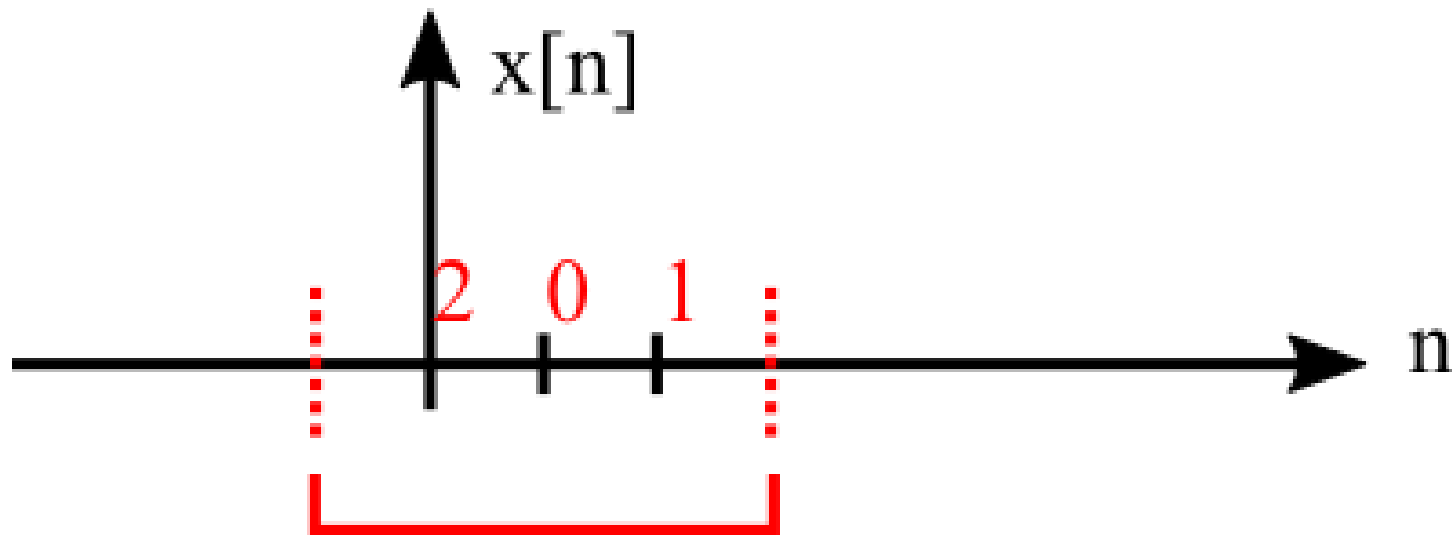
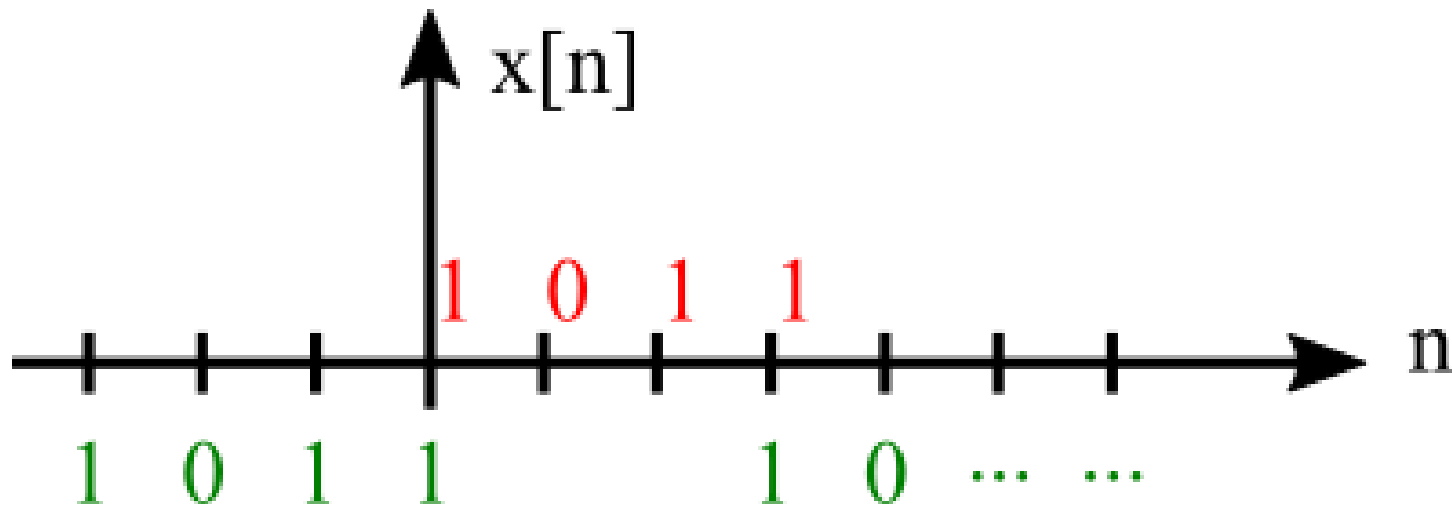
[1,0,2]

[2,0,1]

[1,1,1]

results

vote



SURVEY: SOLUTION

$x=[1,0,1,1]$, $X=\text{DTFT}\{x\}$ sampled with $N=3$,
 $y=\text{IDFT}\{X,3\}=?$


- [1,0,1]
- [1,0,2]
- [2,0,1]
- [1,1,1]

results

vote



DFT MATRIX REPRESENTATION



Notation via
linear algebra

MATRIX REPRESENTATION

$$\text{DFT} \quad X[k] = \sum_{n=0}^{N-1} x[n] W_N^{-kn}, \quad 0 \leq k \leq N-1$$

- Matrix representation: ייצוג מטריציוני

$$x[n] = [x[0], x[1], x[2], \dots, x[N-1]]; \quad X[k] = [X[0], X[1], X[2], \dots, X[N-1]] \quad \text{אם:}$$

נגדיר מטריצה W_N :

$$W_N = \begin{matrix} & \begin{matrix} n=0 & n=1 & n=2 & \dots & n=N-1 \end{matrix} \\ \begin{matrix} k=0 \\ k=1 \\ k=2 \\ \vdots \\ k=N-1 \end{matrix} & \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & \dots & W_N^{-(N-1)} \\ W_N^0 & W_N^{-2} & W_N^{-4} & \dots & W_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ W_N^0 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \dots & W_N^{-(N-1)^2} \end{bmatrix} \end{matrix}$$

נוח לייצג התמרת DFT כהכפלה של אות הרצוי במטריצה W_N :

$$X_N[k] = W_N \cdot x_N[n]$$

$$W_N = e^{j \frac{2\pi}{N}}$$

F is named 'DFT matrix'

$$\underline{X} = \underline{F} \underline{x}$$

Note: it reminds F matrix in non-uniform sampling

DFT IN MATRIX REPRESENTATION

$N \times N$

$$N \times 1 \begin{bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{bmatrix} = \begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & \dots & W_N^{-(N-1)} \\ W_N^0 & W_N^{-2} & W_N^{-4} & \dots & W_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ W_N^0 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \dots & W_N^{-(N-1)^2} \end{bmatrix} \times \begin{bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{bmatrix} N \times 1$$

MATRIX REPRESENTATION: EXAMPLES FOR N=2, 4

$$\begin{bmatrix} W_N^0 & W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^{-1} & W_N^{-2} & \dots & W_N^{-(N-1)} \\ W_N^0 & W_N^{-2} & W_N^{-4} & \dots & W_N^{-2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ W_N^0 & W_N^{-(N-1)} & W_N^{-2(N-1)} & \dots & W_N^{-(N-1)^2} \end{bmatrix}$$

$$W_N = e^{j\frac{2\pi}{N}}$$

נתון N=2: הצגה בצורה מטריציונית.

פתרון: $X_2[k] = W_2 \cdot x_2[n]$

$$\underline{F} = \begin{bmatrix} W_2^0 & W_2^0 \\ W_2^0 & W_2^{-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

DC חיבור דגימות
AC דגימות חיסור

Direct Current
Alternating Current

כאשר $W_N = e^{j\frac{2\pi}{N}}$ ואז

נתון N=4: הצגה בצורה מטריציונית.

פתרון: $X_4[k] = W_4 \cdot x_4[n]$

$$\underline{F} = \begin{matrix} & \begin{matrix} n \\ k \end{matrix} \\ \begin{matrix} 1 \\ 1 \\ 1 \\ 1 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \end{matrix}$$

$$W_N^{-kn} = e^{-j\frac{2\pi}{4}nk} = e^{-j\frac{\pi}{2}nk}$$

THE CURRENT WAR THE TALE OF AN EARLY TECH RIVALRY

DC DIRECT CURRENT

The flow of electricity is in one direction only. The system operates at the same voltage level throughout, and is not as efficient for long-distance transmission.

Direct current runs through:

- Battery-Powered Devices
- Fuel and Solar Cells
- Light Emitting Diodes

"TESLA'S IDEAS ARE SPLENDID, BUT THEY ARE UTTERLY IMPRACTICAL."
-THOMAS EDISON

AC ALTERNATING CURRENT

Electric charges periodically reverse direction and is transmitted to customers by a transformer that continuously made higher voltages.

Alternating current runs through:

- Car Motors
- Radio Signals
- Appliances

"IF EDISON HAD A NEEDLE TO FIND IN A HAYSTACK, HE WOULD PROCEED AT ONCE... UNTIL HE FOUND THE OBJECT OF HIS SEARCH, I WAS A SORRY WITNESS OF SUCH DOINGS, KNOWING THAT A LITTLE THEORY AND CALCULATION WOULD HAVE SAVED HIM 90 PERCENT OF HIS LABOR."
-NIKOLA TESLA

THOMAS EDISON VS. NIKOLA TESLA

You would have never found two geniuses so spiteful of each other beyond turn-of-the-century inventors Nikola Tesla and Thomas Edison. They worked together—and hated each other. Let's compare their life, achievements, and embittered battles.

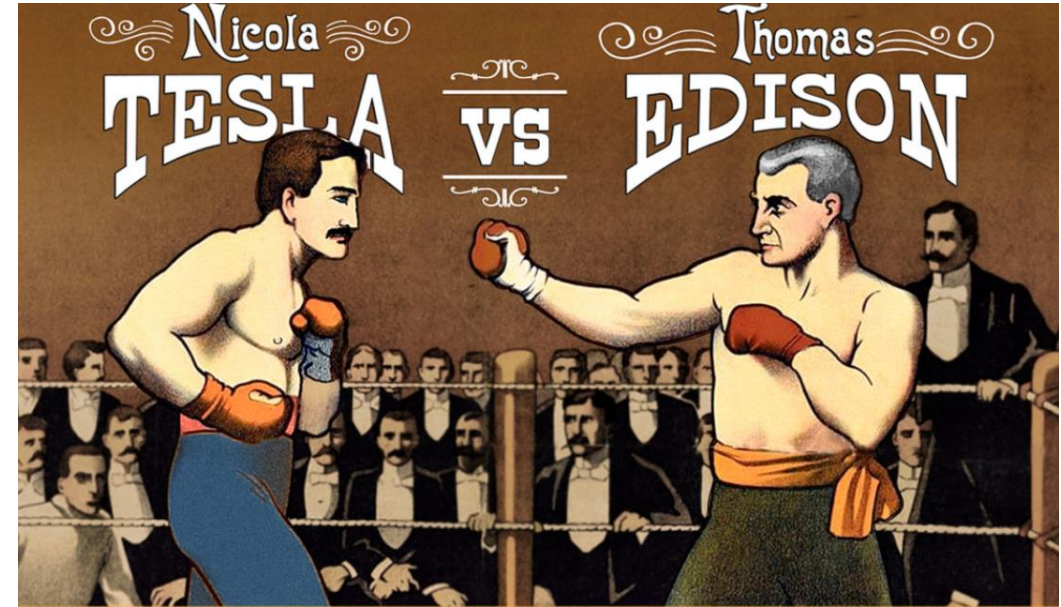
| | |
|---|--|
| 1847 BORN 1858 | 1856 BORN 1878 |
| Milan, Ohio | Smiljan, Croatia |
| NICKNAME Wizard of Menlo Park | Wizard of the West |
| EDUCATION Home-schooled and self-taught | Studied math, physics, and mechanics at The Polytechnic Institute at Graz |
| METHOD Blue and white | Color and green |
| WAR OF CURRENTS, ELECTRICAL TRANSMISSION IDEA AC (Alternating Current) | DC (Direct Current) |
| NOTABLE INVENTIONS Incandescent light bulb, phonograph, record-making technology, motion picture camera, DC motor and electric power | Radio transmitter, radio, radio receiver, alternating current, AC motor and electric power |
| NUMBER OF US PATENTS 112 | 308 |
| NUMBER OF NOBEL PRIZES WON 0 | 1 |
| NUMBER OF ELEPHANTS ELECTROCUTED 0 | 1 |
| DEATH 1931—Passed away peacefully in his home | 1943—Killed by lightning and in 1944 by a plane crash |


FALLING OUT
Edison presented Tesla a generous reward if he could amount to the direct current system. The young engineer took on the challenge and ended up saving Edison more than \$100,000 (millions of dollars by today's standards). When Tesla asked for his rightful compensation, Edison declined to pay him. Tesla resigned shortly after, and the bitter rivalry spanned the rest of his life, culminating in a dramatic final confrontation.

EDISON FRIES AN ELEPHANT
In order to prove the dangers of Tesla's alternating current, Thomas Edison staged a highly publicized demonstration of the streamer system known as "Frying the Elephant." The dead elephants after being electrocuted.

WAR OF CURRENTS OFFICIALLY SETTLED
In 2001, the IEEE named Tesla the inventor of the AC current electricity system that began when Thomas Edison opened the power station in 1882. It changed to only provide alternating current.

NOBEL PRIZE CONTROVERSY
In 1912, both Tesla and Tesla were on the Nobel Prize for their work in physics, but ultimately neither won. It is rumored that Tesla had been promised a Nobel prize, but it never came.





WAR OF CURRENTS

- In 1912, the Nobel Committee announced that Nikola Tesla and Thomas Edison were the recipients of the Physics Prize; instead, the prize went to Gustav Dalen. Details of the reversal are unclear but it is known that **Tesla refused the prize** (and the \$20,000 that came with it).

MATRIX REPRESENTATION: EXAMPLE N=3

H.W.

נתון $N=3$: הצגה בצורה מטרצית.
פתרון: $X_3[k] = W_3 \cdot x_3[n]$

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & W_3^{-1} & W_3^{-2} \\ 1 & W_3^{-2} & W_3^{-4} \end{bmatrix}$$

כאשר $W_N = e^{j\frac{2\pi}{N}}$ ואז

$$W_3 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & e^{-j\frac{2\pi}{3}} & e^{-2 \cdot j\frac{2\pi}{3}} \\ 1 & e^{-2 \cdot j\frac{2\pi}{3}} & e^{-4 \cdot j\frac{2\pi}{3}} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \cos\left(\frac{2\pi}{3}\right) - j \sin\left(\frac{2\pi}{3}\right) & \cos\left(2 \cdot \frac{2\pi}{3}\right) - j \sin\left(2 \cdot \frac{2\pi}{3}\right) \\ 1 & \cos\left(2 \cdot \frac{2\pi}{3}\right) - j \sin\left(2 \cdot \frac{2\pi}{3}\right) & \cos\left(4 \cdot \frac{2\pi}{3}\right) - j \sin\left(4 \cdot \frac{2\pi}{3}\right) \end{bmatrix} =$$
$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -\frac{1}{2} - j\frac{\sqrt{3}}{2} & -\frac{1}{2} + j\frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} + j\frac{\sqrt{3}}{2} & -\frac{1}{2} - j\frac{\sqrt{3}}{2} \end{bmatrix}$$

DEMONSTRATION OF DFT

Exponents here are presented graphically

DFT matrix

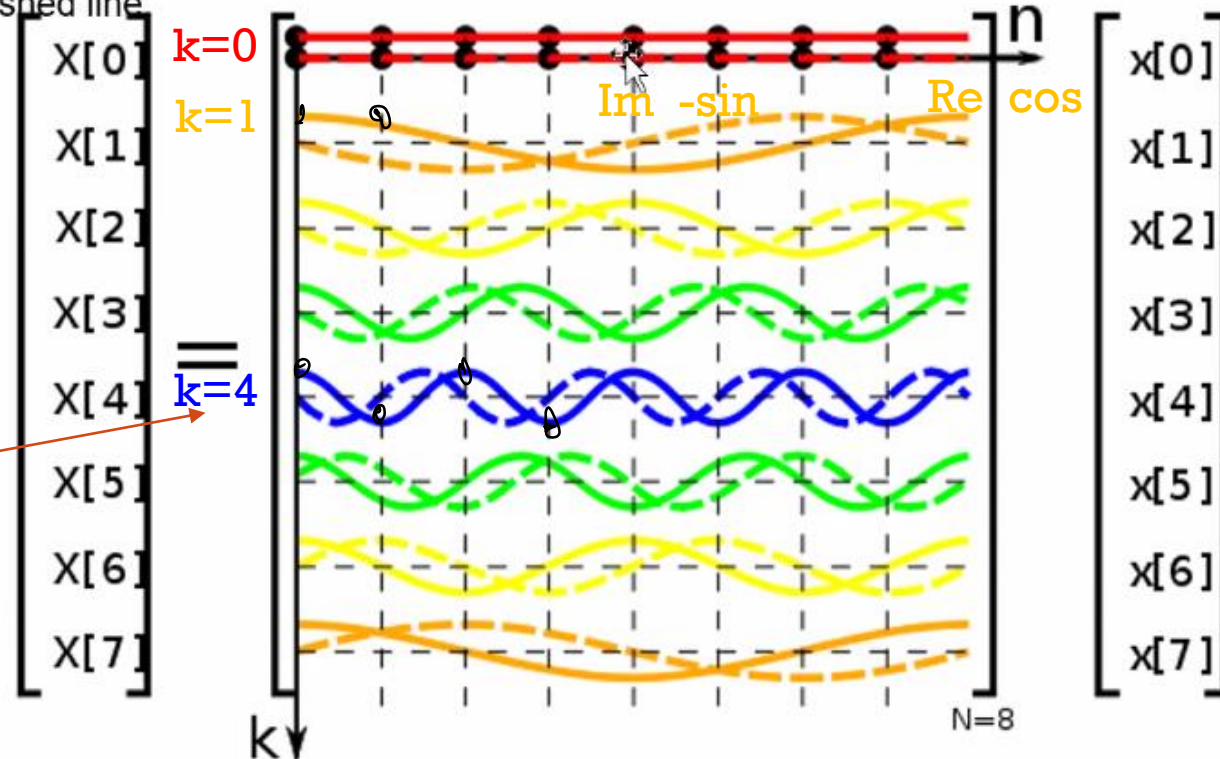
The real part is denoted by a solid line, the imaginary part (sine wave) by a dashed line

N=8

positive k

highest change: change in each sample

negative k



http://en.wikipedia.org/wiki/DFT_matrix

IDFT: MATRIX REPRESENTATION

$$\begin{aligned} \text{From IDFT definition: } \underline{x} &= \frac{1}{N} \underline{F}^* \underline{X} \\ \text{Inverse matrix DFT} &= \underline{F}^{-1} \underline{X} \end{aligned}$$

$$\sum_{k=0}^{N-1} W_N^{kn} W_N^{-km} = N \cdot \delta[n - m]$$

Unitary matrix
(orthonormal and
complex)



$$\left(\frac{1}{\sqrt{N}} \underline{F} \right)^H \left(\frac{1}{\sqrt{N}} \underline{F} \right) = \underline{I}$$
$$(F_N)_{k,m} = W_N^{-km}$$

*In Matlab: X' is both Hermite and transpose
but X.' is just transpose*