LECTURE 1 - INTRODUCTION

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Fundamentals of Fiber Optics Communication 377-2-5060

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SYLLABUS

Topic	Content	Lectures	References
Introduction	Course content and requirements, history and properties, classification of optical fibers	1,	L1
Maxwell equations	Reflection and refraction, Maxwell equations, boundary conditions, Fresnel's equation, Evanescent field	2	L1
Ray optics vs wave optics	Ray optics vs wave optics, critical angle, Brewster angle, V-number, NA	3	L2
Guided modes	Polarizations dependence on an electric field, Helmholtz equation solution, guided modes, polarization, cut-off condition, normalized propagation constant b, SM condition, spot size, polarization, attenuation	3-5	L3
	Single mode fibers, mode properties, Fiber losses	5	L3
Fiber dispersion	Dispersion in Single-Mode Fibers, Polarization-mode dispersion, Dispersion-induced limitations, Limitations on the Bit Rate	6-7	L4
Nonlinear optical effects	Nonlinear Schrodinger equation, nonlinear phase modulation, Four-Wave Mixing (FWM), solitons, stimulated scattering, Raman Scattering	8-10	L5
Students' lectures		11	
Test		12	

Grade composition: 30% Test (בוחן) + 30% Simulation + 20% Presentation + 20% Homework & participation

OUTLINE

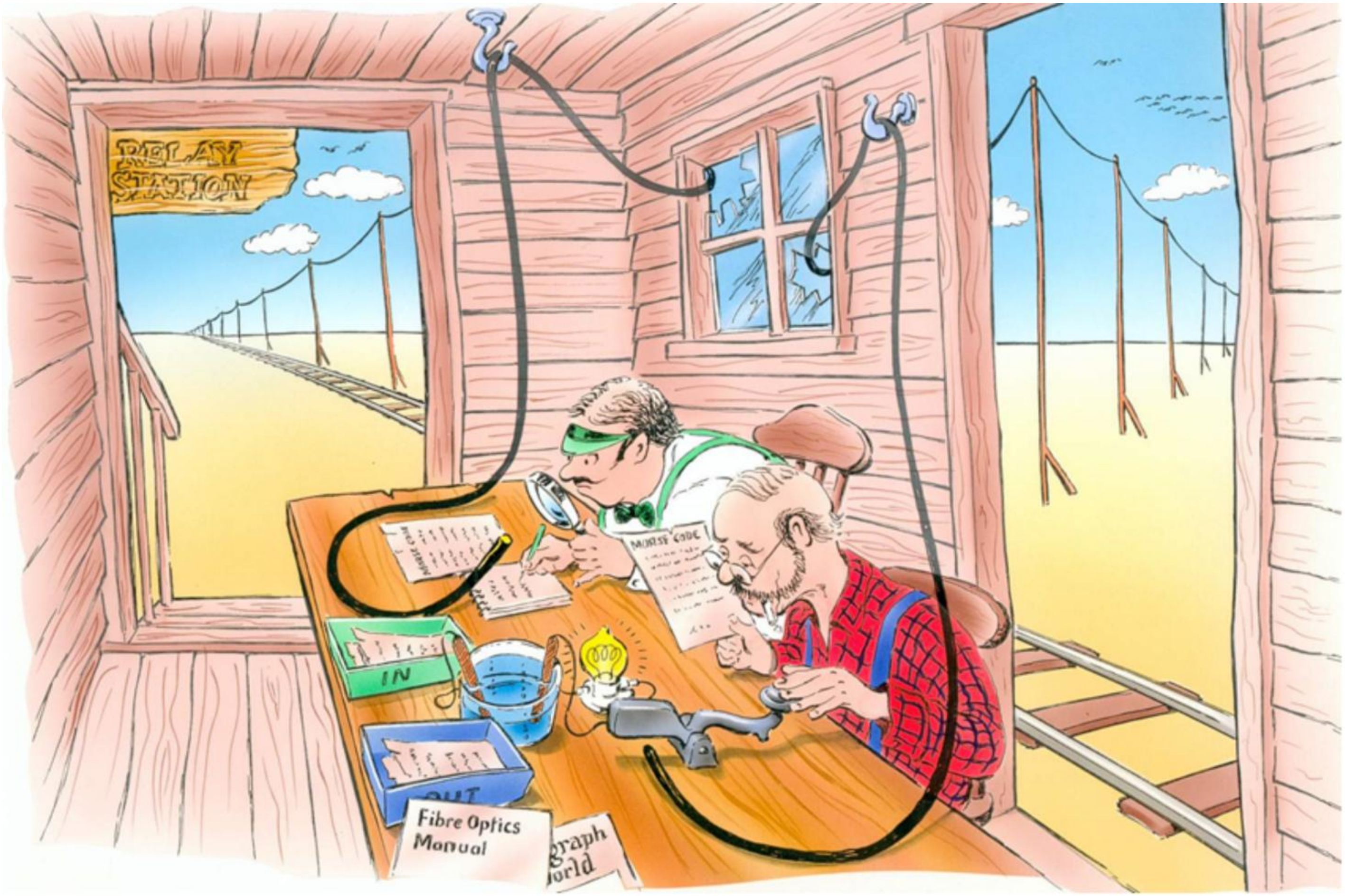
- Introduction
- Reflection and refraction
- Bibliography

TELEPHONE FRENCH PREDICTION FOR THE YEAR 2000

Fig. 17.2 A French prediction for the year 2000.

Le photo-telephone

COMMUNICATION: MORSE



WHY WE CAN'T LIVE WITHOUT FIBER OPTICS

Figure 1: Fiber optics communication

WHY WE CAN'T LIVE WITHOUT FIBER OPTICS

Lightwave communications is a necessity for the information age.

1) Transporting massive amounts of data over long distances.
2) Optical links provide enormous bandwidth.
3) Applications: from global high-capacity networks, which constitute the backbone of the internet, to the massively parallel interconnects that provide data connectivity inside datacenters and supercomputers.
4) Optical communications is a diverse and rapidly changing field merging experts in photonics, communications, electronics, and signal processing.
5) Ever-increasing demands for higher capacity, lower cost, and lower energy consumption, while adapting the system design to novel services and technologies.

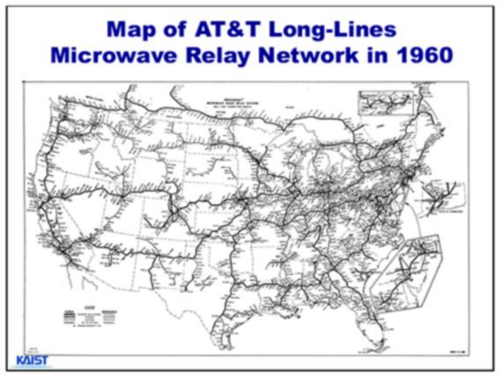
WHY WE CAN'T LIVE WITHOUT FIBER OPTICS

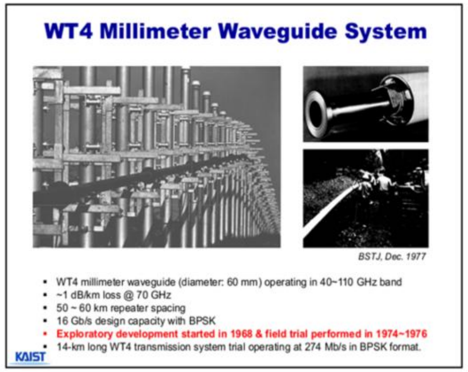
- Optical fibers for next generation optical networks.
- Amplification and regeneration.
- Spatial multiplexing.
- Coherent transceivers.
- Modulation formats.
- Digital signal progressing.
- Optical signal processing.
- Nonlinear channel modeling and mitigation.
- Forward error correction.
- Long-haul networks.
- Optical integration and silicon photonics.
- Optical wireless communications.
- Quantum communication.

HISTORY

- Data transmission in 1960 - MW radio links (300-3000 kHz; includes 525-1715 kHz, AM radio broadcasting).
- Bandwidth limitations of radio links lead to development of mm wave metallic waveguides.
- Charlie Kau and George Hockham 1966 low loss (<20 dB/km) fiber 2009 Nobel Prize (Charlie Kau).

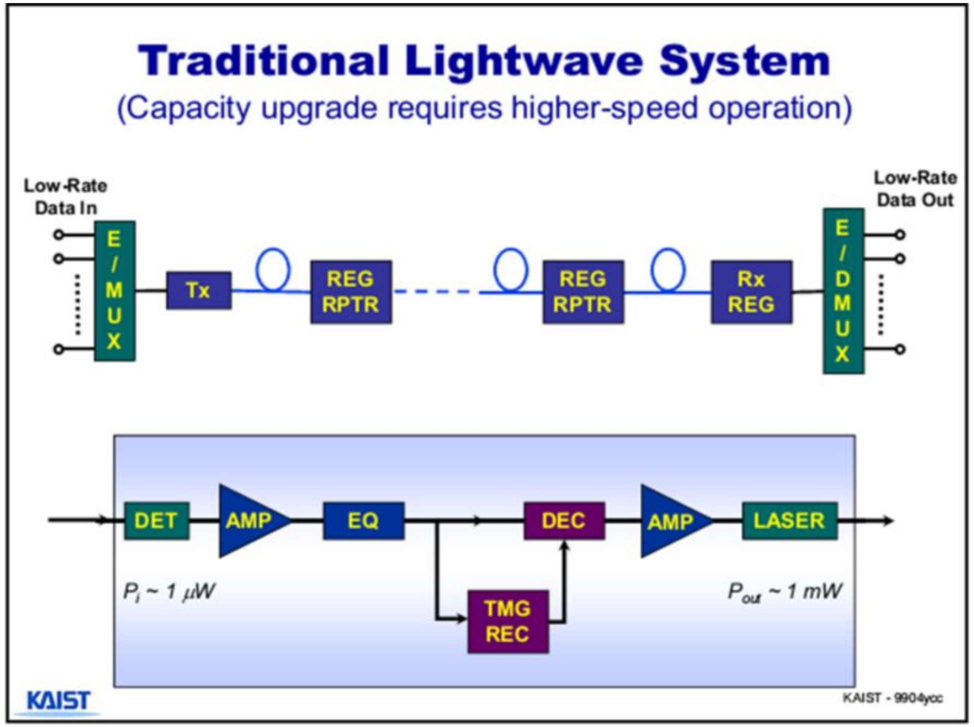
The original 1948 network in Kingston, NY.





HISTORY

- Fiber optics links make use of a transmission medium - the fiber with nearly no loss and a seemingly infinite bandwidth.
- No other communication system has similar properties. Consequence: For the first 25-30 years of the technology, while it supported the major information revolutions, NO real communication principles had to be invoked and all the technology was developed by physicists.



BIT RATE-DISTANCE

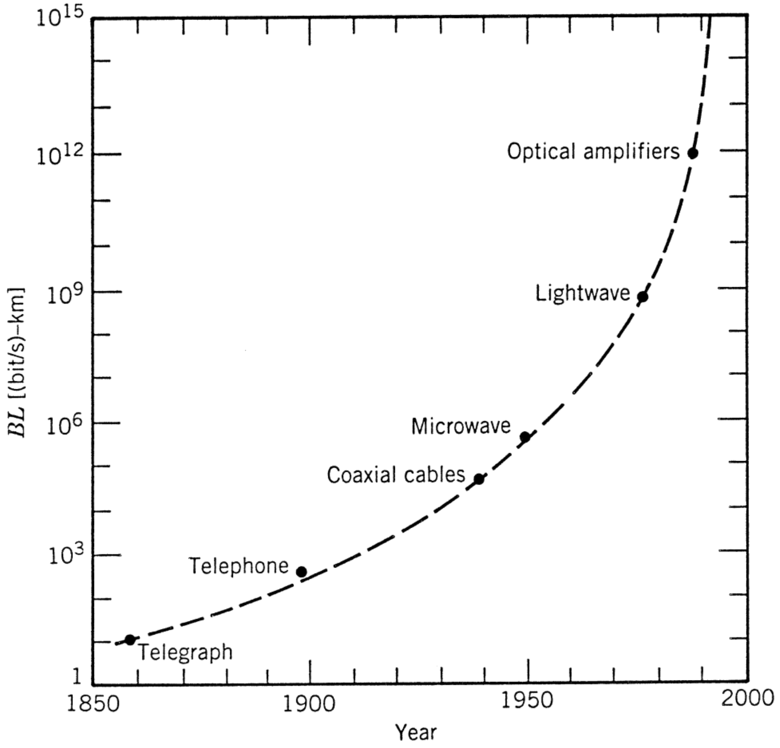


Figure 2: Increase in bit rate-distance product BL during the period 1850-2000. The emergence of a new technology is marked by a solid circle [1].

INTERNET TRAFFIC

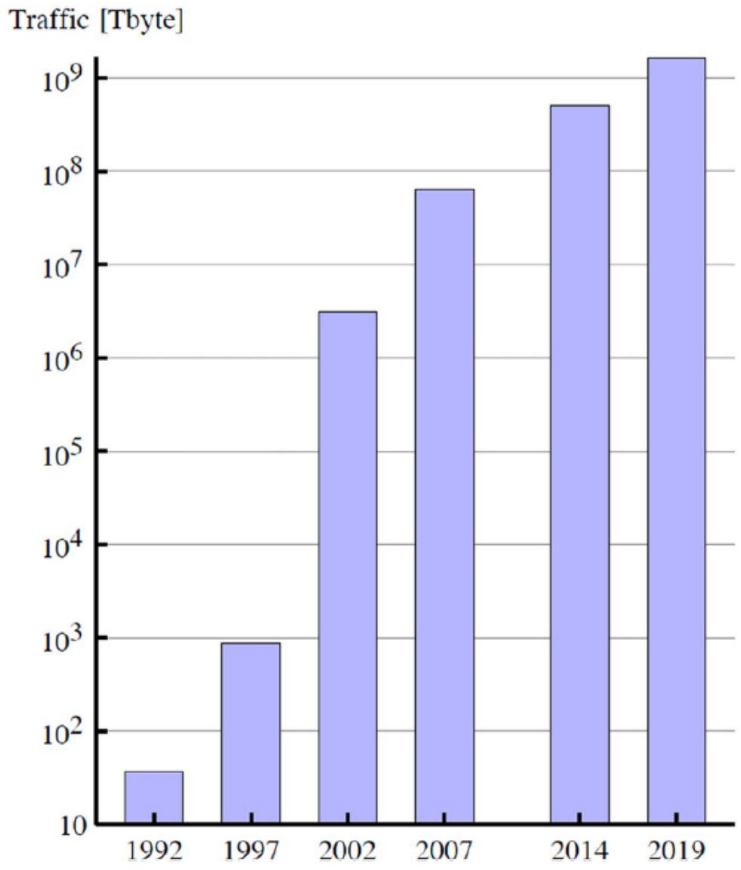


Figure 3: Internet traffic evolution.

HISTORY

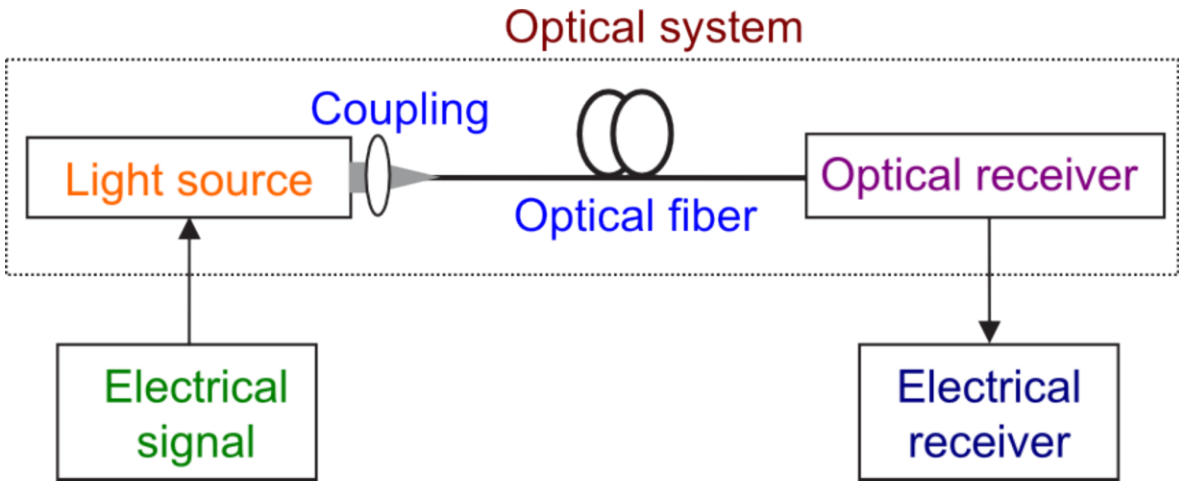
- 1970 - First successfully drawn fiber with a loss of 20 dB/km.
- Soon thereafter loss reaches almost the same level as modern fiber.
- Next major achievement: RT CW operating diode laser in 1970. Wavelength was 850 nm.
- 1970-1971 - all the components of a fiber optics link were available.
- 1980s and 90s - optical fibers were laid down for commercial deployment, replacing the older copper wires and communication satellites for long-distance transmission.
- With society's rapidly growing demands single fiber has been boosted by several orders of magnitude, from a few Gb/s in 1990 to hundreds of Tb/s today.

PRESENT ERA III

- Era III: We are currently in the third major era in the age of fiber communications.
- The Era I, the era of direct-detection, regenerated systems began in 1977 and lasted about 16 years.

COMMUNICATIONS SYSTEM

- An optical fiber communications system is similar in basic concept to any type of communications system.
- The basic function is to convey the signal from the information source over the transmission medium to the destination.
- The communication system consists of a transmitter or modulator linked to the information source, the transmission medium, and a receiver or demodulator at the destination point.



OPTICAL FIBER TRANSMISSION SYSTEM

- Source of light - LASER, LED or broadband.
- Signal - electric signal at entrance converted into the optical signal at transmitter, modulating the light intensity.
- Speed - NIR light illuminating the fiber propagates in the core with the speed of light $v = c/n$. In glass $v = 200$ Mm/s when $n_{\mathrm{glass}} = 1.52$.
- In order to reach a receiver (detector) - PIN photodiode or avalanche photodiode where the optical signal converted back into electrical signal.

ADVANTAGES OF OPTICAL FIBER TRANSMISSION SYSTEM

- Immense binary flow rates with order of several Tb/s (under laboratory conditions reaching the order of 10 Tb/s) which impossible while using copper-based media.
- Low attenuation - the signal can be transmitted over long distances without regeneration.
- Do not create external electromagnetic field, therefore they belong to media hard to be listened in devices.
- No inter-fiber crosstalk.
- Resistance to external electromagnetic field perturbations.
- Reliable signal due to the bit error rate (BER) lower than 10^{10}.

H.W.: SELF-READING

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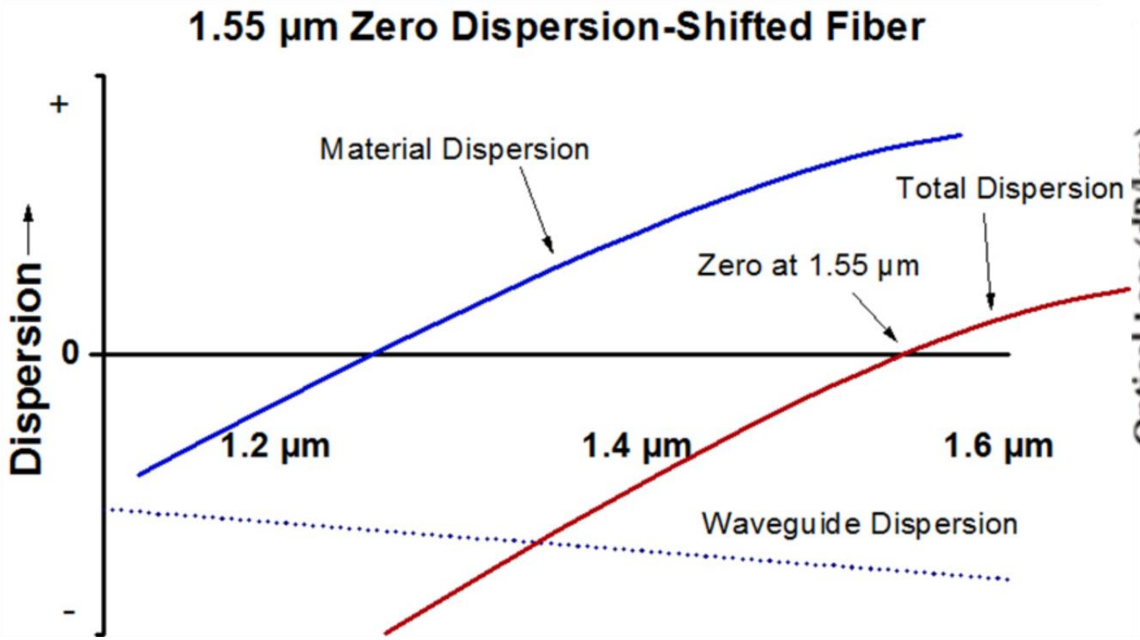
Roadmap of optical communications

Summarize:

[1] Introduction.

[2] History.

LIMITATIONS



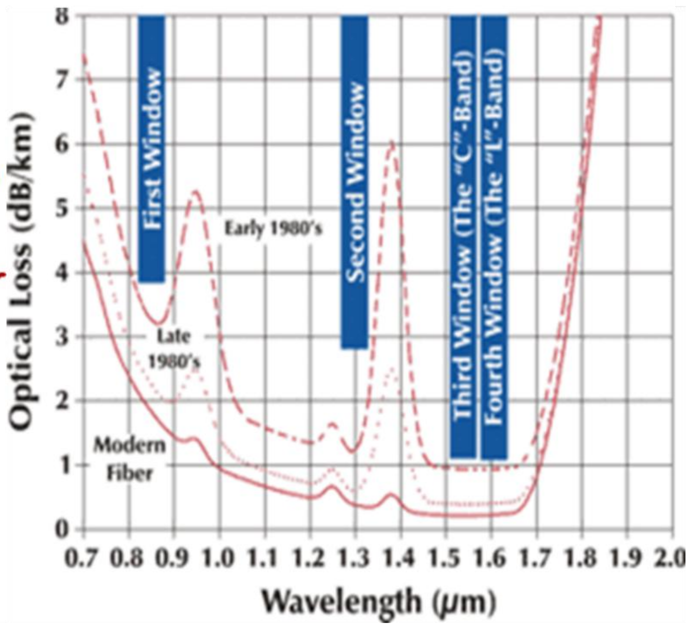


Figure 4: Limitations: fiber loss, fiber dispersion.

FIBER OPTIC NETWORK OPTICAL WAVELENGTH TRANSMISSION BANDS

Table 1: wavelength transmission bands

Band name	Wavelengths	Description
O-band (original)	1260-1360 nm	Original band, PON upstream
E-band (extended)	1360-1460 nm	Water peak band
S-band (short)	1460-1530 nm	PON downstream
C-band (conventional)	1530-1565 nm	Lowest attenuation, original DWDM
L-band (long)	1565-1625 nm	Low attenuation, expanded DWDM
U-band (ultra-long)	1625-1675 nm	Ultra-long wavelength

PHYSICAL CONSTANTS, SI

Table 2: physical constants

Symbol	Term	Value	Units
k	Boltzmann's constant	$1.38 \cdot 10^{-23}$	j/K
h	Planck's constant	$6.626 \cdot 10^{-34}$	j·s
q	Electron charge	$1.602 \cdot 10^{-19}$	Coulomb
c	Speed of light in vacuum	$2.998 \cdot 10^{8}$	m/s
T	Absolute temperature	$T_K = T_C + 273$	Kelvin
ε_0	Vacuum permittivity	$8.854 \cdot 10^{-12}$	F/m
μ_0	Vacuum permeability	$4\pi \cdot 10^{-7}$	H/m

PARAMETERS

Table 3: Parameters, SI

Symbol	Term	Units
λ	Vacuum wavelength	nm
$\Delta\lambda$	Wavelength difference	nm
α	Attenuation	1/km
D	Dispersion parameter	ps/(nm·km)
E_p	Photon energy	J
$\tilde{\nu}$	Wavenumber (spectroscopy)	cm^{-1}
T_C	Temperature	C

CONVERSION TABLE

Table 4: Conversion table

Symbol	Term	Value	Units
f	Frequency	$f = c/\lambda$	Hz
Δf	Frequency difference	$\Delta f = -c\Delta\lambda/\lambda^2$	Hz
α_{dB}	Attenuation	$\alpha_{\mathrm{dB}} = 4.343\alpha$	dB/km
β_2	Dispersion parameter	$D = -\frac{2}{\lambda^2}\beta_2$	ps²/nm
λ	Wavelength	$\lambda = hc/E_p$	nm
λ	Wavelength	$\lambda = 10^7/\tilde{v}$	nm
T_F	Temperature	$T_c = \frac{5}{9}(T_F - 32)$	F

ELECTROMAGNETIC RADIATION

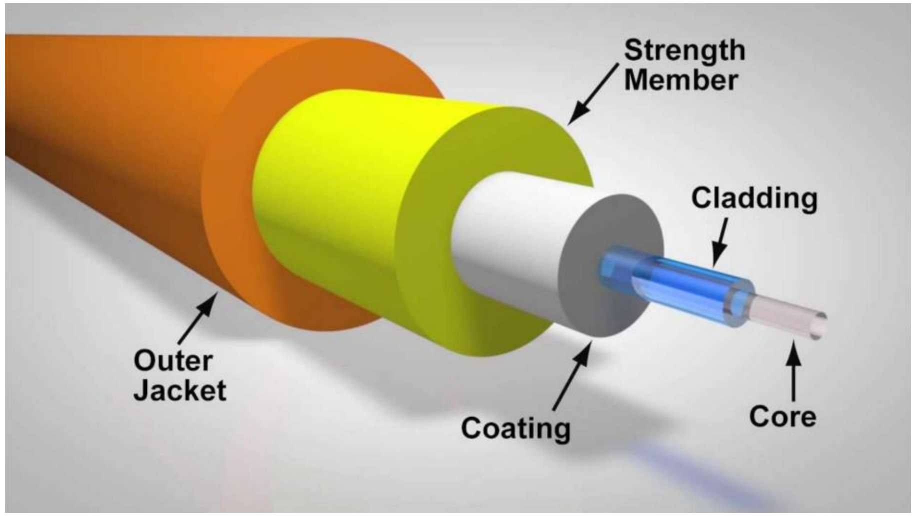
- **Term Light** defines Electromagnetic (EM) radiation.
- **Optical fiber** is a waveguide used for transmission of light. It consists of 1) a dielectric core and 2) surrounded layer named cladding.
- **Guiding condition:** Refractive index of cladding < core.
- **Total Internal Reflection:** The light inside the core is trapped due to TIR.
- **Propagation condition:** TIR occurs at the core-cladding interface when the light inside the core of the fiber is incident at an angle greater than the critical angle θ_c and returns to the core lossless allowing the light propagation along the fiber.

OPTICAL FIBER

In its simplest form, a step-index fiber consists of a cylindrical core surrounded by a cladding layer whose index is slightly lower than the core.

Both core and cladding use silica as the base material; the difference in the refractive indices is realized by doping the core, or the cladding, or both.

- Dopants such as GeO_2 and P_2O_5 increase the refractive index of silica and are suitable for the core.
- Dopants such as B_2O_3 and fluorine decrease the refractive index of silica and are suitable for the cladding.



FABRICATION METHODS

Several methods can be used to make the preform. The three commonly used methods are modified chemical-vapor deposition (MCVD), outside-vapor deposition (OVD), and vapor-axial deposition (VAD).

Fabrication of telecommunication-grade silica fibers involves two stages:

1) First stage - making a cylindrical preform with the desired refractive-index profile. The preform is typically 1 m long and 2 cm in diameter and contains core and cladding layers with correct relative dimensions.
2) Second stage - the preform is drawn into a fiber by using a precision-feed mechanism that feeds the preform into a furnace at the proper speed.

MCVD PROCESS

Peter Schultz, Donald Keck, and Bob Maurer (left to right) at Corning were the first to produce low-loss optical fibers in the 1970s by using the outside vapor deposition method for the fabrication of preforms, which were then used to draw fibers with low losses. (*Courtesy of Corning.*)

MCVD PROCESS

The MCVD process is also known as the inner-vapor-deposition method, as the core and cladding layers are deposited inside a silica tube.

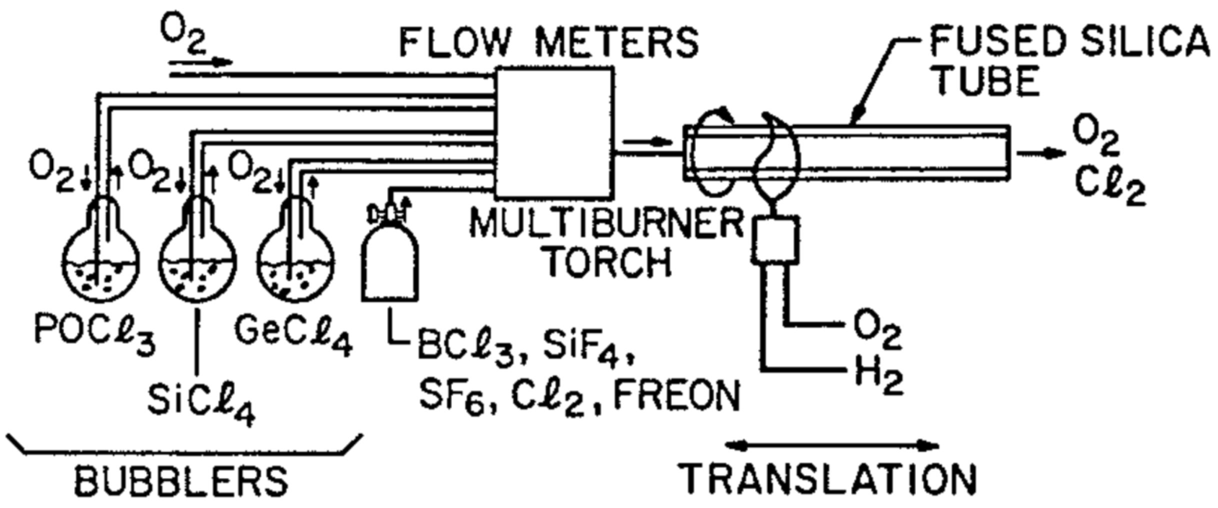


Figure 5: MCVD process commonly used for fiber fabrication [1].

MCVD PROCESS

In this process, successive layers of SiO_2 are deposited on the inside of a fused silica tube by mixing the vapors of $SiCl_4$ and O_2 at a temperature of about 1800C. To ensure uniformity, a multiburner torch is moved back and forth across the tube length using an automatic translation stage. The refractive index of the cladding layers is controlled by adding fluorine to the tube. When a sufficient cladding thickness has been deposited, the core is formed by adding the vapors of $GeCl_4$ or $POCl_3$. These vapors react with oxygen to form the dopants GeO_2 and P_2O_5:

$$GeCl_4 + O_2 \rightarrow GeO_2 + 2Cl_2$$

$$4POCl_3 + 3O_2 \rightarrow 2P_2O_5 + 6Cl_2$$

The flow rate of $GeCl_4$ or $POCl_3$ determines the amount of dopant and the corresponding increase in the refractive index of the core. A triangular-index core can be fabricated simply by varying the flow rate from layer to layer. When all layers forming the core have been deposited, the torch temperature is raised to collapse the tube into a solid rod of preform.

FIBER PREFORMS

(a)

(b)

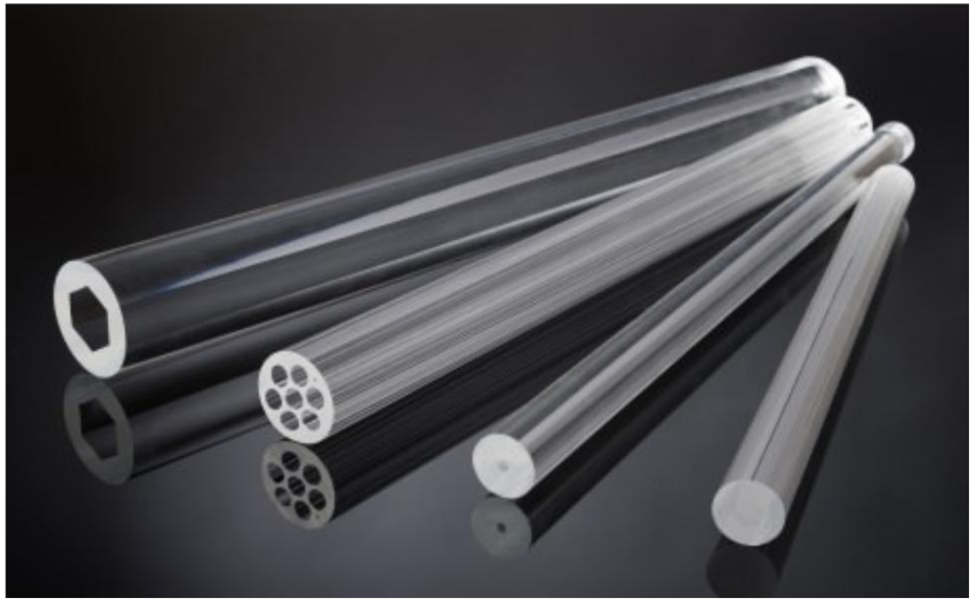


Figure 6: (a) Fiber preform [from LEONI fiber optics]. (b) Fiber special preform [from kohokukogyo]

FIBER DRAWING

- The preform is fed into a furnace in a controlled manner where it is heated to a temperature of about 2000C.
- The melted preform is drawn into a fiber by using a precision-feed mechanism.
- The fiber diameter is monitored optically by diffracting light emitted by a laser from the fiber.
- A polymer coating is applied to the fiber during the drawing step.

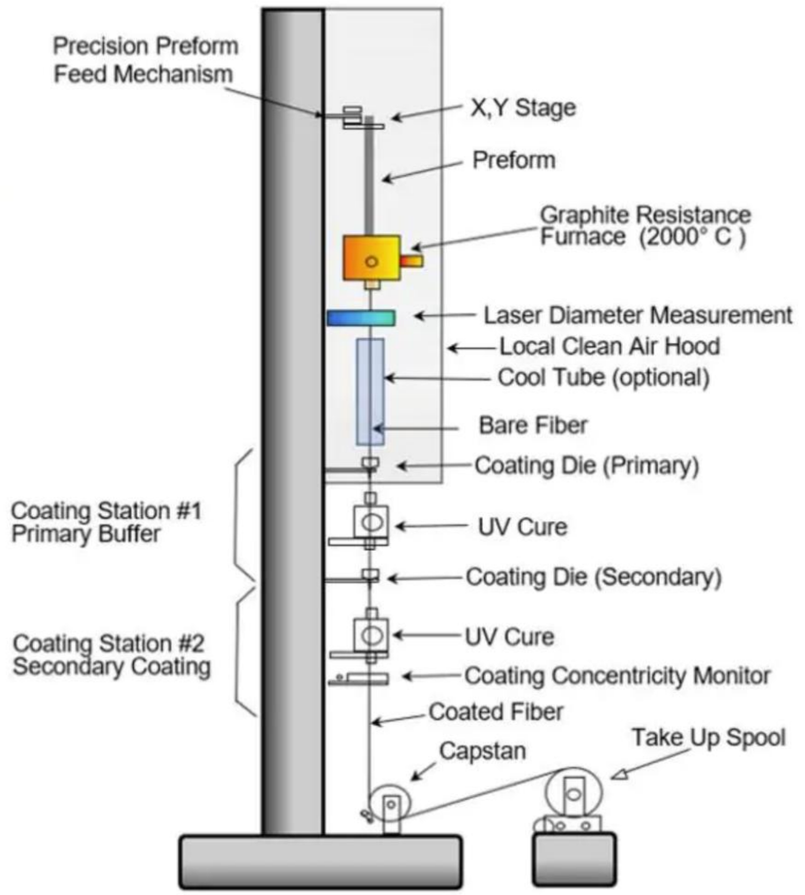


Figure 7: Apparatus used for fiber drawing [from Fiber Optic Center].

FIBER DRAWING

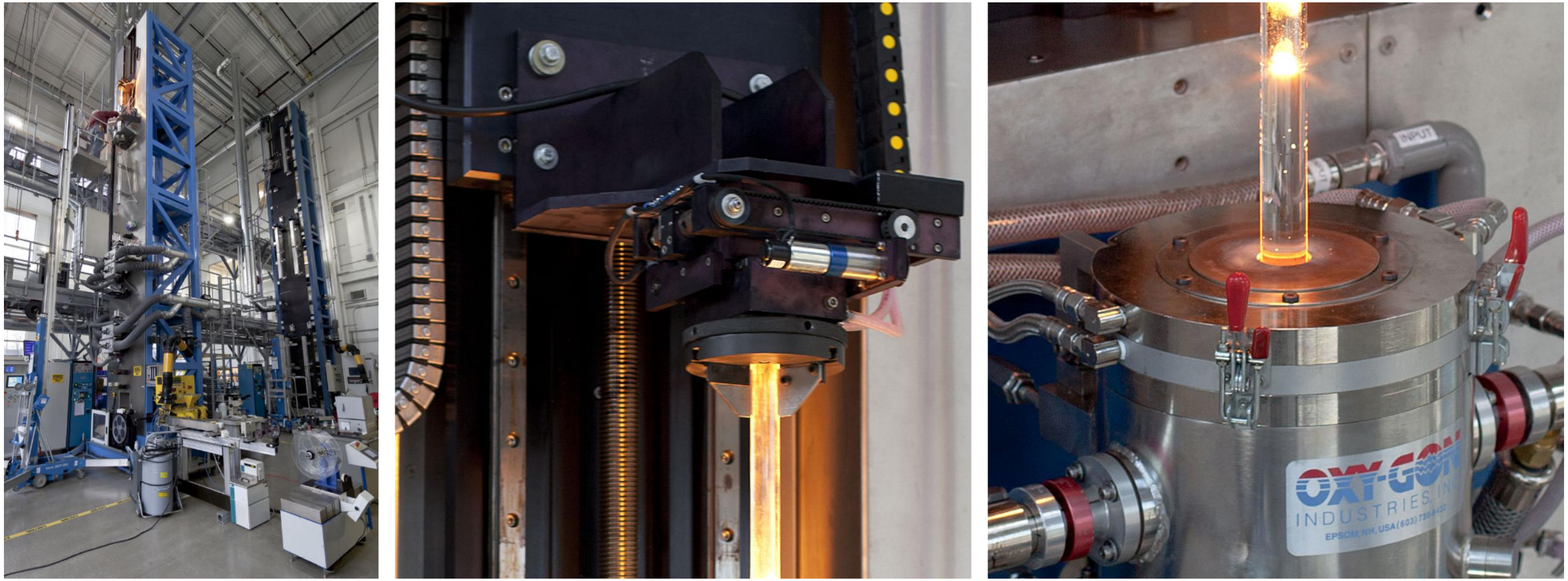


Figure 8: Fiber drawing tower, preform and furnace [from Thorlabs]

TYPES OF OPTICAL FIBERS

- Number of modes (single/multi mode).
- Refraction index profile (step-index or graded index).
- Material (glass, plastic, semiconductor).
- Dispersion (natural, shifted fiber - DSF, dispersion widened fiber - DWF, reverse dispersion).
- Signal processing ability (passive-data transmission, active-amplifier, laser).
- Polarization (classic, maintaining, polarized).

SINGLE MODE VS MULTIMODE

- Single mode fibers (SMF) - the core diameter 5-10 μm. The cladding diameter 125 μm.
- Multimode fibers (MMF) - the core diameter can be 50, 62.5 μm or more. The typical cladding diameter is 125 μm (can be bigger).

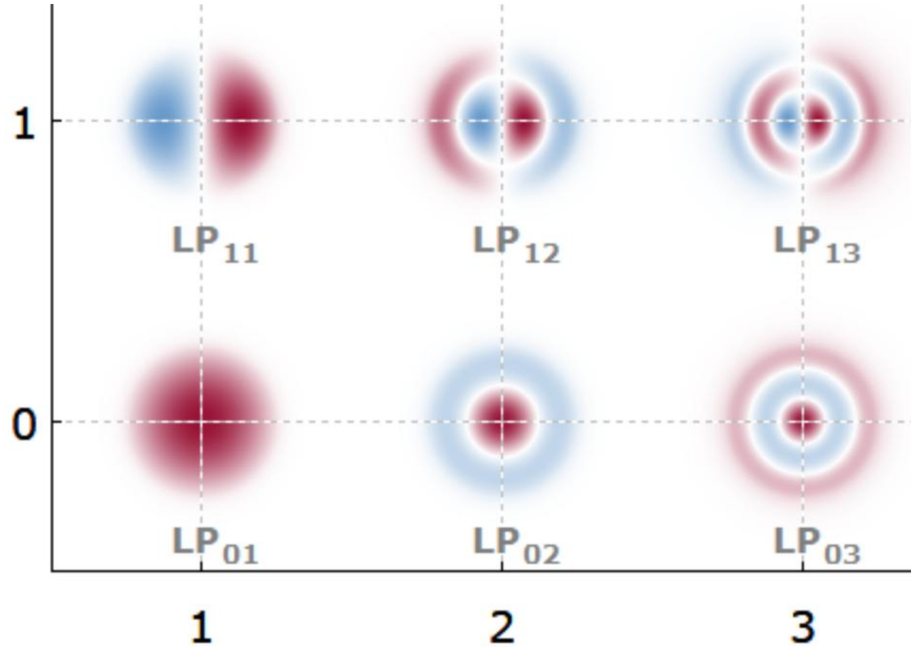


Figure 9: Electric field amplitude profiles for guided modes of a fiber [RP photonics].

TYPES OF OPTICAL FIBERS

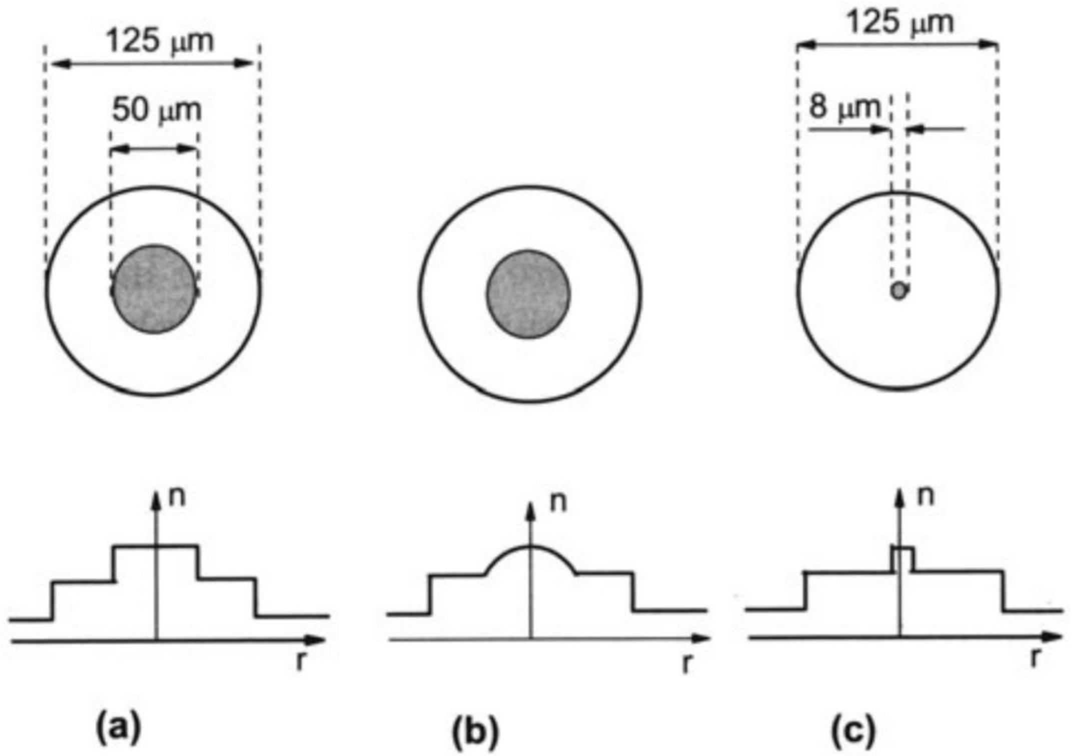


Figure 10: Typical refractive index progression for (a) a step-index, (b) graded index and (c) single mode fiber [2].

DISPERSION SHIFTED FIBER - DSF

- Chromatic dispersion in a single-mode fiber is the sum of material dispersion and waveguide dispersion.
- The waveguide dispersion can be controlled by proper choice of the waveguide parameters, while the material dispersion is almost independent of these parameters.
- The zero-dispersion wavelength can be made coincident with the 1.55 μm minimum loss wavelength of optical fibers.
- In addition, the dispersion can be very small over a wide wavelength range are called dispersion flattened fibers (DFFs).

DISPERSION SHIFTED FIBER - DSF

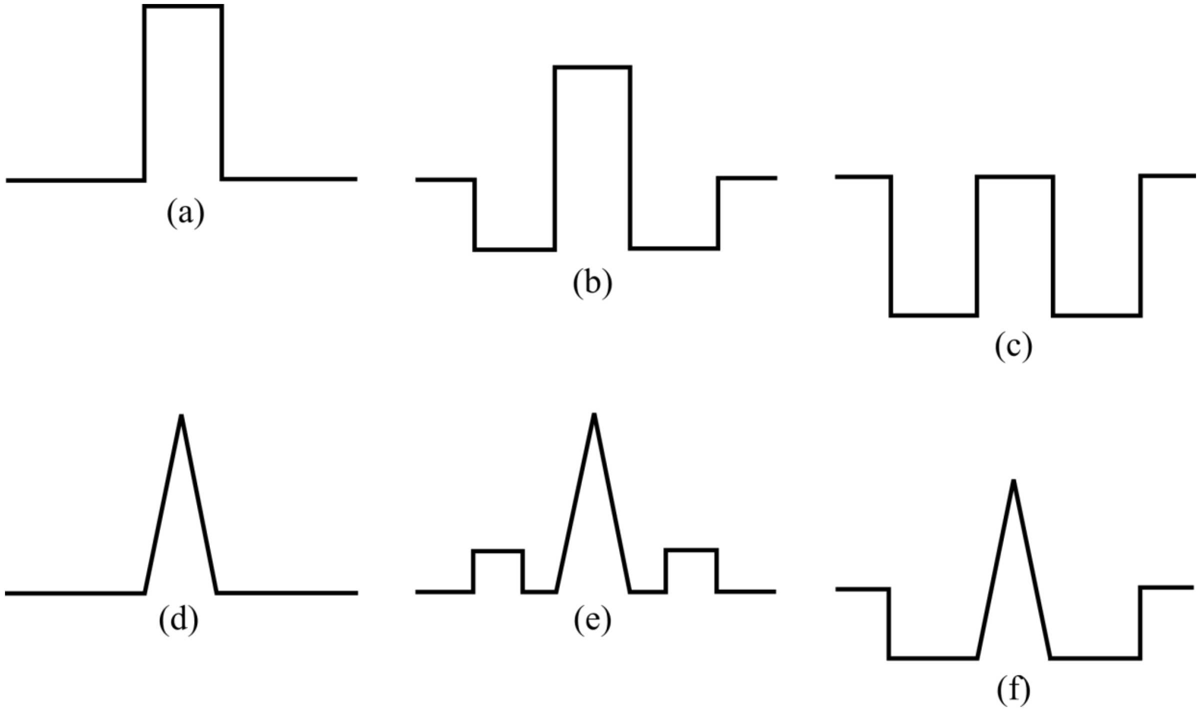


Figure 11: Several index profiles used in the design of single-mode fibers. Upper and lower rows correspond to standard and dispersion-shifted fibers, respectively [1].

CLASSIFICATION OF OPTICAL FIBERS

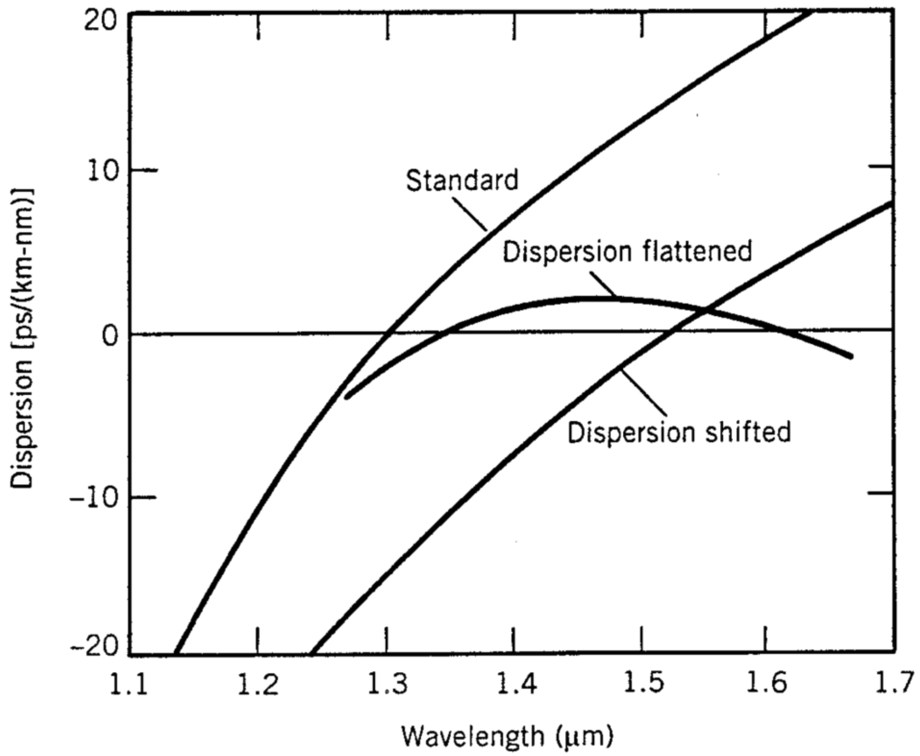


Figure 12: Typical wavelength dependence of the dispersion parameter D for standard, dispersion-shifted, and dispersion-flattened fibers [1].

THE DESIGN OF SINGLE-MODE FIBERS

In the axially symmetric single-mode fiber, there exist two orthogonally polarized modes. They are known as HE_{11}^{x} and HE_{11}^{y} modes in accordance with their polarization directions. If the fiber waveguide structure is truly axially symmetric, these orthogonally polarized modes have the same propagation constants and thus they are degenerate. This is why such fiber is called "single-mode" fiber.

In practical fibers, however, an axial nonsymmetry is generated by the core deformation and/or core eccentricity to the outer diameter, and it causes a slight difference in the propagation constants of the two polarization modes. In such fibers, the state of polarization (SOP) of the output light randomly varies, since the mode coupling take place between HE_{11}^{x} and HE_{11}^{y} modes, which is caused by fluctuations in core diameter along the z-direction, vibration and temperature variations.

THE DESIGN OF SINGLE-MODE FIBERS

Birefringent fibers have been proposed and fabricated to solve such polarization fluctuation problems.

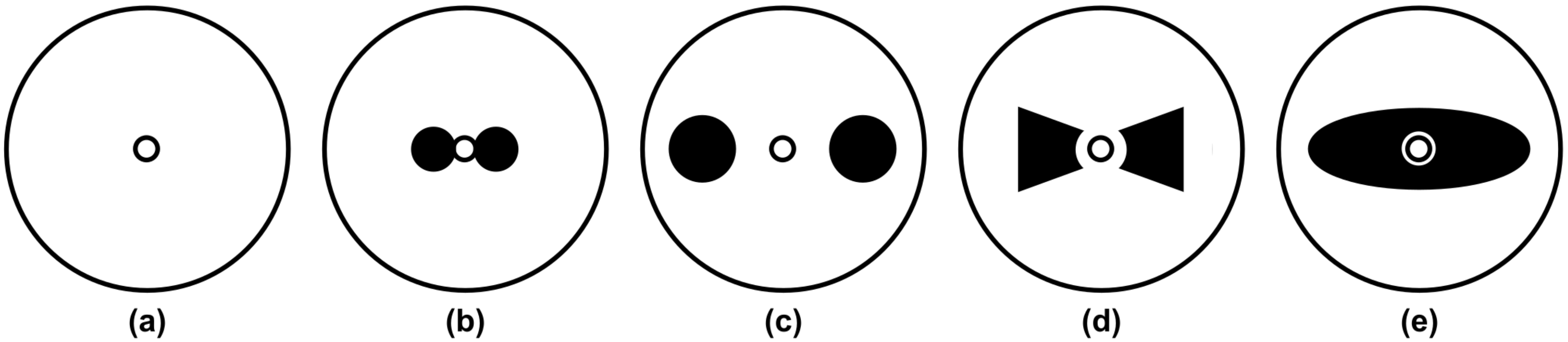
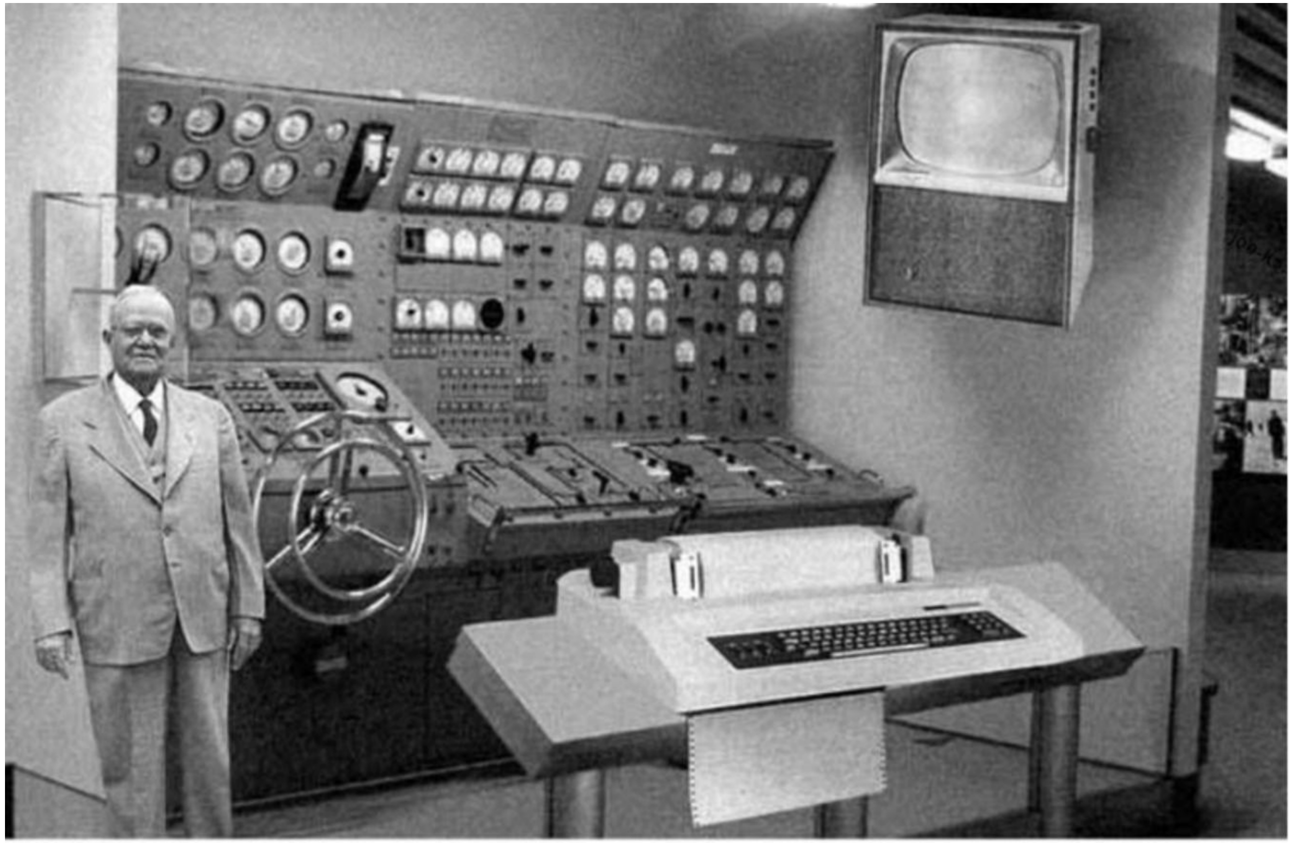


Figure 13: Cross-sectional structures of typical birefringent fibers: (a) elliptic-core fiber, (b) side-pit or side-tunnel fibers, (c) PANDA fiber, (d) Bow-tie fiber, and (e) elliptical-jacket fiber [3].

HOME COMPUTER PREDICTION FOR YEAR 2004



Scientists from the RAND Corporation have created this model to illustrate how a "home computer" could look like in the year 2004. However the needed technology will not be economically feasible for the average home. Also the scientists readily admit that the computer will require not yet invented technology to actually work, but 50 years from now scientific progress is expected to solve these problems. With teletype interface and the Fortran language, the computer will be easy to use.

REFLECTION AND REFRACTION

<u>Assumptions:</u>

- Plane wave propagation.
- Linear medium.
- Isotropic medium.
- Smooth planar optical interface.

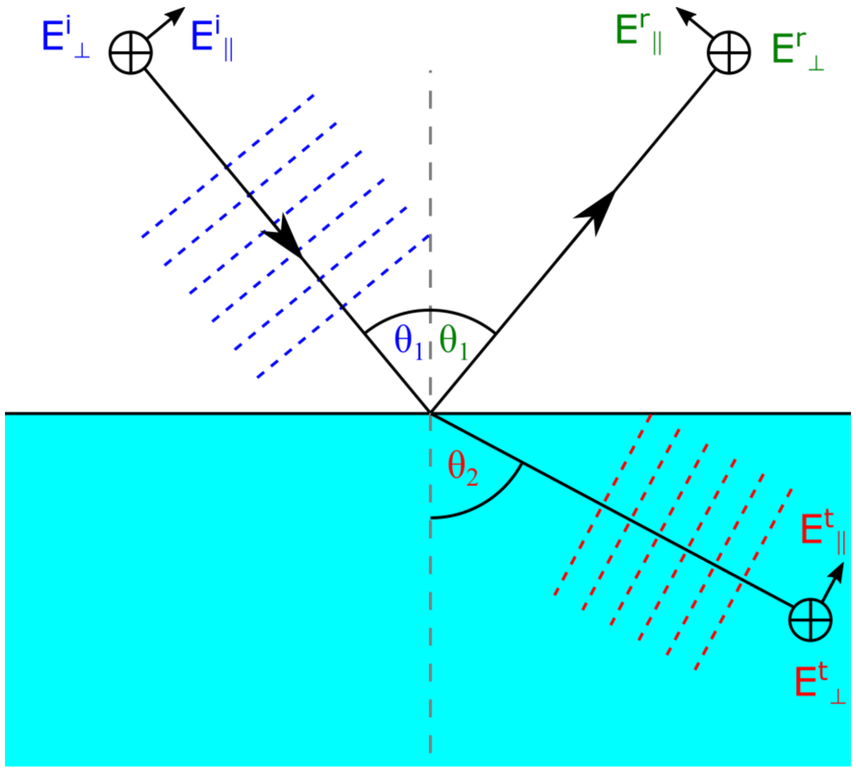
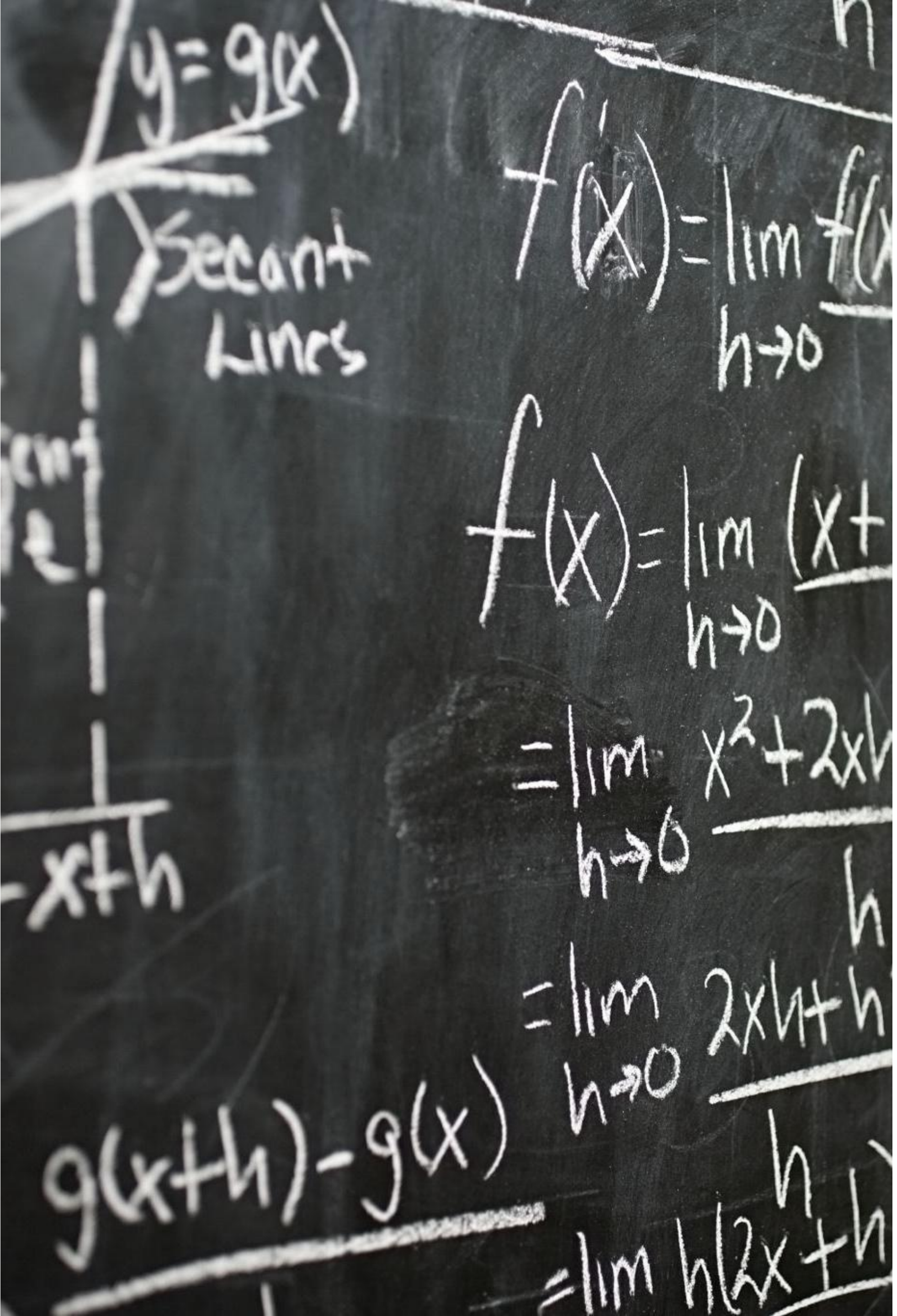


Figure 14: Plane wave reflection and refraction at an optical interface.



MAXWELL'S EQUATIONS

Maxwell's equations, or **Maxwell–Heaviside** equations, are a set of coupled partial differential equations that, together with the **Lorentz force law**, form the foundation of classical electromagnetism, classical optics, and electric circuits. The equations provide a mathematical model for electric, optical, and radio technologies, such as power generation, electric motors, wireless communication, lenses, radar etc.

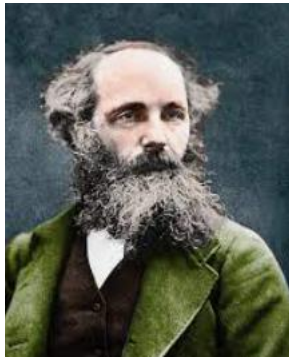
MAXWELL'S EQUATIONS

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\nabla \cdot \vec{D} = \rho_{\text{ext}}$$

$$\nabla \cdot \vec{B} = 0$$



- The link between electricity and magnetism was completed by the work of James Clerk Maxwell.
- He took the four equations made by Gauss (also Coulomb), Faraday, and Amp'ere and by making some corrections he developed mathematically the connection between those equations.
- In 1861, Maxwell presented a set of coupled equations (around 20 equations) that describe electromagnetic phenomena varying in time which are called Maxwell's equations.
- The four equations known today were obtained by Oliver Heaviside, using vector notation to simplify 12 of the 20 equations into the 4 known Maxwell's equations.

MAXWELL'S EQUATIONS

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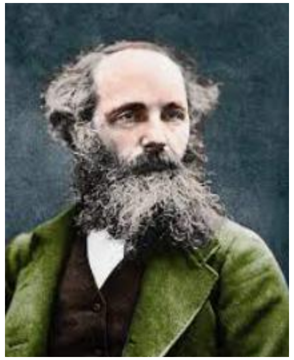
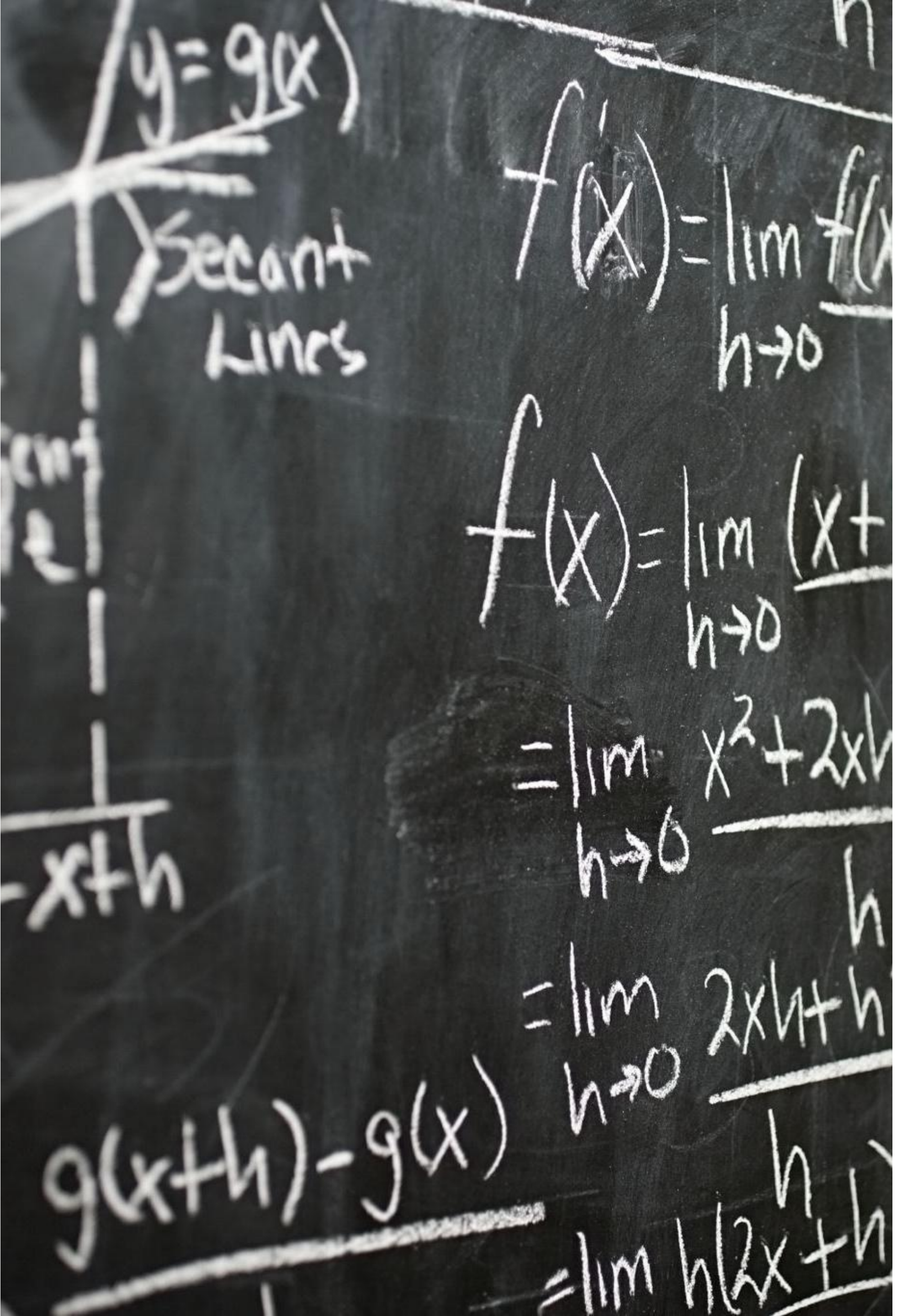


- These equations can be used as a mathematical model for phenomena in nature and for electrical and optical problems.
- In a paper published in 1865, Maxwell has derived a wave equation from his equations thus discovering electromagnetic waves.
- He suggested that light is an electromagnetic wave and showed this hypothesis to be consistent with experimental results. Therefore, he concluded that light is an electromagnetic wave.
- In 1886-1889, German physicist Heinrich Rudolf Hertz performed a series of experiments that proved that light is an electromagnetic wave as was analytically calculated by Maxwell.



MAXWELL'S EQUATIONS

Assumptions:

1. The parameters of the medium in a linear system don't dependent on the electric field E and the magnetic field H: $\varepsilon = \varepsilon_r \varepsilon_0 \quad \mu = \mu_0$.
2. The medium parameters μ and ε are constant and time independent.
3. The medium is isotropic $\Rightarrow \mu$ and ε are direction independent.
4. The medium is dielectric $\Rightarrow J = 0$ and $\rho_{\text{ext}} = 0$

MAXWELL'S EQUATIONS

Assuming linear, homogeneous and isotropic medium, Maxwell's equations are defined as

Faraday's law $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

Ampere-Maxwell law $$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J} \tag{2}$$

Gauss law $$\nabla \cdot \vec{D} = \rho_{\text{ext}} \tag{3}$$

Gauss's law for magnetism $$\nabla \cdot \vec{B} = 0 \tag{4}$$

where E is the electric field vector, D is the electric displacement field vector, H is the magnetic field vector and B is the magnetic flux density vector, ρ_{ext} and J are the charge and current densities, respectively.

MAXWELL'S EQUATIONS

- The current density is defined as $J = \sigma E$, where σ is the electric conductivity, and only exists in ohmic material, such as metals and semiconductors.
- In dielectric medium, $J = 0$ and $\rho_{\mathrm{ext}} = 0$
- D and B are related to the field vectors and are defined as

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \tag{5}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) \tag{6}$$

where ε_0 and μ_0 are the electric permittivity and magnetic permeability of vacuum, respectively, P is the polarization and M is the magnetization.

POLARIZATION AND MAGNETIZATION

In the case of isotropic material, the polarization and the magnetization are given by

$$\vec{P} = \varepsilon_0 \chi \vec{E} \qquad \vec{M} = \chi \vec{H}$$

and

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$
$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

where μ is the permeability, ε is the permittivity, c is the speed of light in vacuum, χ is the electric susceptibility and ε_r is called the relative permittivity and μ_r the relative permeability, which in case of non-magnetic material is $\mu_r = 1$.

MAXWELL'S EQUATIONS

- Maxwell's equations for dielectric waveguide are given as

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (7)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad (8)$$

$$\nabla \cdot \vec{D} = 0 \quad (9)$$

$$\nabla \cdot \vec{B} = 0 \quad (10)$$

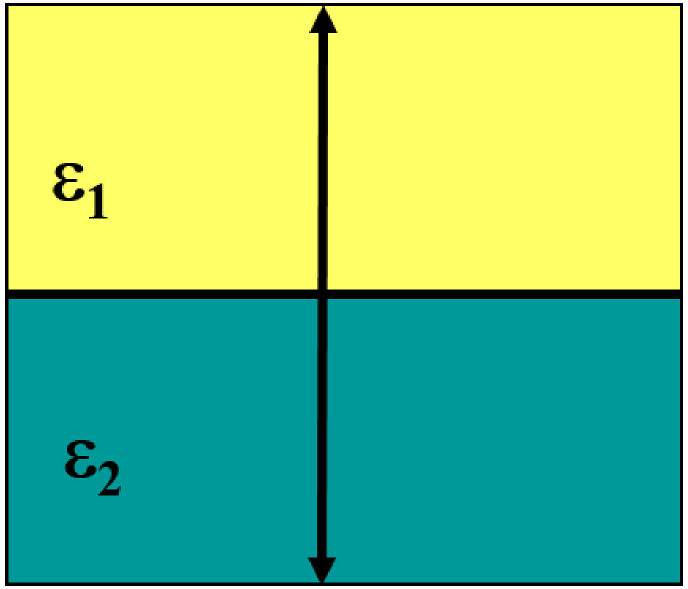
where

$$\vec{B} = \mu \vec{H} \qquad \vec{D} = \varepsilon \vec{E}$$

BOUNDARY CONDITIONS

- The boundary conditions define the behavior of the electric and the magnetic fields on the boundary.
- Assume two different media with permittivity of ε_1 and ε_2, as shown in the figure.
- The electric field - E and the magnetic field H can be decomposed to the tangential (t) and vertical (n) components.

$$E = E_n + E_t \qquad H = H_n + H_t$$



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE ELECTRIC FIELD - E_t

- Assume electric field in medium 1 (ε_1). From Faraday's law:

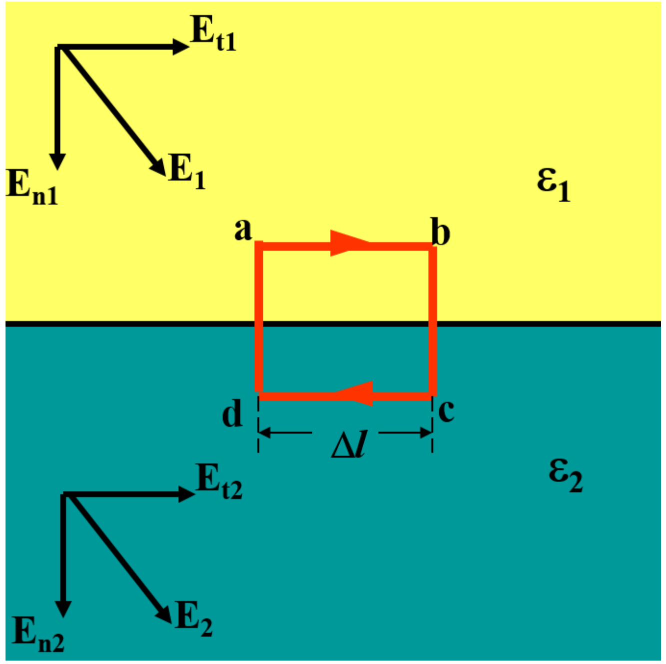
$$\oint E \cdot dl = -\iint \frac{\partial B}{\partial t} dA = 0$$

- Faraday's law for closed loop $a \Rightarrow b \Rightarrow c \Rightarrow d$ is:

$$\oint E \cdot dl = \int_a^b \dots + \int_b^c \dots + \int_c^d \dots + \int_d^a \dots = 0$$

- Assuming $a - d$ and $b - c$ equal 0 then:

$$\oint E \cdot dl = \int_a^b \dots + \int_c^d \dots$$



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE ELECTRIC FIELD - E_t

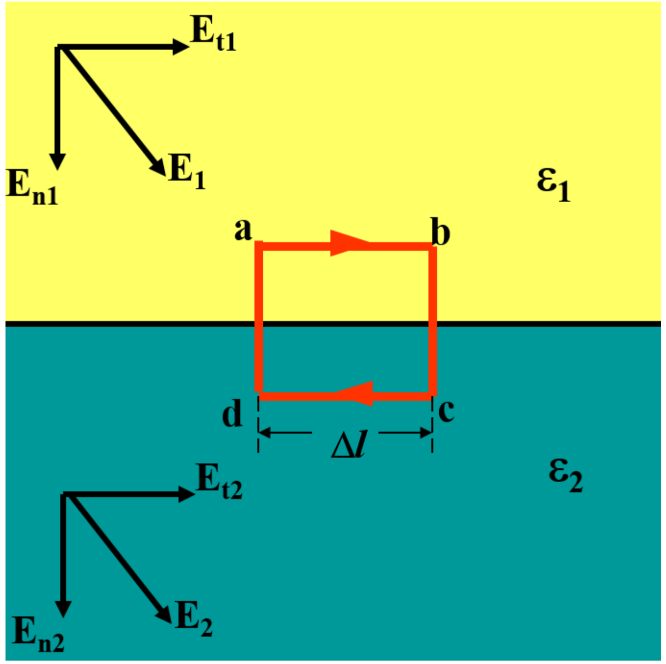
- $\int_a^b \ldots \approx E_{t1}\Delta l$ and $\int_c^d \ldots \approx -E_{t2}\Delta l$ therefore $E_{t1}\Delta l - E_{t2}\Delta l = 0$ and:

$$\boxed{E_{t1} = E_{t2}}$$

- The tangential components of the electric field are continuous on the boundary.
- In addition, $D_i = \varepsilon_{ij}E_j$ therefore:

$$\boxed{\frac{D_{t1}}{\varepsilon_1} = \frac{D_{t2}}{\varepsilon_2}}$$

The tangential components of the electric displacement field are not continuous on the boundary.



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE ELECTRIC FIELD - E_n

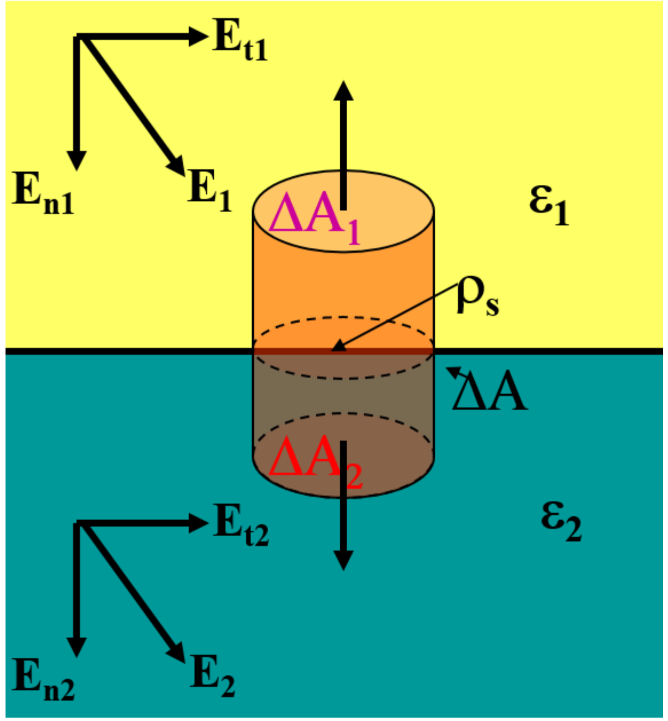
- From Gauss's law:

$$\oint D \cdot dA = \iiint \rho \, dv = Q_{\text{enclosed}}$$

- The figure shows that the surrounded charge is a surface.
- We write Gauss's law as:

$$\oint D \cdot dA = \iint \rho_s \, dA$$

where ρ_s (units of [C/m^2] is the charge on the boundary.



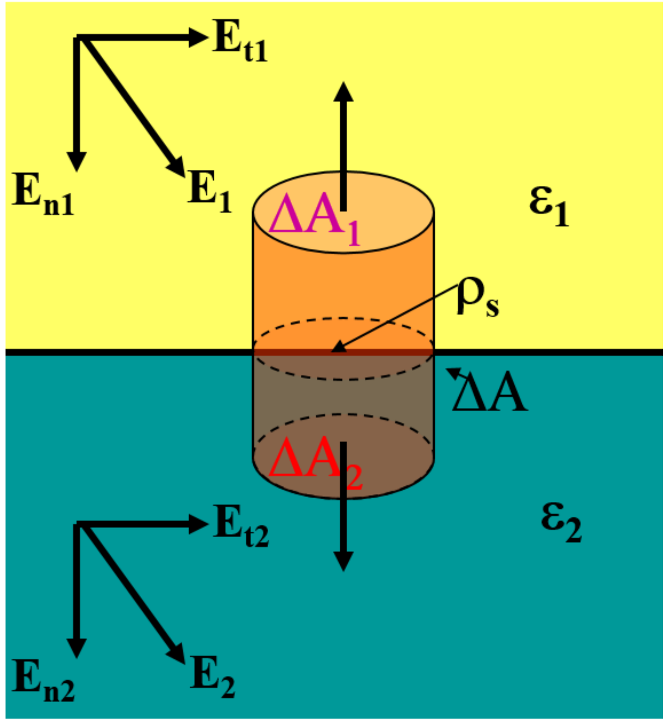
BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE ELECTRIC FIELD - E_n

- We write the equation as:

$$-D_{n1}\Delta A_1 + D_{n2}\Delta A_2 = \rho_s \Delta A$$

- Vertical vectors are defined far from the boundary and the electric field is in medium 1.
- Since $\Delta A_1 = \Delta A_2 = \Delta A$ therefore:

$$-D_{n1} + D_{n2} = \rho_s$$



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE ELECTRIC FIELD - E_n

- In addition:

$$-\varepsilon_1\varepsilon_0 E_{n1} + -\varepsilon_2\varepsilon_0 E_{n2} = \rho_s$$

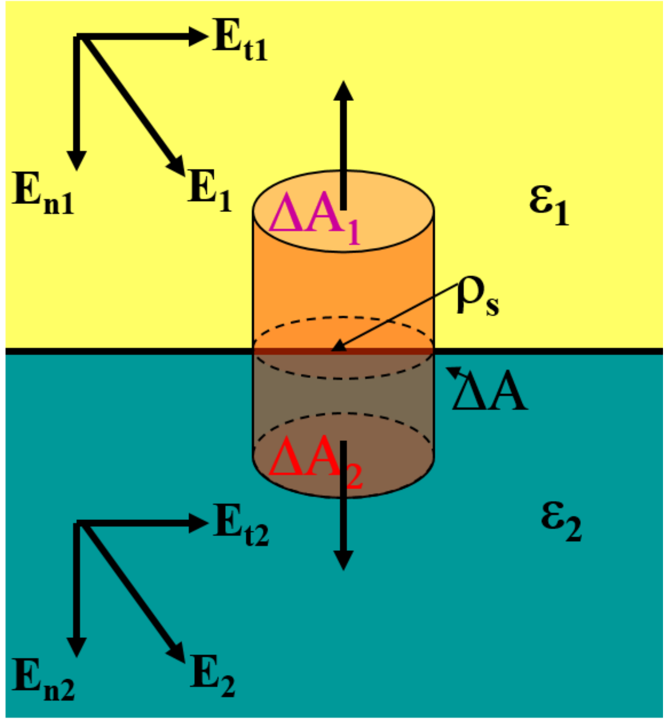
- Without charge on the boundary, we get:

$$\boxed{D_{n1} = D_{n2}}$$

and

$$\boxed{\varepsilon_1 E_{n1} = \varepsilon_2 E_{n2}}$$

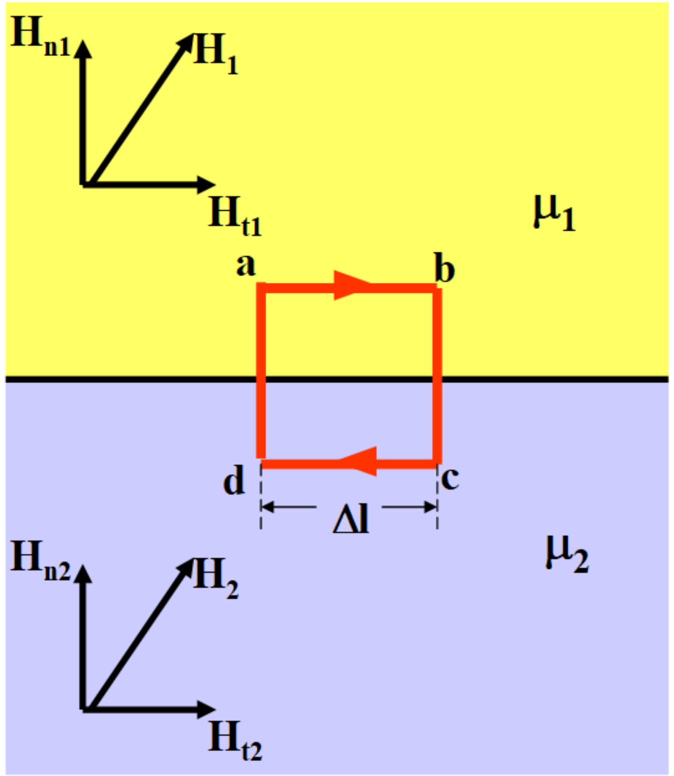
The vertical components of the electric displacement field are continuous between the two media but the vertical components of the electric field are not.



BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE MAGNETIC FIELD - H_t

- Assume boundary between two media with different permeability of μ_1 and μ_2.
- Assume Ampere's law without currents on the boundary:

$$\oint H \cdot dl = \iint \left(J + \frac{\partial D}{\partial t}\right) dA = 0$$



For closed loop is:

$$\oint \ldots = \int_a^b \ldots + \int_b^c \ldots + \int_c^d \ldots + \int_d^a \ldots = 0$$

- Assuming $a - d$ and $b - c$ equal 0 then:

$$\oint H \cdot dl = \int_a^b \ldots + \int_c^d \ldots = 0$$

BOUNDARY CONDITIONS FOR THE TANGENTIAL COMPONENT OF THE MAGNETIC FIELD - H_t

- $\int_a^b \ldots \approx H_{t1}\Delta l$ and $\int_c^d \ldots \approx -H_{t2}\Delta l$ therefore $H_{t1}\Delta l - H_{t2}\Delta l = 0$ and so:

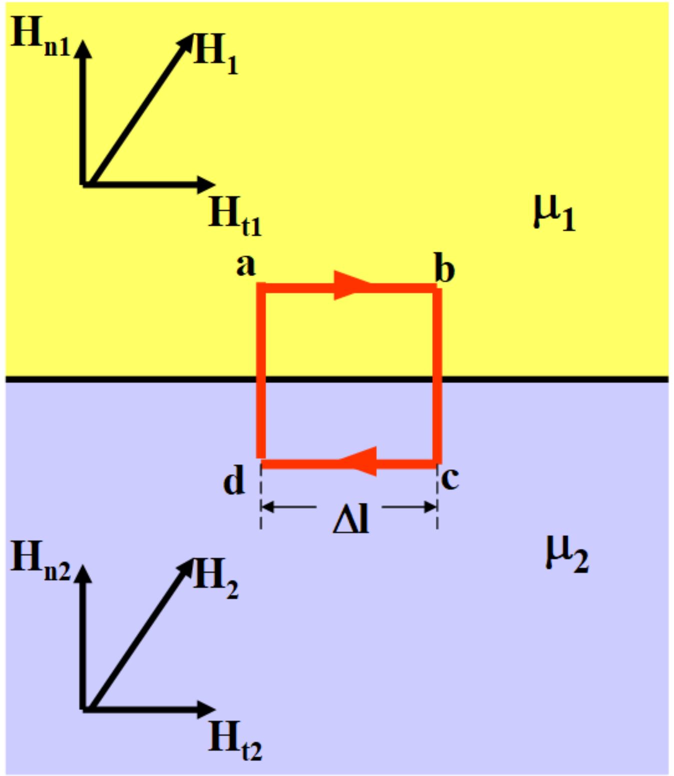
$$\boxed{H_{t1}\Delta l = H_{t2}\Delta l}$$

The tangential components of the magnetic field are continuous on the boundary between the two media.

- From $B = \mu H$ we get:

$$\boxed{\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}}$$

The tangential component of the magnetic flux density are not continuous on the boundary between the two media.



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE MAGNETIC FIELD - H_n

- From magnetic Gauss's law:

$$\oint B \cdot dA = 0$$

- From the figure:

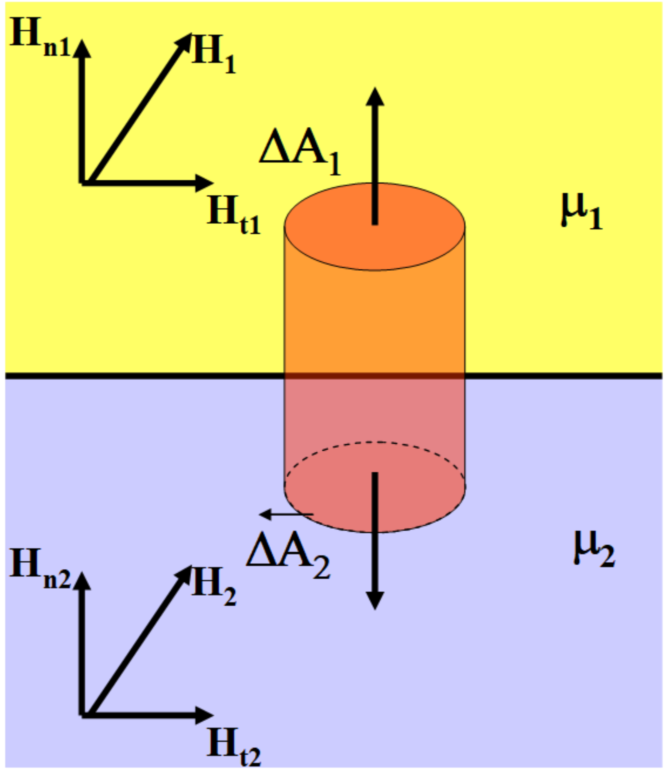
$$\oint \ldots = \oint_{\text{top}} \ldots + \oint_{\text{side}} \ldots + \oint_{\text{bottom}} \ldots = 0$$

- We shrink the cylinder - $\oint_{\text{side}} \ldots \to 0$ therefore:

$$\oint_{\text{top}} \ldots \approx B_{n1}\Delta A \quad \text{and} \quad \oint_{\text{bottom}} \ldots \approx B_{n2}\Delta A$$

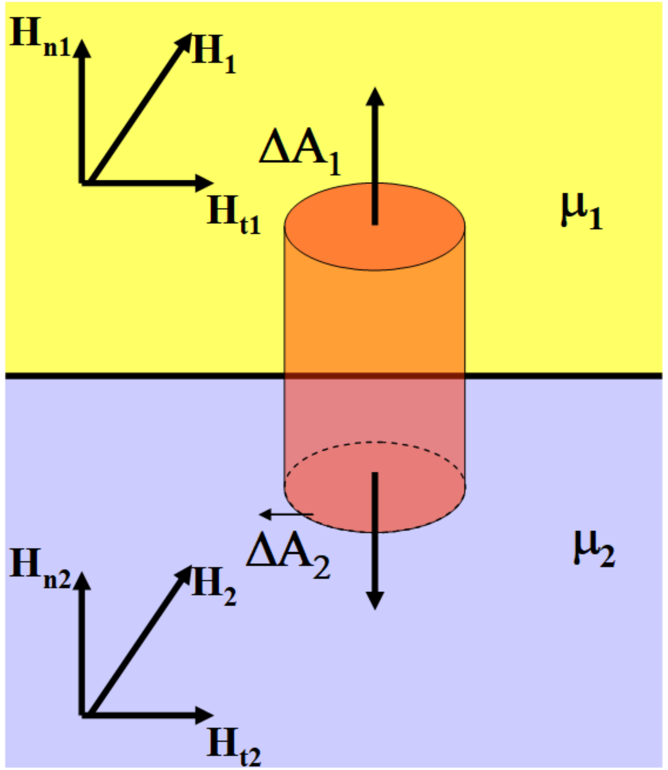
- We get:

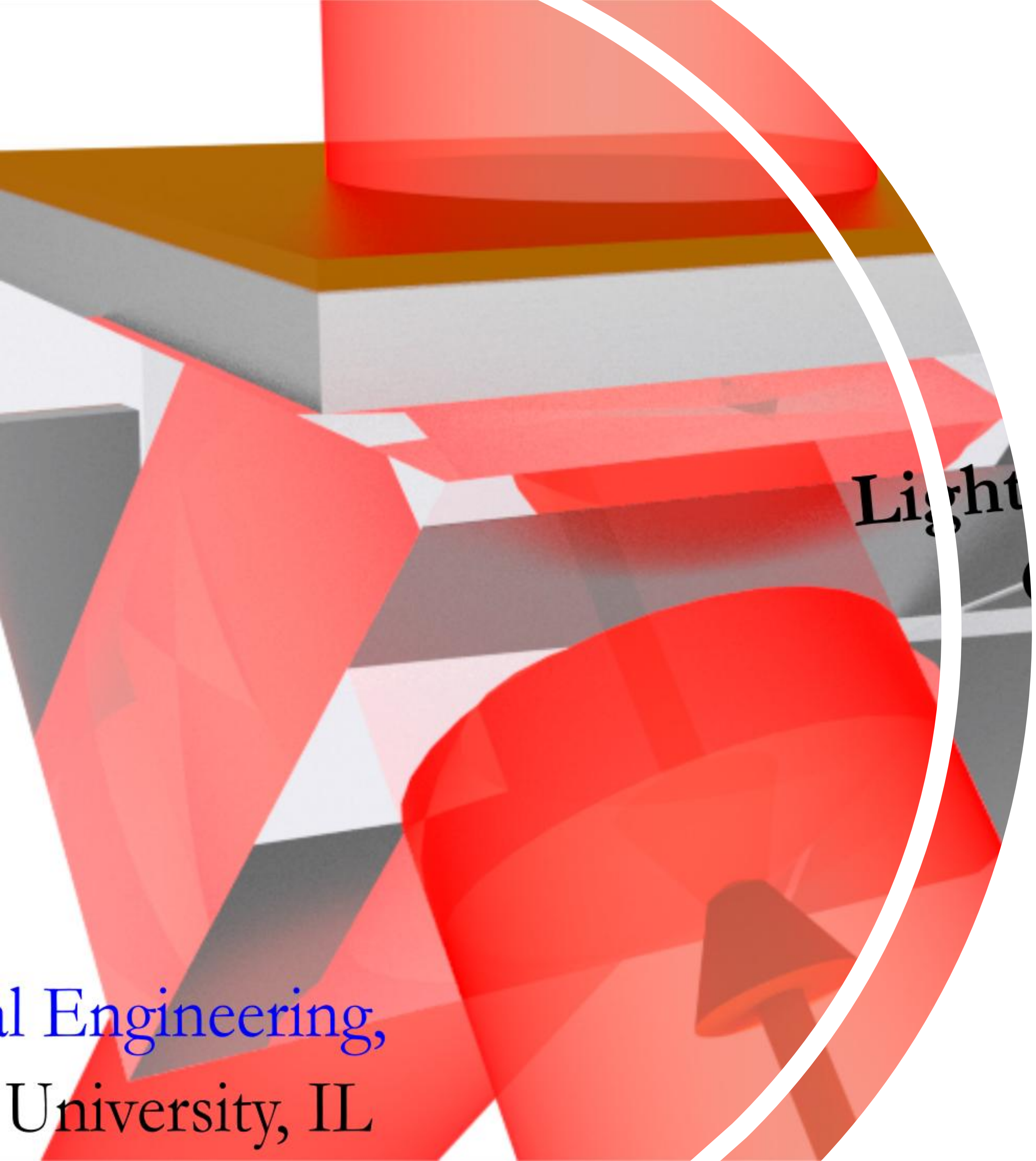
$$\boxed{B_{n1} = B_{n2}} \qquad \boxed{\mu_1 H_{n1} = \mu_2 H_{n2}}$$



BOUNDARY CONDITIONS FOR THE VERTICAL COMPONENT OF THE MAGNETIC FIELD - H_n

The vertical components of the magnetic flux density are continuous on the boundary, but the vertical components of the magnetic field are not.





FRESNEL'S EQUATIONS

- As light hits the boundary of two materials, the power is split and a fraction of the power is refracted while the rest is reflected.
- In 1825, Fresnel derived a set of equations that defines the relation between the reflectance or the transmittance to the incident angle and the indices of the material.

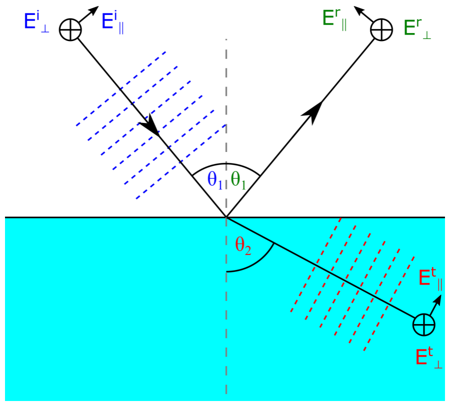
SNELL'S LAW OF REFRACTION

- When light hits the boundary between two materials, the light is reflected and refracted. In the transition from one medium to another medium, the propagation angle changes.
- In 1621, Snell discovered empirically the relationship between the indices of the materials and the propagation angles of the light.
- The refraction angle can be calculated by Snell's law of refraction which is defined as

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \tag{11}$$

where 1 is the incident medium, 2 is the transmitted medium, v is the velocity, θ is the angle of the light in the medium and n is the refractive index.

FRESNEL REFLECTION COEFFICIENT



The amplitude and phase of an optical field reflected back to the same side of the interface:

We will consider parallel field components: $E^{i}{}_{\parallel}$, $E^{t}{}_{\parallel}$ and $E^{r}{}_{\parallel}$.

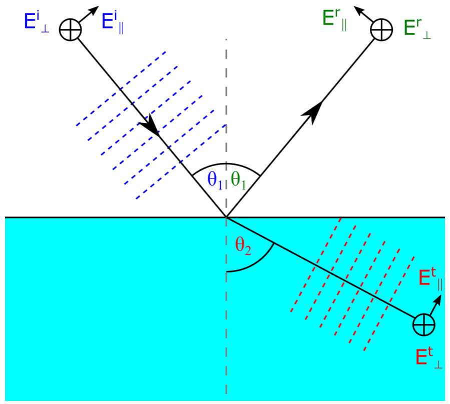
They can be decomposed into components parallel with and perpendicular to the interface; the parallel components are $E^{i}{}_{\parallel}\cos\theta_1$, $E^{t}{}_{\parallel}\cos\theta_2$ and $-E^{r}{}_{\parallel}\cos\theta_1$, respectively, which can be derived from Fig. 13.

Because of the field continuity across the interface, we have:

$$\left(E^{i}{}_{\parallel} - E^{r}{}_{\parallel}\right)\cos\theta_1 = E^{t}{}_{\parallel}\cos\theta_2 \qquad (12)$$

The magnetic field components associated with $E^{i}{}_{\parallel}$, $E^{t}{}_{\parallel}$ and $E^{r}{}_{\parallel}$ are perpendicular to the incident plane.

FRESNEL REFLECTION COEFFICIENT



The magnetic field components associated with $E^i{}_{\parallel}$, $E^t{}_{\parallel}$ and $E^r{}_{\parallel}$ have to be perpendicular to the incident plane. They are:

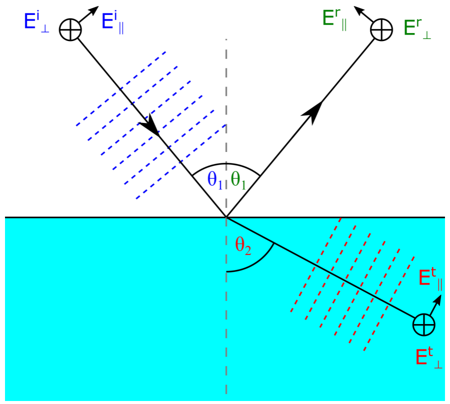
$$H^i{}_{\perp} = \sqrt{\frac{\varepsilon_1}{\mu_1}} E^i{}_{\parallel} \qquad H^t{}_{\perp} = \sqrt{\frac{\varepsilon_2}{\mu_2}} E^t{}_{\parallel} \qquad H^r{}_{\perp} = \sqrt{\frac{\varepsilon_1}{\mu_1}} E^r{}_{\parallel}$$

With electrical permittivities $\varepsilon_{1,2}$ and magnetic permittivities $\mu_{1,2}$ of the optical materials at two sides of the interface.

Since $H^i{}_{\perp}$, $H^r{}_{\perp}$ and $H^t{}_{\perp}$ are all parallel to the interface (although perpendicular to the incident plane), magnetic field continuity requires $H^i{}_{\perp} + H^r{}_{\perp} = H^t{}_{\perp}$. Assume that $\mu_1 = \mu_2$, $\sqrt{\varepsilon_1} = n_1$ and $\sqrt{\varepsilon_2} = n_2$ and then:

$$n_1 E^i{}_{\parallel} + n_1 E^r{}_{\parallel} = n_2 E^t{}_{\parallel} \tag{13}$$

FRESNEL'S FIELD REFLECTIVITY



The reflectivity for optical field components parallel to the incident plane as:

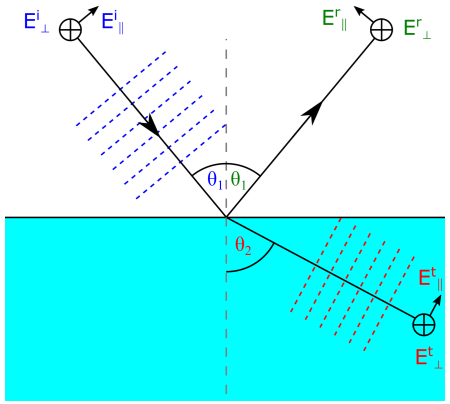
$$\rho_{\parallel} = \frac{E^r{}_{\parallel}}{E^i{}_{\parallel}} = \frac{n_1 \cos \theta_2 - n_2 \cos \theta_1}{n_1 \cos \theta_2 + n_2 \cos \theta_1} \tag{14}$$

In order to eliminate θ_2, we can use Snell's Law:

$$\rho_{\parallel} = \frac{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_1\right)^2} - n_2 \cos \theta_1}{n_1 \sqrt{1 - \left(\frac{n_1}{n_2} \sin \theta_1\right)^2} + n_2 \cos \theta_1} \tag{15}$$

Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as:

FIELD REFLECTIVITY

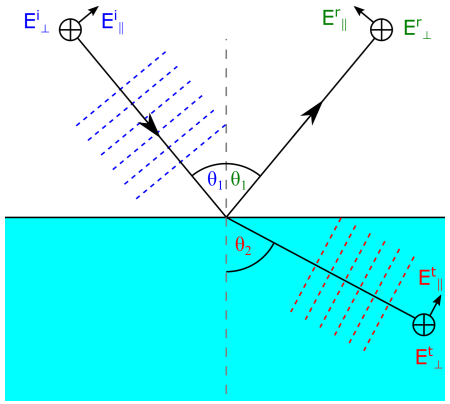


Similar analysis can also find the reflectivity for optical field components perpendicular to the incident plane as:

$$\rho_\perp = \frac{E^r{}_\perp}{E^i{}_\perp} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \quad (16)$$

$$\rho_\perp = \frac{n_1 \cos\theta_1 - n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2}}{n_1 \cos\theta_1 + n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2}} \quad (17)$$

FIELD REFLECTIVITY



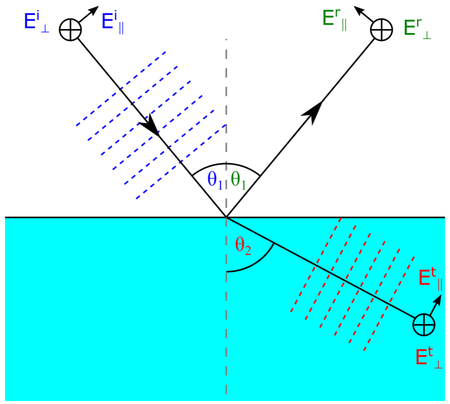
Power reflectivities for parallel and perpendicular field components are therefore:

$$R_{\parallel} = \left|\rho_{\parallel}\right|^2 = \left|\frac{E^r{}_{\parallel}}{E^i{}_{\parallel}}\right|^2 \quad (18)$$

and

$$R_{\perp} = \left|\rho_{\perp}\right|^2 = \left|\frac{E^r{}_{\perp}}{E^i{}_{\perp}}\right|^2 \quad (19)$$

FRESNEL'S POWER TRANSMISSION COEFFICIENTS



According to energy conservation, the power transmission coefficients can be found as:

$$T_{\parallel} = \left| \frac{E^{t}{}_{\parallel}}{E^{i}{}_{\parallel}} \right|^2 = 1 - |\rho_{\parallel}|^2 \qquad (20)$$

and

$$T_{\perp} = \left| \frac{E^{t}{}_{\perp}}{E^{i}{}_{\perp}} \right|^2 = 1 - |\rho_{\perp}|^2 \qquad (21)$$

In practice, for an arbitrary incidence polarization state, the input field can always be decomposed into $E_{\parallel}$ and $E_{\perp}$ components. Each can be treated independently.

TOTAL INTERNAL REFLECTION IN FIBERS

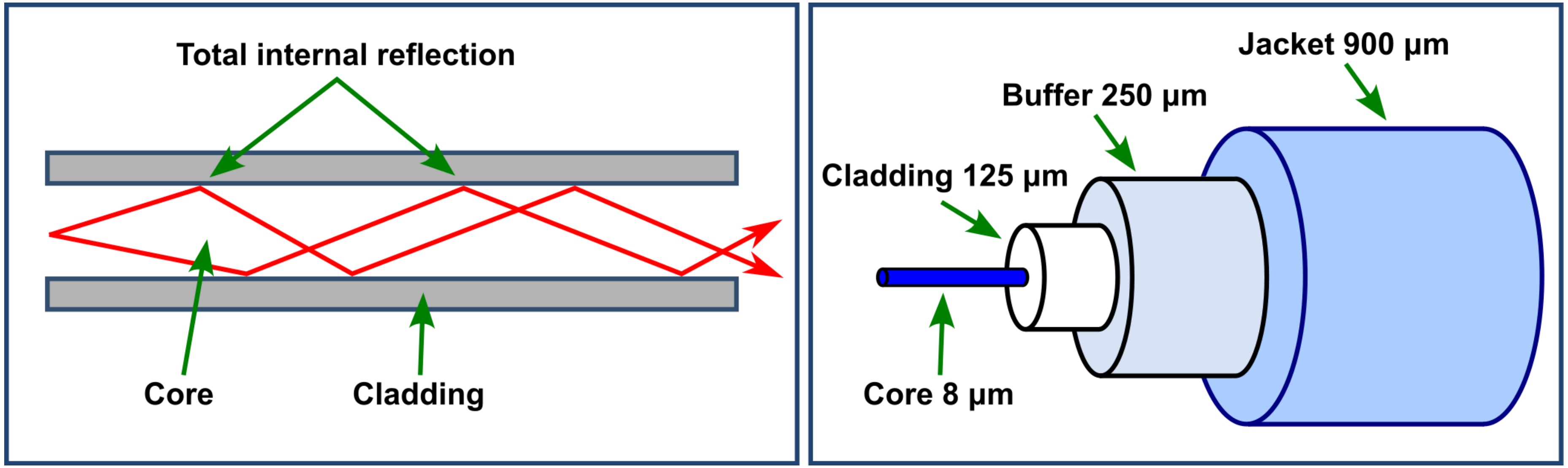


Figure 15: Left: total internal reflection, which happens because the material of the cladding (shown on the right) has a lower index of refraction compared to the core.

CONCEPT OF TOTAL INTERNAL REFLECTION

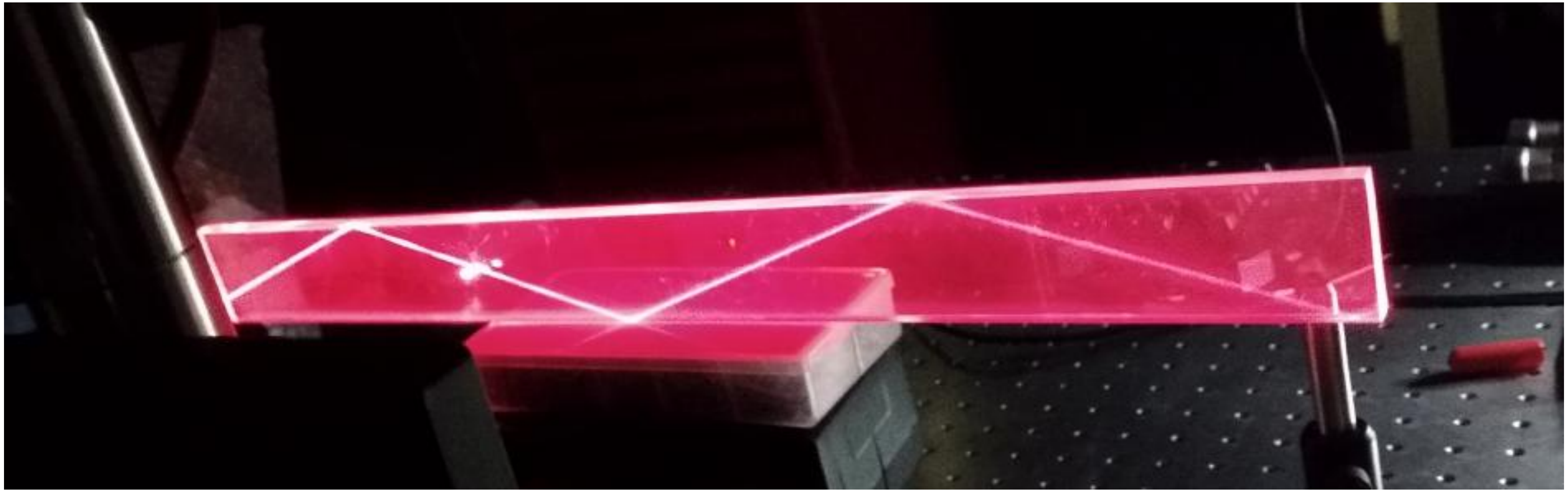


Figure 16: A laser beam through acrylic shows the concept of total internal reflection (the light doesn't continue straight through the edge of the glass but bounces back and forth until exiting at the end).

CRITICAL ANGLE - θ_c

- However, when light hits the boundary between high to low refractive index material, above a specific angle, called the critical angle, the light will be fully reflected.
- This phenomenon is called total internal reflection (TIR). According to Fresnel Equations (15) and (17), total reflection ($|\rho_{\parallel}| = |\rho_{\perp}| = 1$) occurs when $\frac{n1}{n2}\sin(\theta_1) = 1$ and the critical angle is defined as:

$$\theta_c = \theta_1 = sin^{-1}\left(\frac{n_2}{n_1}\right) \tag{22}$$

CRITICAL ANGLE

Obviously, the necessary condition to have a critical angle depends on the interface condition.

- **if $n_1 < n_2$: there is no real solution** for $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$.

It means that when a light beam goes from a low index material to a high index material, total reflection is not possible.

- **if $n_1 > n_2$: there is a real solution** for $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$.

Therefore, total reflection can only happen when a light beam launches from a high index material to a low index material.

CRITICAL ANGLE

- It is important to note that at a larger incidence angle $\theta_1 > \theta_c$, $1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2 < 0$ and $\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$ becomes imaginary.

- Equations (2) and (4) show that if $\sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta_1\right)^2}$ is imaginary, both $\left|\rho_{\parallel}\right|^2$ and $|\rho_{\perp}|^2$ are equal to 1. The important conclusion is that for all incidence angles satisfying $\theta_1 = \theta_c$ total internal reflection will happen with $R = 1$.

CRITICAL ANGLE

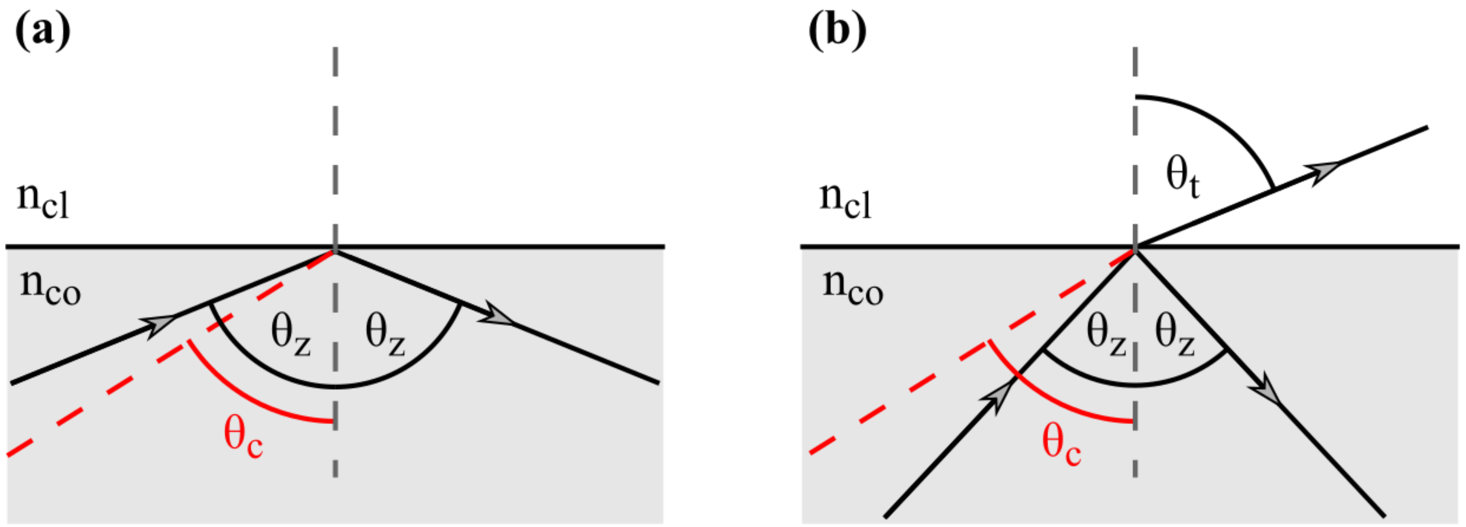


Figure 17: Reflection at a planar interface between unbounded regions of refractive indices n_{co} and n_{cl} showing (a) total internal reflection and (b) partial reflection and refraction.

EVANESCENT FIELD

An evanescent field is a side effect of TIR and appears beyond the boundary surface. Specifically, even though the entire incident wave is reflected back into the originating medium (TIR), a fraction of the field penetrates into the medium with a lower n at the boundary. The evanescent wave is leading to the Goos-Hanchen shift.

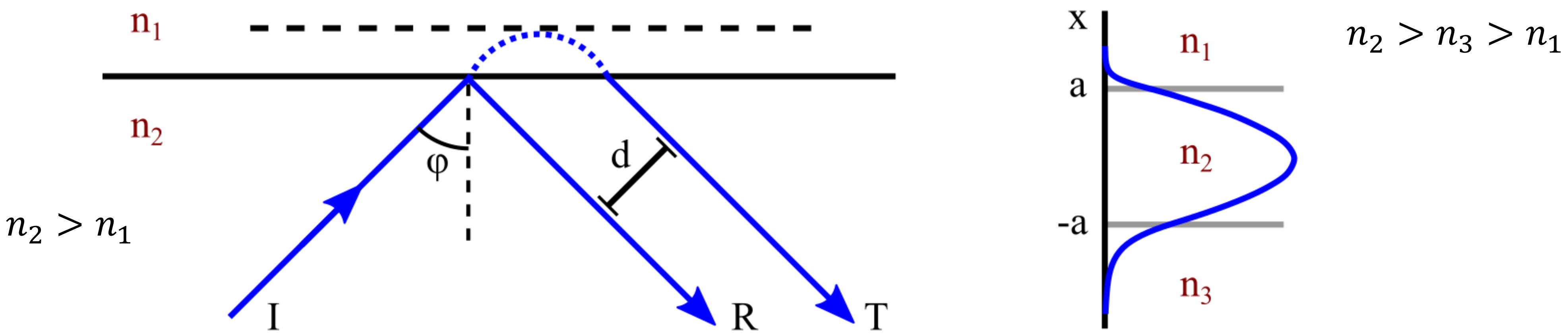
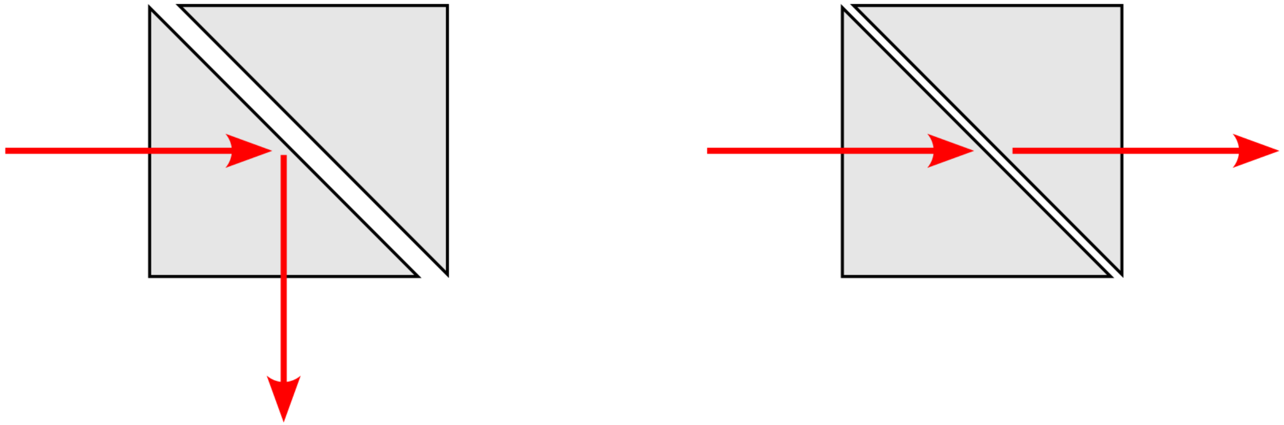


Figure 18: Total internal reflection and Goos-Hänchen shift. R is the behavior of the partially reflected beam, T is the behavior of the total internal reflection beam and d is the Goos-Hänchen shift.

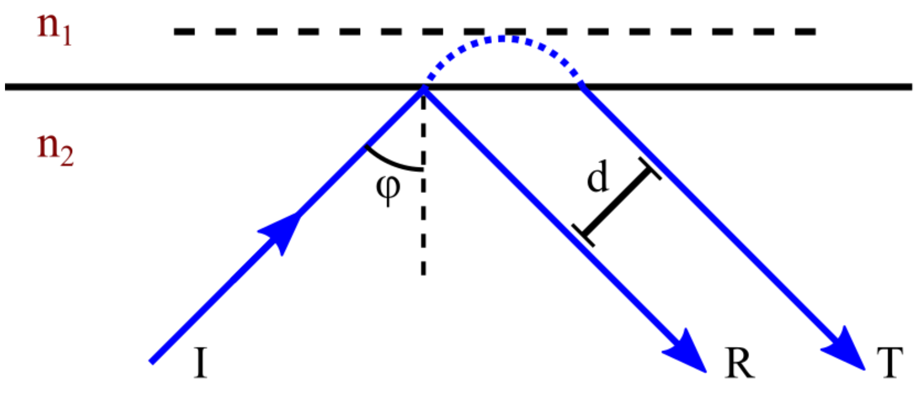
EVANESCENT FIELD

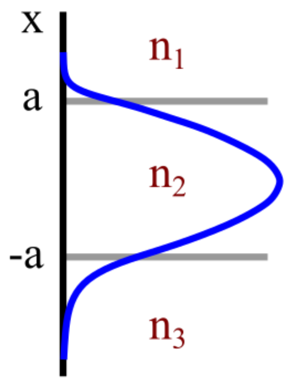
- The first to observe the phenomenon was Isaac Newton in 1726. In the experiment, he used two identical prisms with a distance between them of a few tens of nanometers. While illuminating a prism with an angle bigger than the critical angle he saw that the light pass the gap and move to the second prism. This phenomenon is described as optical tunneling which can be used for beam splitters and filters.



EVANESCENT FIELD

- Although according to the ray optic model, the light is totally reflected inside the guiding layer in the case of guided mode, according to Maxwell's theory, a fraction of the field is penetrating outside the guiding layer to a less dense medium before the total internal reflection occurs. This phenomenon contradicts the total internal reflection.
- In 1947, Goos and Hanchen observed a small lateral phase shift when the light is under total internal reflection. It appears that the wave is reflected from a virtual plane from the medium with the lower refractive index as shown in Figure below.





$n_2 > n_3 > n_1$

EVANESCENT FIELD

The reflected wave phase shift is called Goos-Hanchen shift and is defined as:

$$\Phi = -2\tan^{-1}\left(\frac{\sqrt{{n_2}^2\sin^2\phi - {n_1}^2}}{n_2\cos\phi}\right) \tag{23}$$

where n_2 is the incident medium (higher refractive index), n_1 is the transmitted medium (lower refractive index), and ϕ is the incident angle. Goos-Hanchen shift can be utilized for characterizing materials in optical microscopy and lithography.

EVANESCENT FIELD

Due to the penetration beyond the guiding layer, the evanescent field interacts with its surroundings and can be utilized for plasmons, sensing and near-field microscopy. The penetration depth of the evanescent field to a medium outside the guiding layer is defined as:

$$d_p = \frac{\lambda}{2\pi\sqrt{{n_2}^2 \sin^2\phi - {n_1}^2}} \tag{24}$$

where ϕ is the incident angle inside the guiding layer. The equation shows that the smaller the incident angle, the larger the penetration depth of the evanescent field into the medium.

EVANESCENT FIELD

- The transmitted wavevector is: $k_t = k_t \sin \theta_t \, \hat{x} + k_t \cos \theta_t \, \hat{z}$
- If $n_1 > n_2$, then $\sin \theta_t > 1$.
- Since $\sin \theta_t = \frac{n_1}{n_2} \sin \theta_i$ (Snell's law), $\frac{n_1}{n_2} \sin \theta_i > 1$ for $\theta_i > \theta_c$ therefore $\cos \theta_t$ becomes complex:

$$\cos \theta_t = \sqrt{1 - \sin^2 \theta_t} = j\sqrt{\sin^2 \theta_t - 1}$$

EVANESCENT FIELD

The electric field of the transmitted plane wave is given by $E_t = E_0 e^{j(\bar{k}_t \cdot \bar{r} - \omega t)}$

$$E_t = E_0 e^{j(\bar{k}_t \cdot \bar{r} - \omega t)} = E_0 e^{j[k_t \sin(\theta_t) x + k_t \cos(\theta_t) z - \omega t]}$$

$$E_t = E_0 e^{j\left[x k_t \sin(\theta_t) + z j k_t \sqrt{\sin^2(\theta_t) - 1} - \omega t\right]}$$

By substituting $k_t = \frac{\omega n_2}{c}$ we obtain:

$$E_t = E_0 e^{-\kappa z} e^{j(kx - \omega t)} \qquad (25)$$

where $k = \frac{\omega n_1}{c}$ and $\kappa = \frac{\omega}{c}\sqrt{{n_1}^2 \sin^2 \theta_i - {n_2}^2}$

EVANESCENT FIELD

The evanescent field has few unique properties:

- First, the evanescent field doesn't propagate in the medium but is a localized oscillating electric and/or magnetic field. The Poynting vector normal to the surface is equal to zero since it is the average Poynting vector over an oscillation cycle and, therefore, equal to zero.
- However, the evanescent field can still interact with the surroundings. The evanescent field can be also converted back into radiation/guiding mode.
- By placing two waveguides close to each other the power of mode from one waveguide can be transferred to the second waveguide by evanescent field coupling. This is a near-field optics effect which is called optical tunneling or tunneling effect. The effect can be utilized for couplers such as ring resonators of a chip.
- In addition, by manipulating the evanescent field we can control and affect the guided mode, with nanomaterial or metasurface overlayer.

EVANESCENT FIELD - $E_t = E_0 e^{-\kappa \hat{z}} e^{j(k\hat{x}-\omega t)}$

1) Appears in the optically less dense medium.
2) Characterized by its propagation in the x direction.
3) Characterized by its exponential attenuation in the z direction.
4) No energy flows across the boundary.
5) The component of Poynting vector in the direction normal to the boundary is finite, but its time average vanishes (what is Poynting vector? what is time average Poynting vector?).
6) The Goos-Hanchen effect only occurs for linearly polarized light.
7) If the light is circularly or elliptically polarized, it will undergo the analogous Imbert–Fedorov effect.

H.W.: DERIVATION

Goos-Hanchen shift

[1] Derive Goos-Hanchen shift

[2] Schematically illustrate the concept underpinned by your derivations

[3] Summaries the historical evidence of formulation of the Snell's law.

[3] Overview the history of Imbert-Fedorov effect and explain it.

Deadline: Next class

BIBLIOGRAPHY

[1] Agrawal, Govind P. "Fiber-optic communication systems" Vol. 222. John Wiley & Sons, 2012.

[2] Schneider, Thomas. "Nonlinear optics in telecommunications" Springer Science & Business Media, 2013.

[3] Okamoto, K. "Fundamentals of optical waveguides" Academic press, 2006.