LECTURE 4 - FIBER DISPERSION

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OUTLINE

Dispersion in Single-Mode Fibers

- Group-velocity dispersion
- Material dispersion
- Waveguide dispersion
- Higher-order dispersion

Polarization-mode dispersion

Dispersion-induced limitations

Limitations on the Bit Rate

Bibliography

DISPERSION POWER PENALTY

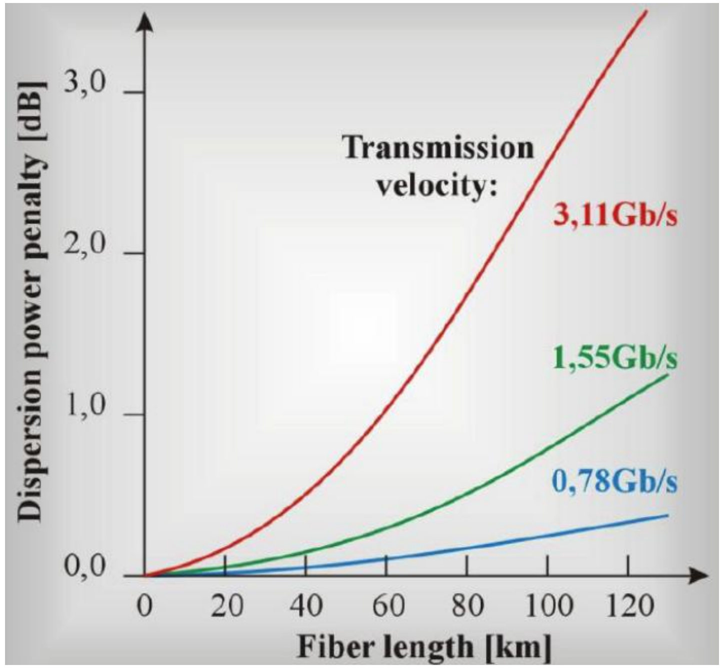
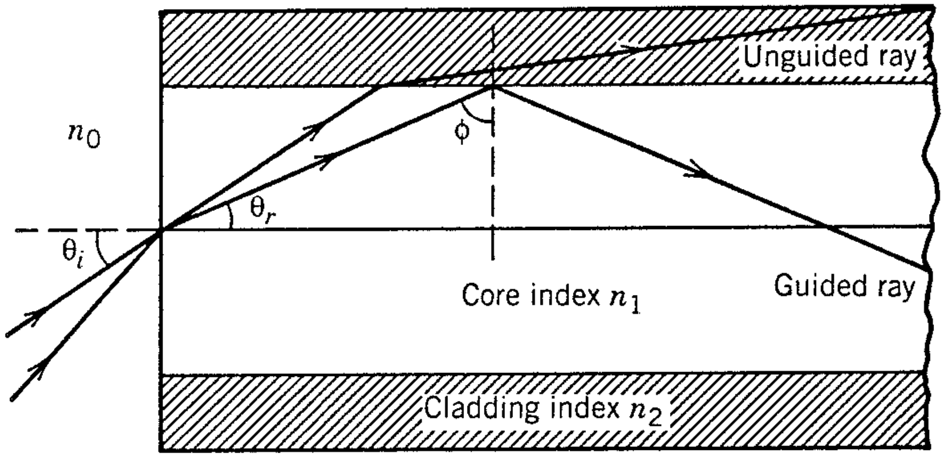


Figure 1: Attenuation caused by dispersion at transmission speed a) 0.78 Gb/s, b) 1.55 Gb/s, c) 3.11 Gb/s for the optical fiber characterized by the chromatic dispersion of 17 ps/nm/km and propagating the light from the single-mode laser DFB at spectral width of 0.1 nm.

DISPERSION IN SINGLE-MODE FIBERS (SMF)

- The intermodal dispersion in multimode fibers leads to considerable broadening of short optical pulses (10 ns/km).
- In the geometrical-optics description, such broadening was attributed to different paths followed by different rays.
- In the modal description it is related to the different mode indices (or group velocities) associated with different modes.



From [1]

DISPERSION IN SINGLE-MODE FIBERS (SMF)

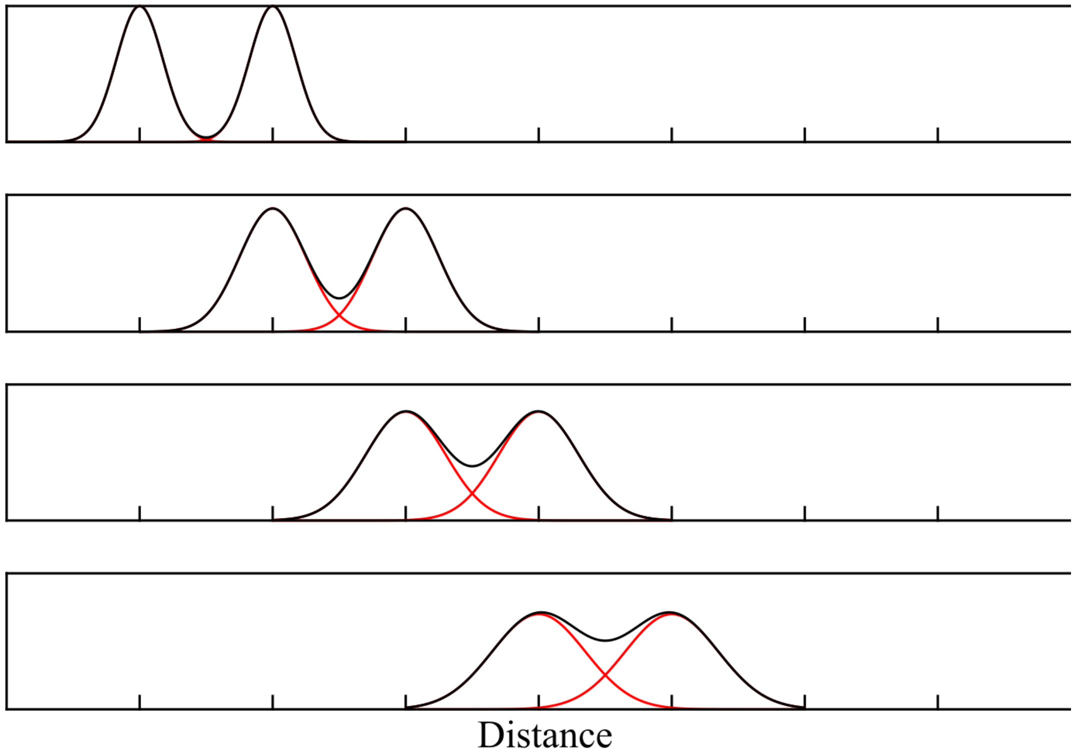


Figure 2: Broadening and attenuation of two adjacent pulses as they propagate in a fiber.

DISPERSION IN SINGLE-MODE FIBERS (SMF)

- The main advantage of SMFs is that intermodal dispersion is absent simply because the energy of the injected pulse is transported by a single mode. However, pulse broadening does not disappear altogether.
- The group velocity associated with the fundamental mode is frequency dependent because of chromatic dispersion.
- As a result, different spectral components of the pulse travel at slightly different group velocities, a phenomenon referred to as group-velocity dispersion (GVD), intramodal dispersion, or simply fiber dispersion.

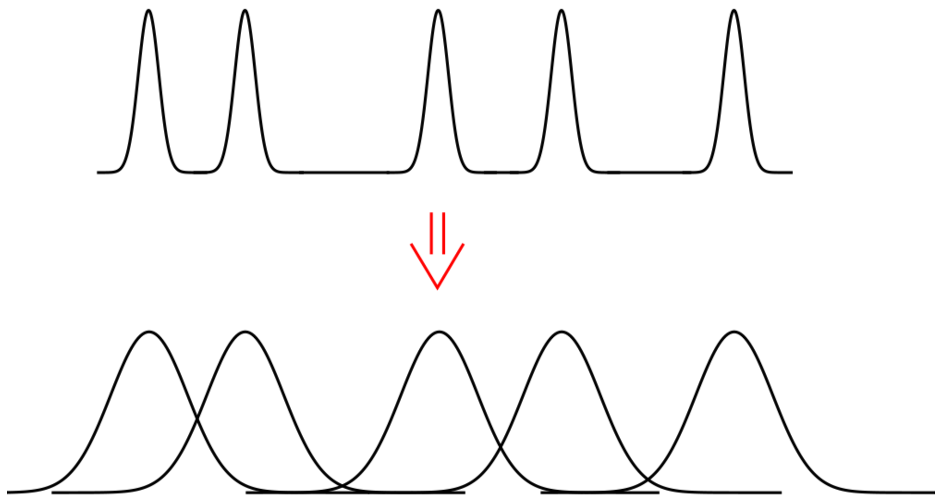


Figure 3: Broadening of pulses as they propagate in a fiber along many kilometers.

GROUP-VELOCITY DISPERSION

Assume a single-mode fiber of length L. The frequency time delay at the end of the fiber is $T = L/v_g$, where v_g is the group velocity which is defined as:

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} \tag{1}$$

By using $\beta = \bar{n}k_0 = \bar{n}\omega/c$ and $v_g = c/\bar{n}_g$ in Eq. (1), the group index $\bar{n}_g$ is defined as:

$$\bar{n}_g = \bar{n} + \omega\left(\frac{d\bar{n}}{d\omega}\right) \tag{2}$$

where $\bar{n}$ is the mode index (shown in lecture 3).

The frequency dependence of the group velocity leads to pulse broadening simply because different spectral components of the pulse disperse during propagation and do not arrive simultaneously at the fiber output.

REFRACTIVE INDEX n AND GROUP INDEX n_g

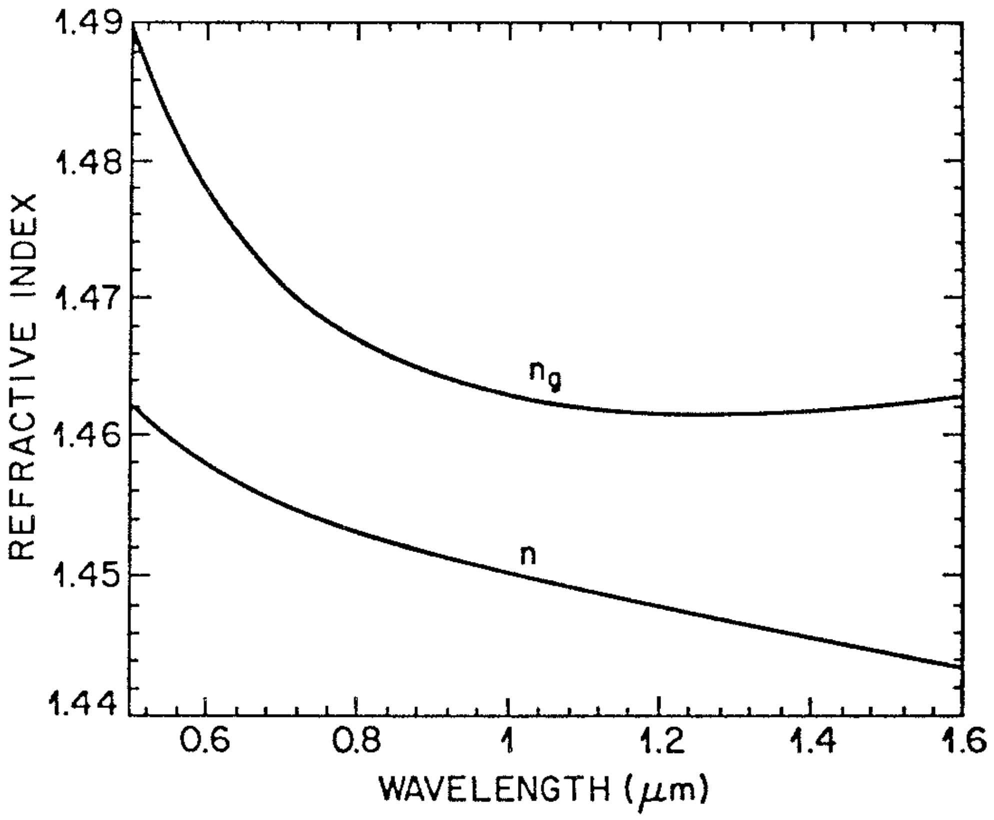


Figure 4: The refractive index n and group index n_g for fused silica [1].

GROUP-VELOCITY DISPERSION

The pulse broadening for a fiber of length L is defined as:

$$\Delta T = \frac{dT}{d\omega}\Delta\omega = \frac{d}{d\omega}\left(\frac{L}{v_g}\right)\Delta\omega = L\frac{d^2\beta}{d\omega^2}\Delta\omega = L\beta_2\Delta\omega \qquad (3)$$

where $\Delta\omega$ is the spectral width of the pulse and β_2 is the GVD parameter which defines the broadening of the pulse in the fiber.

In some optical systems, wavelength is used instead of frequency - $\omega = 2\pi c/\lambda$.

$$\Delta T = \frac{d}{d\lambda}\left(\frac{L}{v_g}\right)\Delta\lambda = DL\Delta\lambda \qquad (4)$$

and

$$D = -\frac{2\pi c}{\lambda^2}\beta_2 \qquad (5)$$

where D is the dispersion parameter and is expressed in units of ps/(nm·km).

DISPERSION PARAMETER - D

The effect of dispersion on the bit rate B can be estimated by using the criterion $B\Delta T < 1$. Using ΔT this condition becomes:

$$BL|D|\Delta\lambda < 1 \tag{6}$$

It provides an order-of-magnitude estimate of the BL product offered by single-mode fibers. For standard silica fibers, D is relatively small in the wavelength region near 1.3 μm [$D \sim 1\ \mathrm{ps/(nm \cdot km)}$].

For a semiconductor laser, the spectral width $\Delta\lambda$ is 2-4 nm even when the laser operates in several longitudinal modes. The BL product of such lightwave systems can exceed 100 (Gb/s)km. Indeed, 1.3 μm telecommunication systems typically operate at a bit rate of 2Gb/s with a repeater spacing of 40-50 km. The BL product of single-mode fibers can exceed 1 (Tb/s)km when SM semiconductor lasers are used to reduce $\Delta\lambda$ below 1 nm.

DISPERSION PARAMETER - *D*

The dispersion parameter D can vary considerably when the operating wavelength is shifted from 1.3 μm. The wavelength dependence of D is governed by the frequency dependence of the mode index $\bar{n}$.

$$D = -\frac{2\pi c}{\lambda^2}\frac{d}{d\omega}\left(\frac{1}{v_g}\right) = -\frac{2\pi}{\lambda^2}\left(2\frac{d\bar{n}}{d\omega} + \omega\frac{d^2\bar{n}}{d\omega^2}\right) = D_M + D_W \qquad (7)$$

D_M is the material dispersion and D_W is the waveguide dispersion defined as:

$$D_M = -\frac{2\pi}{\lambda^2}\frac{dn_{2g}}{d\omega} = \frac{1}{c}\frac{dn_{2g}}{d\lambda} \qquad (8)$$

$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{{n_{2g}}^2}{n_2\omega}\frac{Vd^2(Vb)}{dV^2} + \frac{dn_{2g}}{d\omega}\frac{d(Vb)}{dV}\right] \qquad (9)$$

where n_{2g} is the group index of the cladding.

MATERIAL DISPERSION

Material dispersion occurs because the refractive index of the material changes with the frequency ω. On a fundamental level, the material dispersion is related to the resonance frequencies where the material absorbs the radiation. Far from the medium resonances, the refractive index $n(\omega)$ is approximated by the Sellmeier equation:

$$n^2(\omega) = 1 + \sum_{i=1}^{M} \frac{B_i {\omega_i}^2}{{\omega_i}^2 - \omega^2} \qquad (10)$$

$$\lambda_i = 2\pi c/\omega_i$$

where ω_i is the resonance frequency and B_i is the oscillator strength.

SILICA DISPERSION

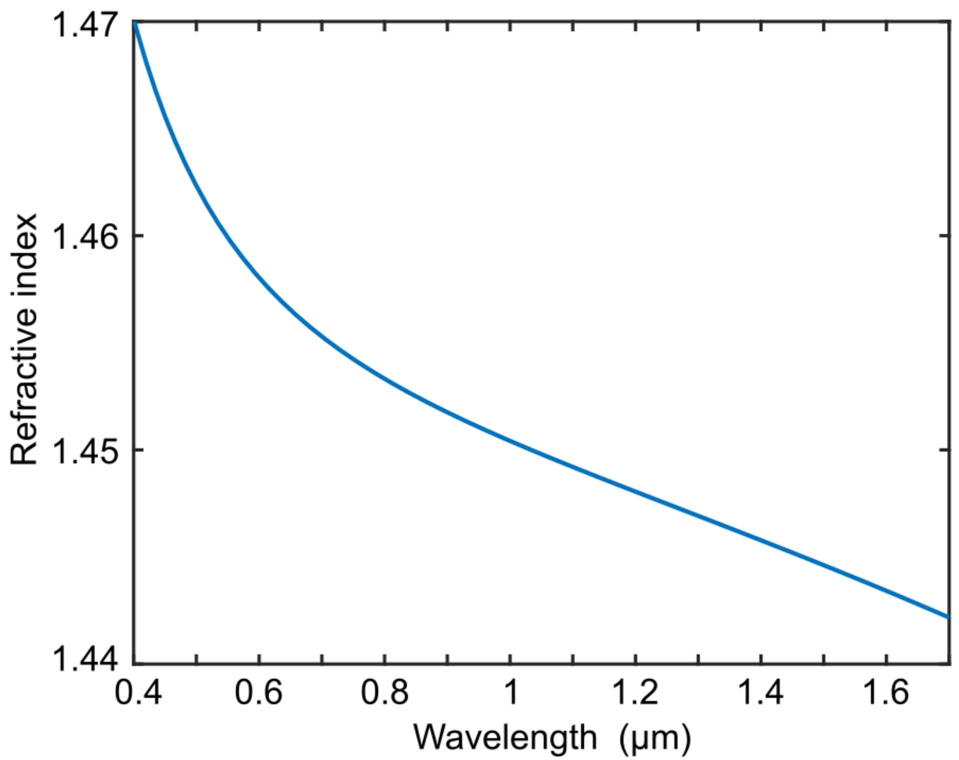


Figure 5: Dependence of refraction index and wavelength for fused silica [Malitson 1965].

MATERIAL DISPERSION

The sum in Eq. (10) extends over all material resonances that contribute to the frequency range of interest. In optical fibers, the parameters are obtained by fitting the dispersion curves to the equation. They depend on the amount of dopants and have been tabulated for several kinds of fibers.

For pure silica:

$B_1 = 0.6961663, B_2 = 0.4079426, B_3 = 0.8974794,$

$\lambda_1 = 0.0684043\ \mu\text{m}, \lambda_2 = 0.1162414\ \mu\text{m},$ and $\lambda_3 = 9.896161\ \mu\text{m}.$

The group index $n_g = n + \omega(dn/d\omega)$ can be obtained by using these parameter values.

ZERO-DISPERSION WAVELENGTH

- Material dispersion DM is related to the slope of n_g by the relation

$$D_M = c^{-1}(dn_g/d\lambda)$$

- At $\lambda = 1.276\ \mu\text{m}$ the dispersion is zero which named zero-dispersion wavelength - λ_{ZD}.
- $D_M < 0$ when $\lambda < \lambda_{ZD}$ and $D_M > 0$ when $\lambda > \lambda_{ZD}$.

In the wavelength range 1.25-1.66 μm it can be approximated by an empirical relation:

$$D_M \approx 122\left(1 - \frac{\lambda_{ZD}}{\lambda}\right) \tag{11}$$

WAVEGUIDE DISPERSION

The waveguide dispersion D_W depends on the V parameter of the fiber. For fiber, both derivatives are positive, as shown in Fig. 6. Therefore, D_W is negative for wavelength range of 0-1.6 µm.

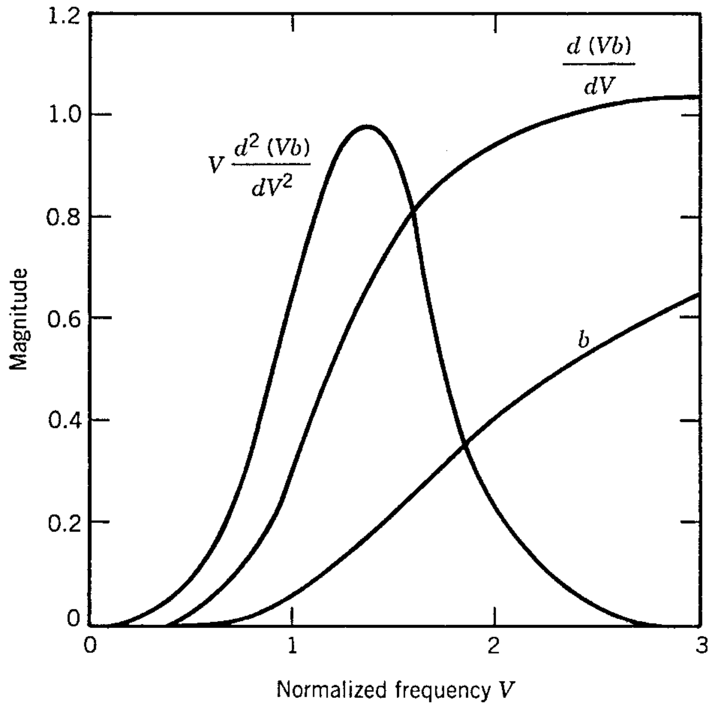
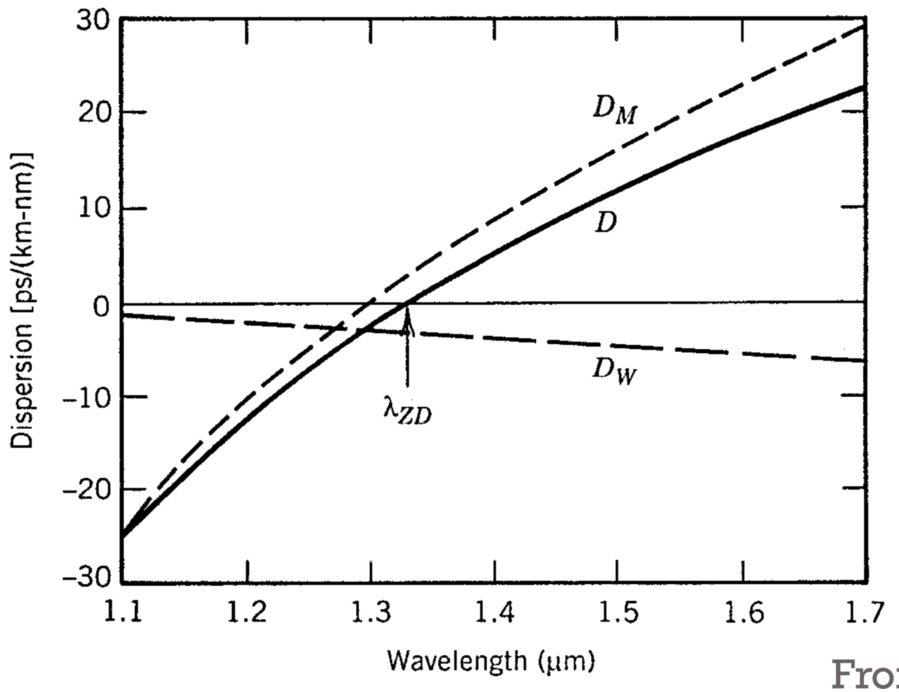


Figure 6: b and its derivatives $d(Vb)/dV$ and $V[d^2(Vb)/dV^2]$ as function of the V parameter [1].

TOTAL DISPERSION FOR FIBER

- D_M is negative below λ_{ZD} and positive above it.
- The waveguide dispersion shifts λ_{ZD} by 30-40 nm to near 1.31 μm.
- D_M also reduces D from its material value D_M in the wavelength range 1.3-1.6 μm which is of interest for optical communication systems.
- $D = 15 - 18\ \text{ps/(nm}\cdot\text{km)}$ around 1.55 μm.



From [1]

This region is of considerable interest for lightwave systems, since the fiber loss is minimum near 1.55 μm. High values of D limit the performance of 1.55 μm lightwave systems.

WAVEGUIDE DISPERSION

- It is possible to design the fiber parameters, such as the core radius a and the index difference Δ and change the waveguide dispersion D_W. λ_{ZD} is shifted to 1.55 μm. Such fibers are called dispersion shifted fibers.
- In addition, It is possible to adjust the waveguide dispersion causing the total dispersion D to be relatively small over a wide wavelength range from 1.3 to 1.6 μm. Such fibers are called dispersion-flattened fibers.

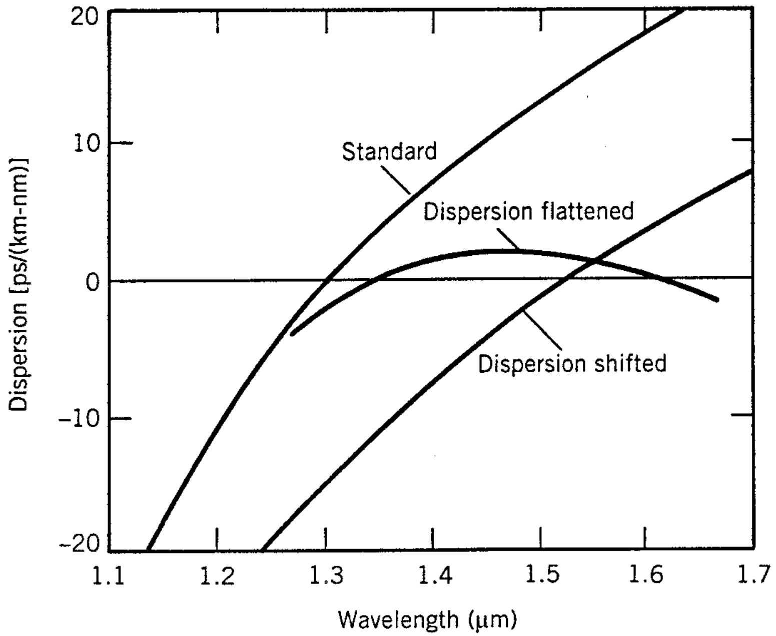


Figure 7: Typical wavelength dependence of the dispersion parameter D for standard, dispersion-shifted, and dispersion-flattened fibers [1].

DISPERSION-SHIFTED FIBERS

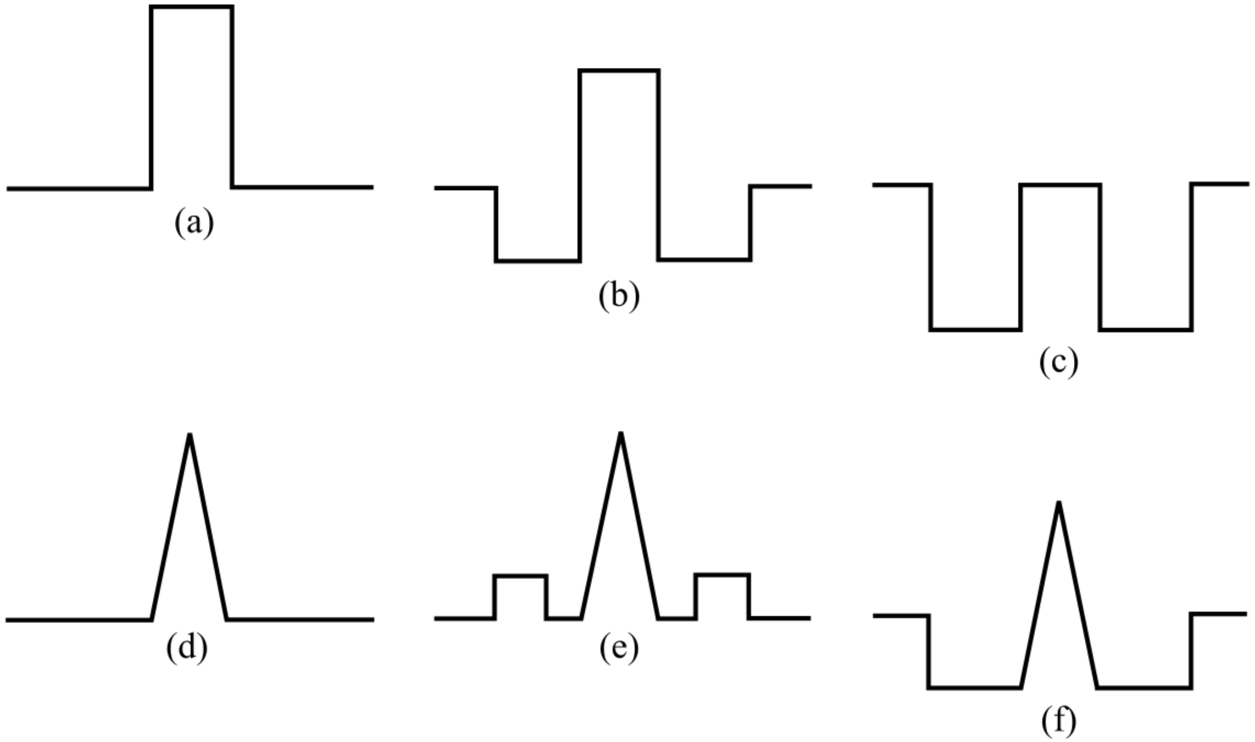
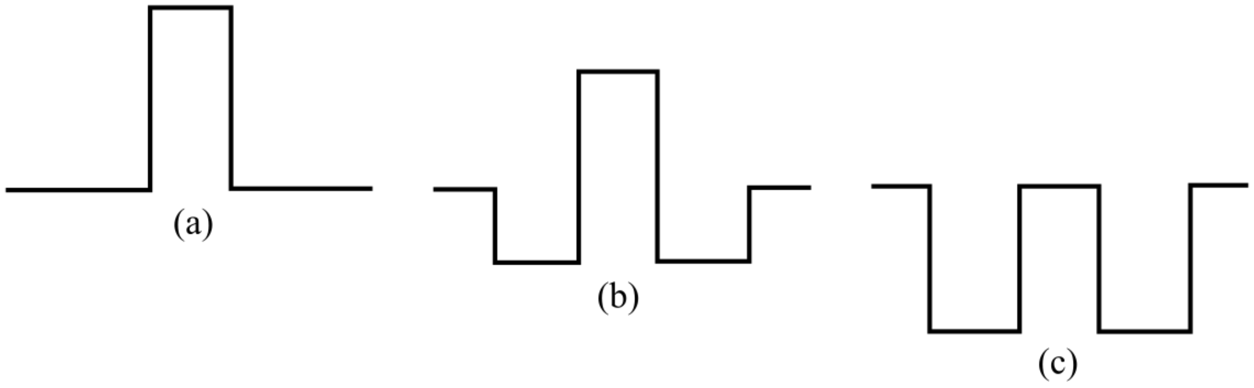


Figure 8: Several index profiles used in the design of single-mode fibers. Upper and lower rows correspond to standard and dispersion-shifted fibers, respectively [1].

DISPERSION-SHIFTED FIBERS



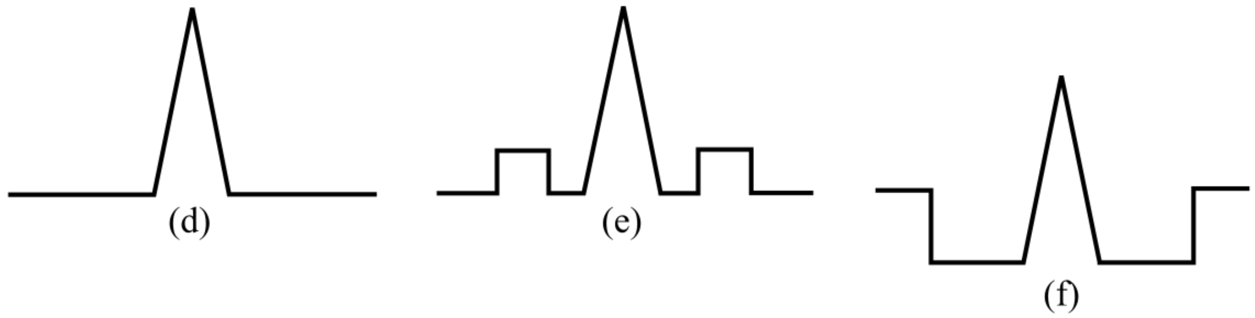
The top row corresponds to standard fibers which are designed to have minimum dispersion near 1.3 μm with a cutoff wavelength in the range 1.1-1.2 μm.

(a) A pure-silica cladding and a core doped with GeO_2 to obtain $\Delta \approx 3 \cdot 10^{-3}$.

(b) Lowers the cladding index over a region adjacent to the core by doping it with fluorine.

(c) An undoped core - doubly clad or depressed-cladding fibers and W fibers, reflecting the shape of the index profile.

DISPERSION-SHIFTED FIBERS



The bottom row shows three index profiles used for dispersion-shifted fibers for which the zero-dispersion wavelength is chosen in the range 1.45-1.60 μm. A triangular index profile with a depressed or raised cladding is often used for this purpose. Sometimes as many as four cladding layers are used for dispersion-flattened fibers.

DISPERSION-SHIFTED FIBERS

- The design of dispersion modified fibers involves the use of multiple cladding layers and a tailoring of the refractive-index profile.
- Waveguide dispersion can be used to produce dispersion-decreasing fibers in which GVD decreases along the fiber length because of axial variations in the core radius.
- In another kind of fibers, known as the dispersion compensating fibers, GVD is made normal and has a relatively large magnitude.

HIGHER-ORDER DISPERSION

- It appears from Eq. (6) that the BL product of a single-mode fiber can be increased indefinitely by operating at the zero-dispersion wavelength λ_{ZD} where $D = 0$.
- Optical pulses still experience broadening because of higher-order dispersive effects. This feature can be understood by noting that D cannot be made zero at all wavelengths contained within the pulse spectrum centered at λ_{ZD}. Clearly, the wavelength dependence of D will play a role in pulse broadening.
- Higher-order dispersive effects are governed by the dispersion slope $S = dD/d\lambda$. The parameter S is also called a differential-dispersion parameter. By using Eq. (5) it can be written as:

$$S = (2\pi c/\lambda^2)^2 \beta_3 + (4\pi c/\lambda^3)\beta_2 \qquad (12)$$

where $\beta_3 = d\beta_2/d\omega \equiv d^3\beta/d\omega^3$ is the third-order dispersion parameter. At $\lambda = \lambda_{ZD}$, $\beta_2 = 0$ and S is proportional to β_3.

HIGHER-ORDER DISPERSION

- The numerical value of the dispersion slope S plays an important role in the design of modern WDM systems. Since $S > 0$ for most fibers, different channels have slightly different GVD values. This feature makes it difficult to compensate dispersion for all channels simultaneously.
- To solve this problem, new kind of fibers have been developed for which S is either small (reduced-slope fibers) or negative (reverse-dispersion fibers). Table 1 lists the values of dispersion slopes for several commercially available fibers.
- It may appear from Eq. (6) that the limiting bit rate of a channel operating at $\lambda = \lambda_{ZD}$ will be infinitely large. However, this is not the case since S or β_3 becomes the limiting factor in that case.

CHARACTERISTICS OF SEVERAL COMMERCIAL FIBERS

Table 1: Characteristics of several commercial fibers

Fiber type and trade name	A_{eff} (μm²)	λ_{ZD} (nm)	D (C band) [ps/(km·nm)]	Slope S [ps/(km·nm²)]
Corning SMF-28	80	1302-1322	16 to 19	0.09
Lucent AllWave	80	1300-1322	17 to 20	0.088
Alcatel ColorLock	80	1300-1322	16 to 19	0.09
Corning Vascade	101	1300-1310	18 to 20	0.06
Lucent TrueWave-RS	50	1470-1490	2.6 to 6	0.05
Corning LEAF	72	1490-1500	2 to 6	0.06
Lucent TrueWave-XL	72	1570-1580	-1.4 to -4.6	0.112
Alcatel TeraLight	65	1440-1450	5.5 to 10	0.058

HIGHER-ORDER DISPERSION

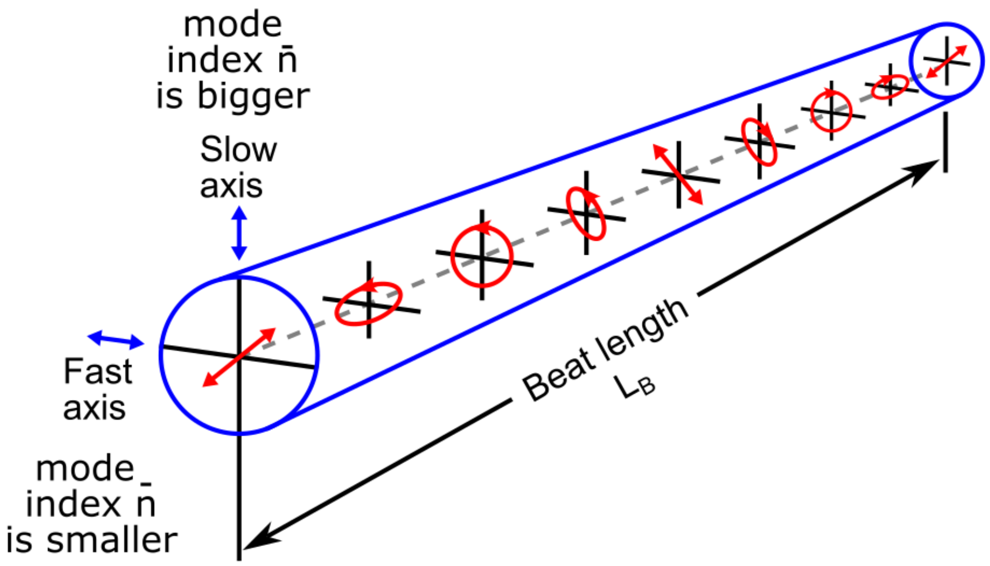
We can estimate the limiting bit rate by noting that for a source of spectral width , the effective value of dispersion parameter becomes $D = S\Delta\lambda$. The limiting bit rate-distance product can now be obtained by using Eq. (6) with this value of D. The resulting condition becomes:

$$BL|S|(\Delta\lambda)^2 < 1 \qquad (13)$$

For a multimode semiconductor laser with $\Delta\lambda = 2$ nm and a dispersion-shifted fiber with $S = 0.05\ \text{ps}/(\text{nm}^2 \cdot \text{km})$ at $\lambda = 1.55\ \mu\text{m}$, the BL product approaches 5 (Tb/s)·km. Further improvement is possible by using single-mode semiconductor lasers.

POLARIZATION-MODE DISPERSION

- A potential source of pulse broadening is related to fiber birefringence. Small departures from perfect cylindrical symmetry lead to birefringence because of different mode indices associated with the orthogonally polarized components of the fundamental fiber mode.
- If the input pulse excites both polarization components, it becomes broader as the two components disperse along the fiber because of their different group velocities. This phenomenon is called the PMD and has been studied extensively because it limits the performance of modern lightwave systems.



POLARIZATION-MODE DISPERSION

In fibers with constant birefringence (e.g., polarization-maintaining fibers), pulse broadening can be estimated from the time delay ΔT between the two polarization components during propagation of the pulse. For a fiber of length L, ΔT is given by:

$$\Delta T = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right| = L\left|\beta_{1x} - \beta_{1y}\right| = L(\Delta\beta_1) \qquad (14)$$

where the subscripts x and y identify the two orthogonally polarized modes and $\Delta\beta_1$ is related to the difference in group velocities along the two principal states of polarization.

Similar to the case of intermodal dispersion, the quantity $\Delta T/L$ is a measure of PMD. For polarization-maintaining fibers, $\Delta T/L$ is quite large (~1 ns/km) when the two components are equally excited at the fiber input but can be reduced to zero by launching light along one of the principal axes.

POLARIZATION-MODE DISPERSION

- The situation is somewhat different for conventional fibers in which birefringence varies along the fiber in a random fashion. The polarization state of light propagating in fibers will generally be elliptical and would change randomly along the fiber during propagation.
- The polarization state will also be different for different spectral components of the pulse. The final polarization state is not of concern for most lightwave systems as photodetectors used inside optical receivers are insensitive to the state of polarization unless a coherent detection scheme is employed.
- What affects such systems is not the random polarization state but pulse broadening induced by random changes in the birefringence. This is referred to as PMD-induced pulse broadening.

POLARIZATION-MODE DISPERSION

The analytical treatment of PMD is quite complex in general because of its statistical nature. A simple model divides the fiber into a large number of segments. Both the degree of birefringence and the orientation of the principal axes remain constant in each section but change randomly from section to section. In effect, each fiber section can be treated as a phase plate using a Jones matrix.

Jones calculus is a way to describe polarized light in optics discovered by R. C. Jones in 1941.

- Jones vector - describe the polarized light

$$E = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} E_x e^{j\phi_x} \\ E_y e^{j\phi_y} \end{pmatrix} \quad (15)$$

- Jones matrix - describe linear optical elements. For example: rotation and polarization.

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \quad (16)$$

POLARIZATION-MODE DISPERSION

- Propagation of each frequency component associated with an optical pulse through the entire fiber length is then governed by a composite Jones matrix obtained by multiplying individual Jones matrices for each fiber section.
- The composite Jones matrix shows that two principal states of polarization exist for any fiber such that, when a pulse is polarized along them, the polarization state at fiber output is frequency independent to first order, in spite of random changes in fiber birefringence. These states are analogous to the slow and fast axes associated with polarization-maintaining fibers.
- An optical pulse not polarized along these two principal states splits into two parts which travel at different speeds. The differential group delay ΔT is largest for the two principal states of polarization.

POLARIZATION-MODE DISPERSION

The principal states of polarization provide a convenient basis for calculating the moments of ΔT. The PMD-induced pulse broadening is characterized by the root-mean-square (RMS) value of ΔT, obtained after averaging over random birefringence changes. Several approaches have been used to calculate this average. The variance ${\sigma_T}^2 \equiv \langle(\Delta T)^2\rangle$ turns out to be the same in all cases and is given by:

$$ {\sigma_T}^2(T) = 2(\Delta\beta_1)^2 {l_c}^2\left[e^{-z/l_c} + z/l_c - 1\right] \quad (17) $$

where l_c is the correlation length defined as the length over which two polarization components remain correlated; its value can vary over a wide range from 1 m to 1 km for different fibers, typical values being ~10 m.

PULSE BROADENING

For short distances such that $z \ll l_c$, $\sigma T = (\Delta\beta_1)z$ from Eq. (17), as expected for a polarization-maintaining fiber. For distances $z > 1$ km, a good estimate of pulse broadening is obtained using $z \gg l_c$. For a fiber of length L, ΔT in this approximation becomes

$$\sigma T \approx (\Delta\beta_1)\sqrt{l_c L} \equiv D_p\sqrt{L} \tag{18}$$

where D_p is the PMD parameter.

Measured values of D_p vary from fiber to fiber in the range $D_p = 0.01 - 10\ \mathrm{ps}/\sqrt{\mathrm{km}}$. Fibers installed during the 1980s have relatively large PMD such that $D_p > 0.1\ \mathrm{ps}/\sqrt{\mathrm{km}}$. In contrast, modern fibers are designed to have low PMD, and typically $D_p < 0.1\ \mathrm{ps}/\sqrt{\mathrm{km}}$ for them. Because of the $\sqrt{L}$ dependence, PMD-induced pulse broadening is relatively small compared with the GVD effects. Indeed, $\sigma T \sim 1$ ps for fiber lengths ~100 km and can be ignored for pulse widths > 10 ps.

BASIC PROPAGATION EQUATION

The discussion in previous slides of pulse broadening is based on an intuitive phenomenological approach. It provides a first-order estimate for pulses whose spectral width is dominated by the spectrum of the optical source.

In general, the extent of pulse broadening depends on the width and the shape of input pulses. Each frequency component of the optical field propagates in a single-mode fiber as:

$$\tilde{E}(r,\omega) = \hat{x}F(x,y)\tilde{\beta}(0,\omega)\exp(j\beta z) \qquad (19)$$

where $\hat{x}$ is the polarization unit vector, $\tilde{\beta}(0,\omega)$ is the initial amplitude, and β is the propagation constant.

BASIC PROPAGATION EQUATION

- The field distribution $F(x, y)$ of the fundamental fiber mode can be approximated by the Gaussian distribution.
- In general, $F(x, y)$ also depends on ω, but this dependence can be ignored for pulses whose spectral width $\Delta\omega$ is much smaller than ω_0 – a condition satisfied by pulses used in lightwave systems.
- Here ω_0 is the frequency at which the pulse spectrum is centered; it is referred to as the carrier frequency.

BASIC PROPAGATION EQUATION

Different spectral components of an optical pulse propagate inside the fiber according to the simple relation

$$\tilde{B}(z,\omega) = \tilde{B}(0,\omega)\exp(j\beta z) \tag{20}$$

The amplitude in the time domain is obtained by taking the inverse Fourier transform and is given by

$$B(z,t) = \int_{-\infty}^{\infty} \tilde{B}(z,\omega)\exp(-j\omega t)\; d\omega \tag{21}$$

The initial spectral amplitude $\tilde{B}(0,\omega)$ is just the Fourier transform of the input amplitude $B(0,\omega)$.

BASIC PROPAGATION EQUATION

Pulse broadening results from the frequency dependence of β. For pulses for which $\Delta\omega \ll \omega_0$, it is useful to expand $\beta(\omega)$ in a Taylor series around the carrier frequency ω_0 and retain terms up to third order. In this quasi-monochromatic approximation:

$$\beta(\omega) = \bar{n}(\omega)\frac{\omega}{c} \approx \beta_0 + \beta_1(\Delta\omega) + \frac{\beta_2}{2}(\Delta\omega)^2 + \frac{\beta_3}{6}(\Delta\omega)^3 \quad (22)$$

where $\Delta\omega = \omega - \omega_0$ and $\beta_m = (d^m\beta/d\omega^m)_{\omega=\omega_0}$. From Eq. (1) $\beta_1 = 1/v_g$, where v_g is the group velocity. The GVD coefficient β_2 is related to the dispersion parameter D by Eq. (5), whereas β_3 is related to the dispersion slope S through Eq. (12). We substitute Eqs. (20) and (22) in Eq. (21) and introduce a slowly varying amplitude $A(z,t)$ of the pulse envelope as:

$$B(z,t) = A(z,t)\exp[j(\beta_0 z - \omega_0 t)] \quad (23)$$

BASIC PROPAGATION EQUATION

The amplitude $A(z,t)$ is found to be given by:

$$A(z,t) = \frac{1}{2\pi} d(\Delta\omega)\tilde{A}(0,\Delta\omega) \cdot \exp\left[j\beta_1 z\Delta\omega + \frac{j}{2}\beta_2 z(\Delta\omega)^2 + \frac{j}{5}\beta_3 z(\Delta\omega)^3 - j(\Delta\omega)t \right] \quad (24)$$

where $\tilde{A}(0,\Delta\omega) \equiv \tilde{A}(0,\omega)$ is the Fourier transform of $A(0,t)$.

By calculating $\partial A/\partial z$ and noting that $\Delta\omega$ is replaced by $j(\partial A/\partial t)$ in the time domain, Eq. (24) can be written as:

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{j\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial x^3} = 0 \quad (25)$$

This is the basic propagation equation that governs pulse evolution inside a single-mode fiber. In the absence of dispersion ($\beta_2 = \beta_3 = 0$), the optical pulse propagates without change in its shape such that $A(z,t) = A(0, t - \beta_1 z)$.

BASIC PROPAGATION EQUATION

Transforming to a reference frame moving with the pulse and introducing the new coordinates

$$t' = t - \beta_1 z \quad \text{and} \quad z = z' \tag{26}$$

β_1 can be eliminated in Eq. (25) to yield

$$\frac{\partial A}{\partial z'} + \frac{j\beta_2}{2}\frac{\partial^2 A}{\partial t'^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t'^3} = 0 \tag{27}$$

For simplicity of notation, we drop the primes over z' and t'.

CHIRPED GAUSSIAN PULSES

Lets consider the propagation of chirped Gaussian pulses inside optical fibers by choosing the initial field as:

$$A(0,t) = A_0 \exp\left[-\frac{1-jC}{2}\left(\frac{t}{T_0}\right)^2\right] \tag{28}$$

where A_0 is the peak amplitude. The parameter T_0 represents the half-width at $1/e$ intensity point. It is related to the full-width at half-maximum (FWHM) of the pulse by the relation

$$T_{\text{FWHM}} = 2\sqrt{\ln(2)}\,T_0 \approx 1.665T_0 \tag{29}$$

The parameter C governs the frequency chirp imposed on the pulse. A pulse is said to be chirped if its carrier frequency changes with time.

CHIRPED GAUSSIAN PULSES

The frequency change is related to the phase derivative and is given by:

$$\delta\omega(t) = -\frac{\partial\phi}{\partial t} = \frac{C}{{T_0}^2} \qquad (30)$$

where ϕ is the phase of $A(0,t)$. The time-dependent frequency shift $\delta\omega$ is called the chirp. The spectrum of a chirped pulse is broader than that of the unchirped pulse. This can be seen by taking the Fourier transform of Eq. (28) so that

$$\tilde{A}(0,\omega) = A_0\sqrt{\frac{2\pi {T_0}^2}{1+jC}}\exp\left[-\frac{\omega^2 {T_0}^2}{2(1+jC)}\right] \qquad (31)$$

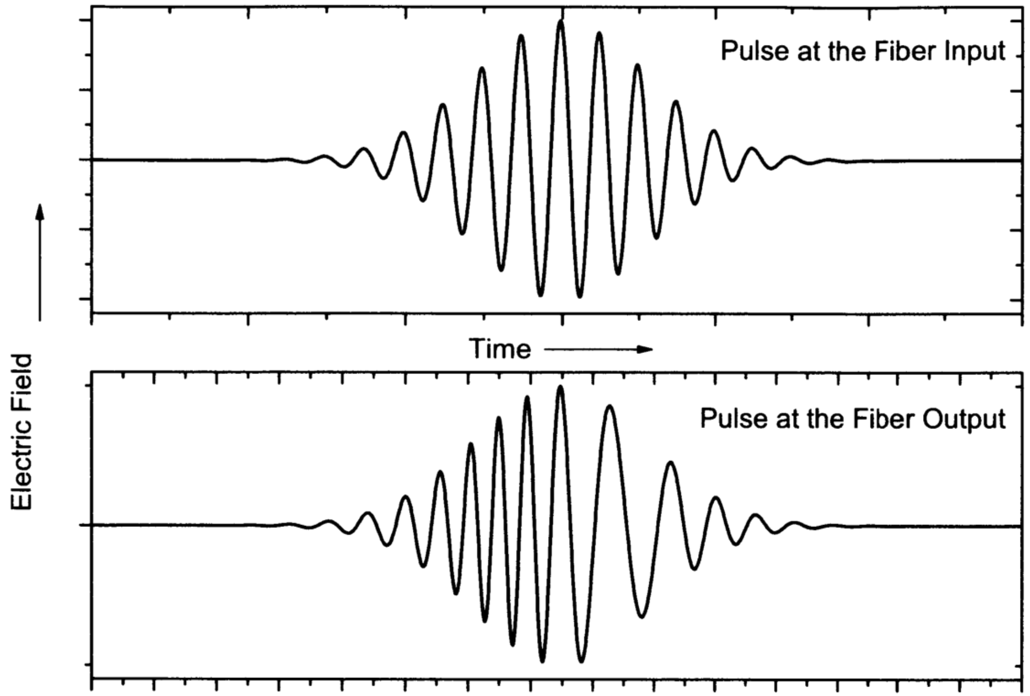


Figure 9: Electric field of a pulse at the fiber input and output. Note that the pulse here propagates from the left to the right [1].

CHIRPED GAUSSIAN PULSES

The spectral half-width (at $1/e$ intensity point) is given by:

$$\Delta\omega_0 = \sqrt{1 + C^2} \cdot {T_0}^{-1} \tag{32}$$

In the absence of frequency chirp ($C = 0$), the spectral width satisfies the relation $\Delta\omega_0 T_0 = 1$. Such a pulse has the narrowest spectrum and is called transform-limited. The spectral width is enhanced by a factor of $\sqrt{1 + C^2}$ in the presence of linear chirp, as seen in Eq. (32). The pulse-propagation equation (27) can be easily solved in the Fourier domain. Its solution is [see Eq. (24)]:

$$A(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(0,\omega) \exp\left(\frac{j}{2}\beta_2 z\omega^2 + \frac{j}{6}\beta_3 z\omega^3 - j\omega t\right) d\omega \tag{33}$$

where $\tilde{A}(0,\omega)$ is given by Eq. (31) for the Gaussian input pulse. Let us first consider the case in which the carrier wavelength is far away from the zero-dispersion wavelength so that the contribution of the β_3 term is negligible.

CHIRPED GAUSSIAN PULSES

The integration in Eq. (33) can be performed analytically with the result:

$$A(z,t) = \frac{A_0}{\sqrt{Q(z)}} \exp\left[\frac{(1+jC)t^2}{2{T_0}^2 Q(z)}\right] \qquad (34)$$

where $Q(z) = 1 + (C - j)\beta_2 z/{T_0}^2$. It shows that a Gaussian pulse remains Gaussian on propagation but its width, chirp, and amplitude change as dictated by the factor $Q(z)$. For example, the chirp at a distance z changes from its initial value C to become

$$C_1(z) = C + (1 + C^2)\beta_2 z/{T_0}^2$$

Changes in the pulse width with z are quantified through the broadening factor given by:

$$\frac{T_1}{T_0} = \sqrt{\left(1 + \frac{C\beta_2 z}{{T_0}^2}\right)^2 + \left(\frac{\beta_2 z}{{T_0}^2}\right)^2} \qquad (35)$$

where T_1 is the half-width defined similar to T_0.

CHIRPED GAUSSIAN PULSES

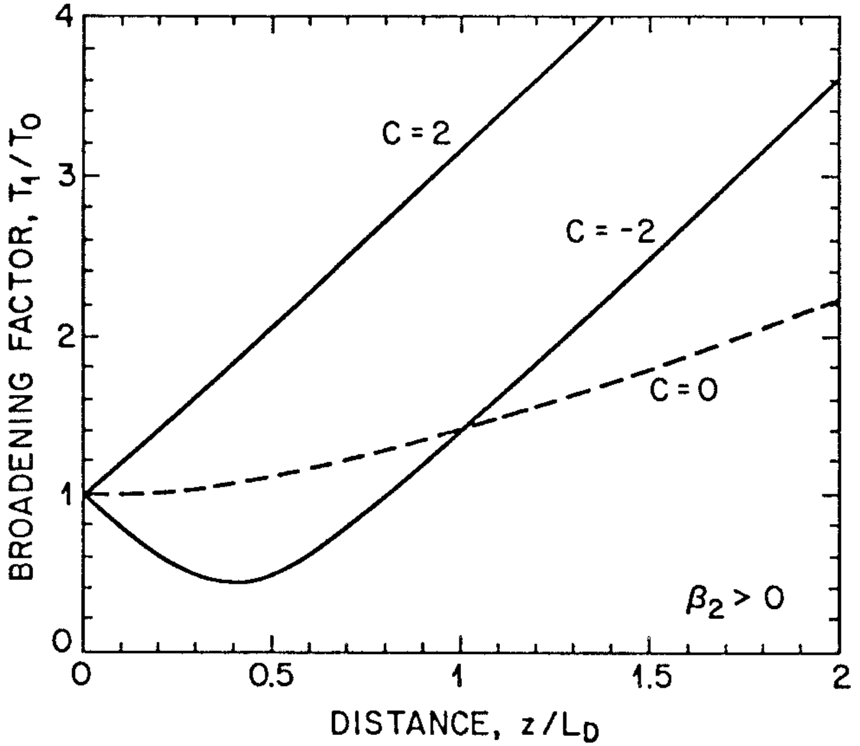
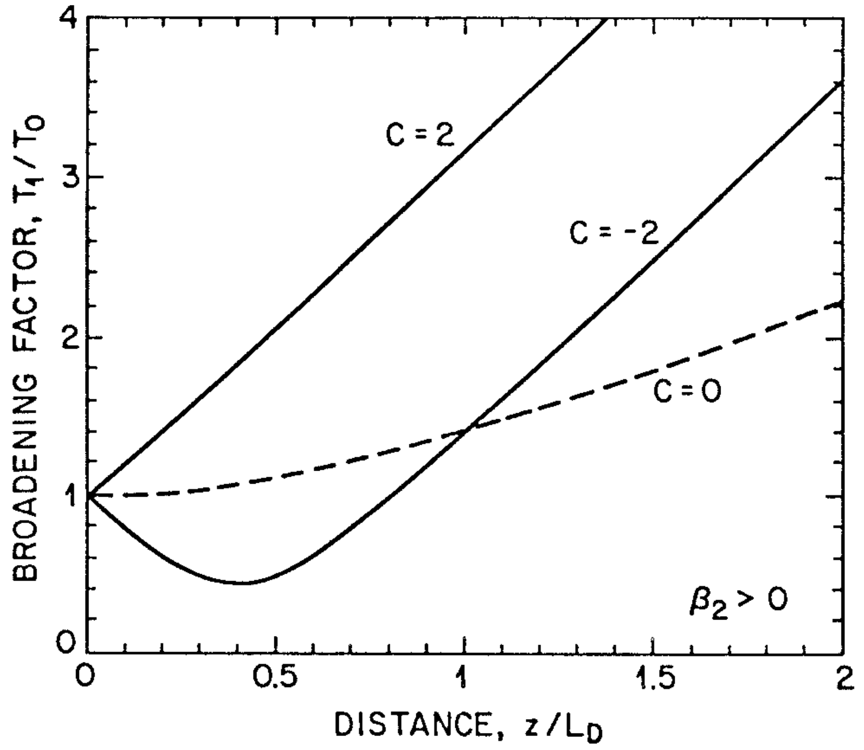


Figure 10: Variation of broadening factor with propagated distance for a chirped Gaussian input pulse. Dashed curve is for an unchirped Gaussian pulse. For $\beta_2 < 0$ the same curves are obtained if the sign of the chirp parameter C is reversed [1].

CHIRPED GAUSSIAN PULSES

Figure 10 shows the broadening factor T_1/T_0 as a function of the propagation distance z/L_D, where $L_D = {T_0}^2/|\beta_2|$ is the dispersion length.

An unchirped pulse ($C = 0$) broadens as $\sqrt{1 + (z/L_D)^2}$ and its width increases by a factor of $\sqrt{2}$ at $z = L_D$.



CHIRPED GAUSSIAN PULSES

The chirped pulse, on the other hand, may broaden or compress depending on whether β_2 and C have the same or opposite signs.

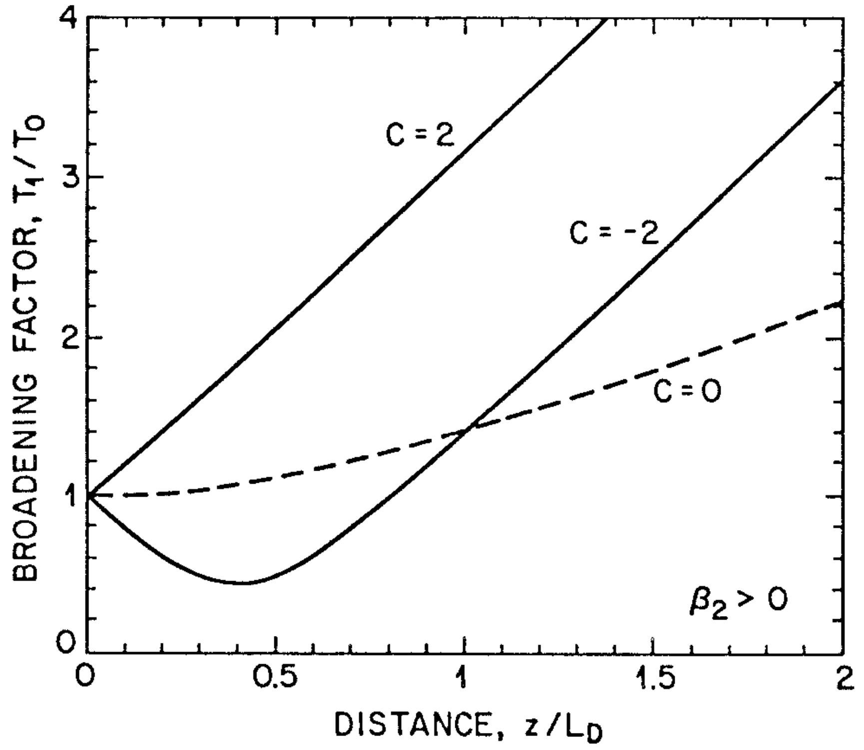
- $\boldsymbol{\beta_2 C > 0}$ - the chirped Gaussian pulse broadens monotonically at a rate faster than the unchirped pulse.
- $\boldsymbol{\beta_2 C < 0}$ - the pulse width initially decreases and becomes minimum at a distance

$$z_{\min} = [|C|/(1 + C^2)]L_D \qquad (36)$$

The minimum value depends on the chirp parameter as

$$T_1^{\min} = T_0/\sqrt{1 + C^2} \qquad (37)$$

Physically, when $\beta_2 C < 0$, the GVD-induced chirp counteracts the initial chirp, and the effective chirp decreases until it vanishes at $z = z_{\min}$.



CHIRPED GAUSSIAN PULSES

The pulse no longer remains Gaussian on propagation and develops a tail with an oscillatory structure. Such pulses cannot be properly characterized by their FWHM.

The RMS width of the pulse defined as

$$\sigma = \sqrt{\langle t^2 \rangle - \langle t \rangle^2} \tag{38}$$

where the angle brackets denote averaging with respect to the intensity profile, i.e.,

$$\langle t^m \rangle = \frac{\int_{-\infty}^{\infty} t_m |A(z,t)|^2 \, dt}{\int_{-\infty}^{\infty} |A(z,t)|^2 \, dt} \tag{39}$$

CHIRPED GAUSSIAN PULSES

The broadening factor is defined as σ/σ_0, where σ_0 is the RMS width of the input Gaussian pulse ($\sigma_0 = T_0/\sqrt{2}$) is given by

$$\frac{\sigma^2}{{\sigma_0}^2} = \left(1 + \frac{C\beta_2 L}{2{\sigma_0}^2}\right)^2 + \left(\frac{\beta_2 L}{2{\sigma_0}^2}\right)^2 + (1 + C^2)^2\left(\frac{\beta_3 L}{4\sqrt{2}{\sigma_0}^3}\right)^2 \qquad (40)$$

where L is the fiber length.

We assumes that the optical source used to produce the input pulses is nearly monochromatic. To account for the source spectral width, we must treat the optical field as a stochastic process and consider the coherence properties of the source through the mutual coherence function.

CHIRPED GAUSSIAN PULSES

When the source spectrum is Gaussian with the RMS spectral width σ_ω, the broadening factor is

$$\frac{\sigma^2}{\sigma_0{}^2} = \left(1 + \frac{C\beta_2 L}{2\sigma_0{}^2}\right)^2 + \left(1 + V_\omega{}^2\right)\left(\frac{\beta_2 L}{2\sigma_0{}^2}\right)^2 + \left(1 + C^2 + V_\omega{}^2\right)^2\left(\frac{\beta_3 L}{4\sqrt{2}\sigma_0{}^3}\right)^2 \qquad (41)$$

where V_ω is defined as $V_\omega = 2\sigma_\omega\rho_0$.

It provides an expression for dispersion-induced broadening of Gaussian input pulses under quite general conditions.

LIMITATIONS ON THE BIT RATE

The limitation imposed on the bit rate by fiber dispersion can be quite different depending on the source spectral width. It is instructive to consider the following two cases separately.

- Optical Sources with a Large Spectral Width.
- Optical Sources with a Small Spectral Width.

Bit rate is the number of bits that are conveyed or processed per unit of time.

OPTICAL SOURCES WITH A LARGE SPECTRAL WIDTH

This case corresponds to $V_\omega \gg 1$.

Assume operating away from the zero-dispersion wavelength so that the β_3 term can be neglected. For source with a large spectral width, the effects of frequency chirp are negligible ($C = 0$). We obtain

$$\sigma^2 = {\sigma_0}^2 + (\beta_2 L \sigma_\omega)^2 \equiv {\sigma_0}^2 + (DL\sigma_\lambda)^2 \qquad (42)$$

where σ_λ is the RMS source spectral width in wavelength units. The output pulse width is thus given by

$$\sigma = \sqrt{{\sigma_0}^2 + {\sigma_D}^2} \qquad (43)$$

where $\sigma_D \equiv |D|L\sigma_\lambda$ provides a measure of dispersion-induced broadening.

OPTICAL SOURCES WITH A LARGE SPECTRAL WIDTH

The criterion is that the broadened pulse should remain inside the allocated bit slot, $T_B = 1/B$. A commonly used criterion is $\sigma \leq T_B/4$ (for Gaussian pulses at least 95%).

The limiting bit rate is given by $4B\sigma \leq 1$. In the limit $\sigma_D \gg \sigma_0$, $\sigma \approx \sigma_D = |D|L\sigma_\lambda$, and the condition becomes

$$BL|D|\sigma_\lambda \leq \frac{1}{4} \tag{44}$$

At the zero-dispersion wavelength, $\beta_2 = 0$. We obtain

$$\sigma^2 = {\sigma_0}^2 + \frac{1}{2}(\beta_3 L{\sigma_\omega}^2)^2 \equiv {\sigma_0}^2 + \frac{1}{2}(SL{\sigma_\lambda}^2)^2 \tag{45}$$

where S is the differential-dispersion parameter and defined in Eq. (12).

OPTICAL SOURCES WITH A LARGE SPECTRAL WIDTH

The output pulse width is $\sigma_D \equiv |S|L\sigma_\lambda{}^2 \leq \frac{1}{\sqrt{2}}$. When $\sigma_D \gg \sigma_0$, the limitation on the bit rate is governed by

$$BL|S|\sigma_\lambda{}^2 \leq \frac{1}{\sqrt{8}} \qquad (46)$$

Example:

In case of a light-emitting diode for which $\sigma_\lambda \approx 15\, nm$.

By using $D = 17\ \mathrm{ps/(nm \cdot km)}$ at 1.55 μm, we get $BL < 1\ \mathrm{(Gb/s) \cdot km}$.

At the zero-dispersion wavelength, BL can be increased to 20 (Gb/s)·km for a typical value $S = 0.08\ \mathrm{ps/(nm^2 \cdot km)}$.

OPTICAL SOURCES WITH A SMALL SPECTRAL WIDTH

This case corresponds to $V_\omega \ll 1$.

we neglect the β_3 term and set $C = 0$

$$\sigma = \sigma_0 + \left(\frac{\beta_2 L}{2\sigma_0}\right)^2 \equiv {\sigma_0}^2 + {\sigma_D}^2 \qquad (47)$$

In the case of a narrow source spectrum, dispersion-induced broadening depends on the initial width σ_0, whereas it is independent of σ_0 when the spectral width of the optical source dominates.

By choosing an optimum value of σ_0, σ can be minimized. For $\sigma_0 = \sigma_D = \sqrt{|\beta_2|L/2}$, it is given by $\sigma = \sqrt{|\beta_2|L}$.

OPTICAL SOURCES WITH A SMALL SPECTRAL WIDTH

The limiting bit rate is obtained by using $4B\sigma \leq 1$ and leads to the condition

$$B\sqrt{|\beta_2|L} \leq \frac{1}{4} \tag{48}$$

The difference between a small spectral width to a large spectral width that the bit rate (B) is scales as $L^{-1/2}$ rather than L^{-1}.

OPTICAL SOURCES WITH A SMALL SPECTRAL WIDTH

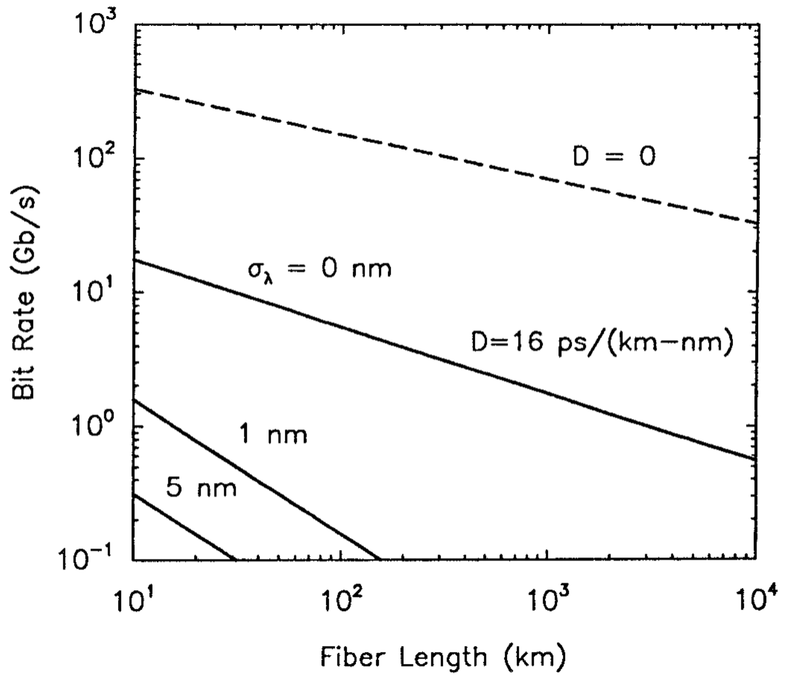


Figure 11: Limiting bit rate of single-mode fibers as a function of the fiber length for $\sigma_\lambda = 0,1,5$ nm. The case of $\sigma_\lambda = 0$ corresponds to the case of an optical source whose spectral width is much smaller than the bit rate [1].

OPTICAL SOURCES WITH A SMALL SPECTRAL WIDTH

Close to the zero-dispersion wavelength, $\beta_2 \approx 0$, we obtain

$$\sigma^2 = {\rho_0}^2 + \frac{1}{2}(\beta_3 L/4{\rho_0}^2)^2 \equiv {\rho_0}^2 + {\rho_D}^2 \tag{49}$$

The limiting bit rate is obtained by using the condition $4B\sigma \leq 1$,

$$B(|\beta_3|L)^{1/3} \leq 0.324 \tag{50}$$

The dispersive effects are most forgiving in this case. When $\beta_3 = 0.1\ \mathrm{ps}^3/\mathrm{km}$, the bit rate can be as large as 150 Gb/s for $L = 100\ \mathrm{km}$.

The performance of a lightwave system can be improved considerably by operating it near the zero-dispersion wavelength of the fiber and using optical sources with a relatively narrow spectral width.

EFFECT OF FREQUENCY CHIRP

Optical pulses are often non-Gaussian and may exhibit considerable chirp.

In a super-Gaussian model, the initial field is given by

$$A(0,T) = A_0 \exp\left[-\frac{1+jC}{2}\left(\frac{t}{T_0}\right)^{2m}\right] \qquad (51)$$

where the parameter m controls the pulse shape.

- $m = 1$ - chirped Gaussian pulse.
- Large value of m - nearly rectangular with sharp leading and trailing edges.

The limiting bit rate-distance product BL is found by requiring that the RMS pulse width does not increase above a tolerable value.

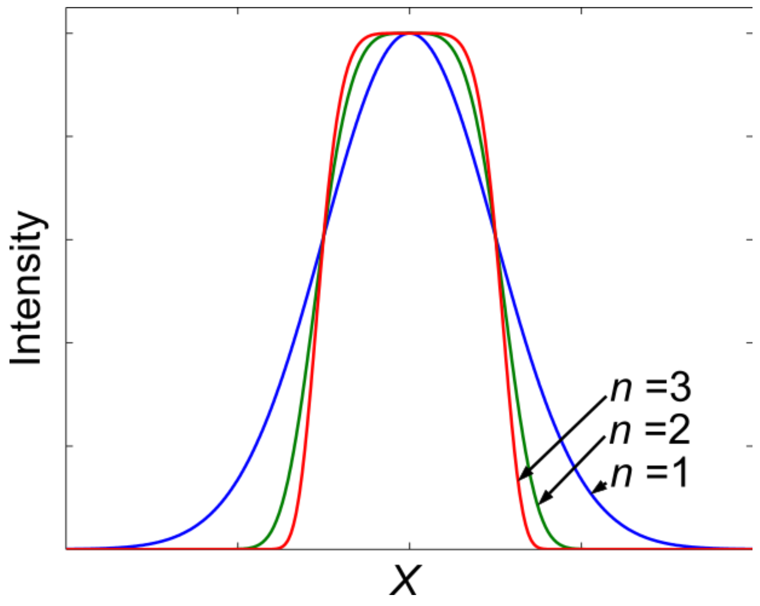


Figure 12: Profile of the super-Gaussian beam for various values of parameter n [2].

EFFECT OF FREQUENCY CHIRP

- The BL product is smaller for super-Gaussian pulses because such pulses broaden more rapidly than Gaussian pulses.
- The BL product is reduced dramatically for negative values of the chirp parameter C due to enhanced broadening occurring when $\beta_2 C$ is positive.

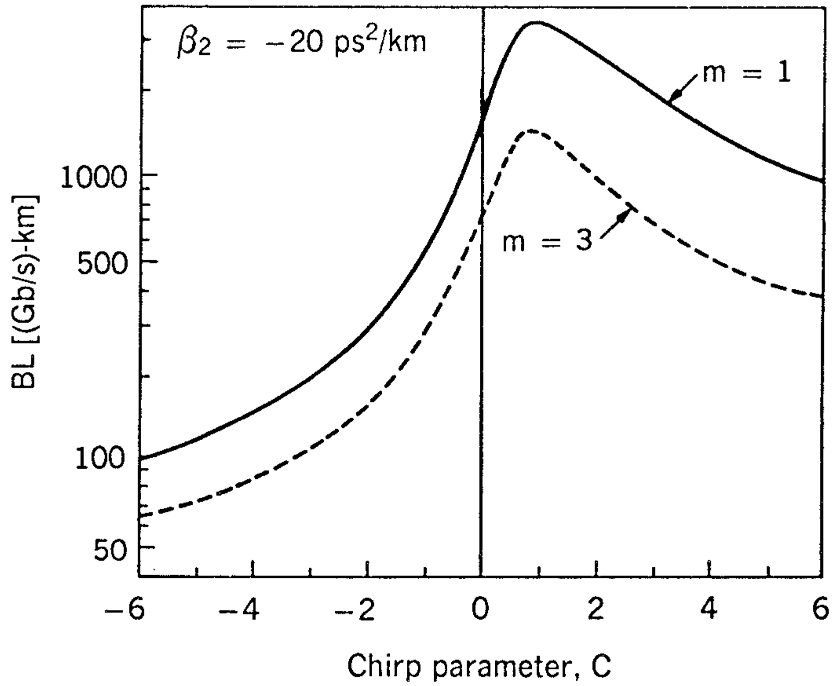


Figure 13: Dispersion-limited BL product as a function of the chirp parameter for Gaussian (solid curve) and super-Gaussian (dashed curve) input pulses [1].

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[2] Manoj Mishra and Woo-Pyo Hong. Investigation on propagation characteristics of super-gaussian beam in highly nonlocal medium. Progress In Electromagnetics Research, 31:175-188, 2011.