Ben-Gurion University of the Negev

BELARUSIAN STATE UNIVERSITY

ROGUE WAVES IN OPTICAL FIBERS

Presenter:

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Outline

1. What is a rogue wave?
    - definition, properties and examples
2. Rogue waves formation in fibers: conditions and process
    - interplay of dispersion and nonlinearity in fibers
    - formation of solitons
    - modulation instability
    - supercontinuum generation and generation of rogue waves
3. Optical rogue waves in fibers: first report
    - experimental observation
    - numerical simulations
4. Numerical simulation of rogue waves in fibers
    - solving NLSE with the use of split-step Fourier method

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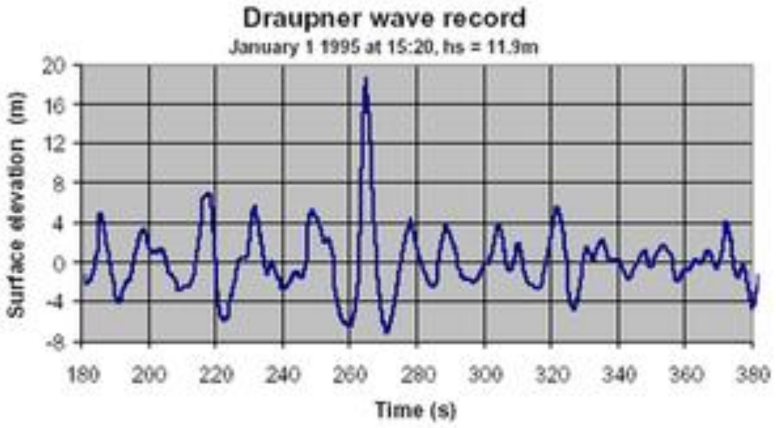
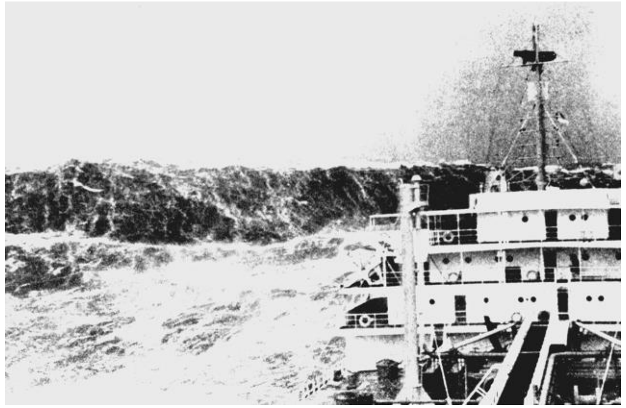
What is a rogue wave?

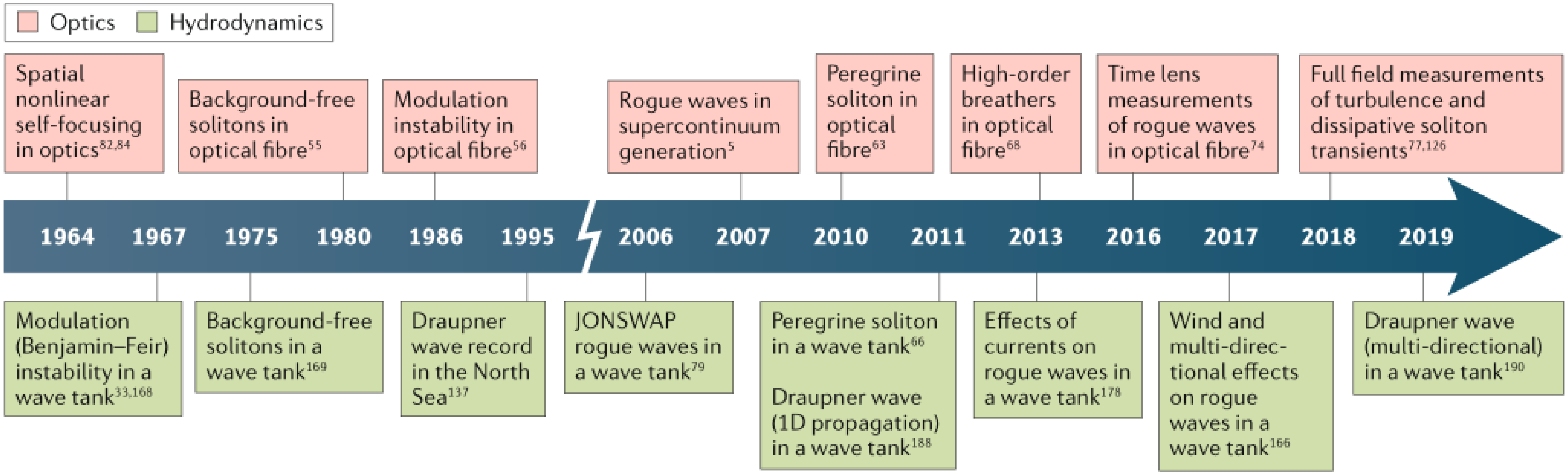
- Rogue waves (RWs) are events with extremely high amplitude, which form spontaneously and disappear without a trace in some time.
- First reports of rogue waves correspond to oceanography:
  The first documented rogue wave (Draupner wave) was recorded in 1995 in the North sea, it was 25.6 m in height.
- Although RWs are elusive and intrinsically difficult to monitor because of their fleeting existences, satellite surveillance has confirmed that rogue waves roam the open oceans, occasionally encountering a ship or seaplatform, sometimes with devastating results.





What is a rogue wave?

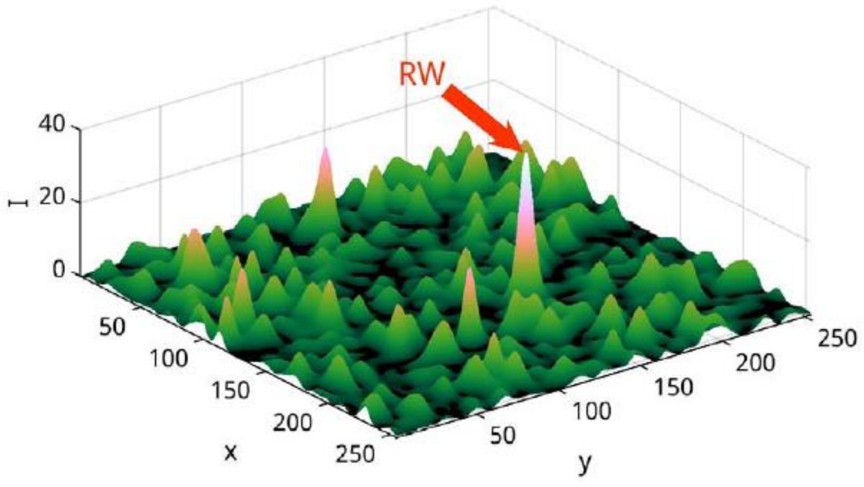
- Optical RWs are analogous to the oceanic RWs – rare anomalous events with large amplitude.
- Deep research into the optical rogue waves has developed considerably since 2007 [4], following the introduction of an analogy between the generation of large ocean waves and the propagation of light in optical fibers.

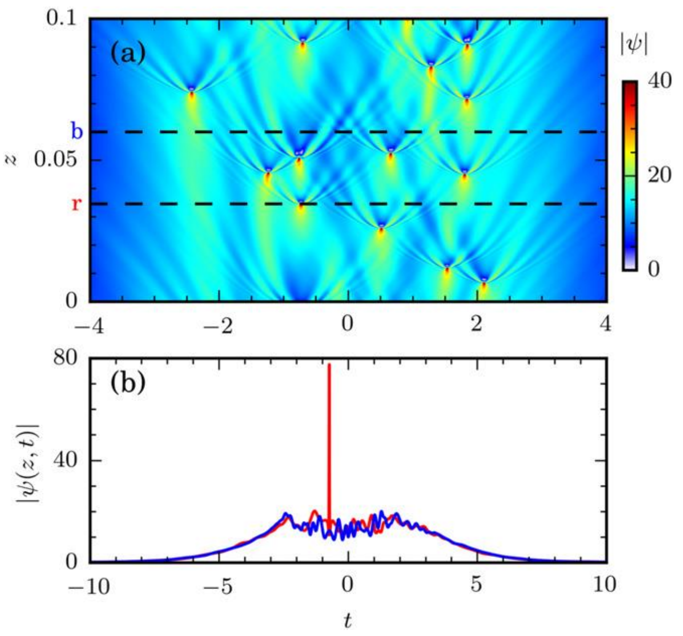


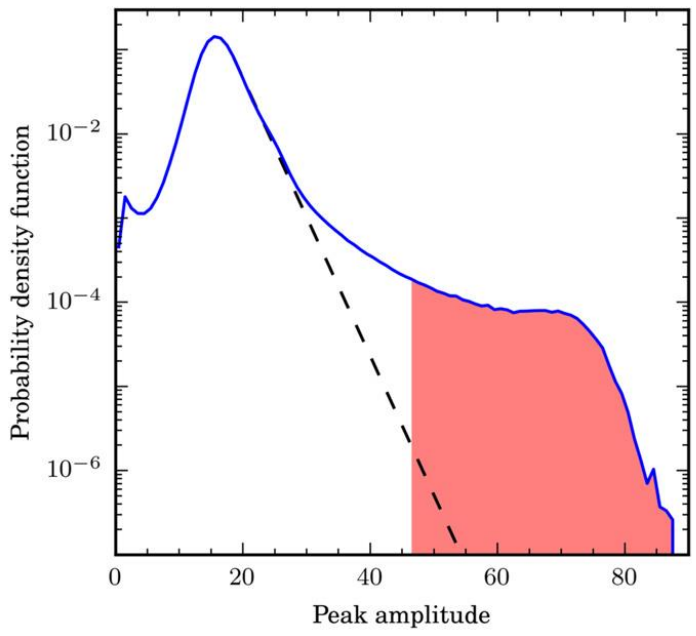
Timeline illustrating the parallel developments in fibre optics (top) and hydrodynamics (bottom) [3].

What is a rogue wave?

- The unusual statistics of rogue waves represent one of their defining characteristics.
- Conventional models of ocean waves indicate that the probability of observing large waves should diminish extremely rapidly with wave height, suggesting that the likelihood of observing even a single freak wave in hundreds of years should be essentially non-existent.
- In reality, however, ocean waves, as well as optical RWs, appear to follow ‘L-shaped’ statistics: most waves have small amplitudes, but extreme outliers also occur much more frequently than expected in ordinary (Gaussian) wave statistics.

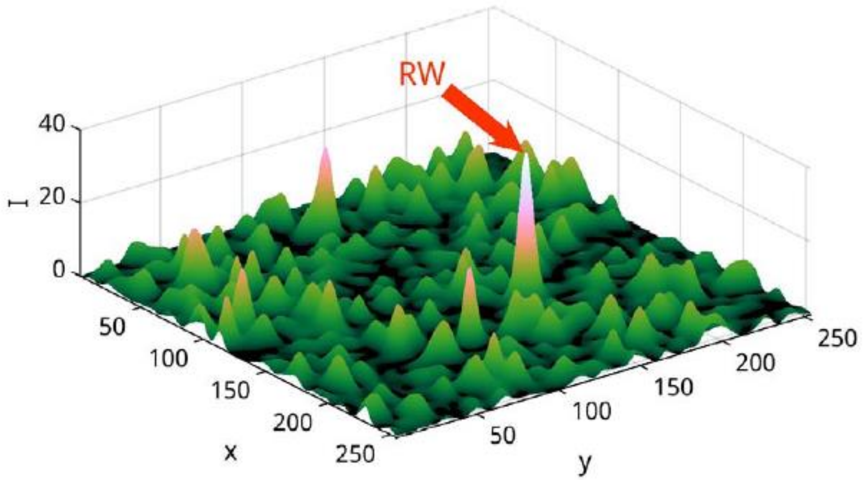


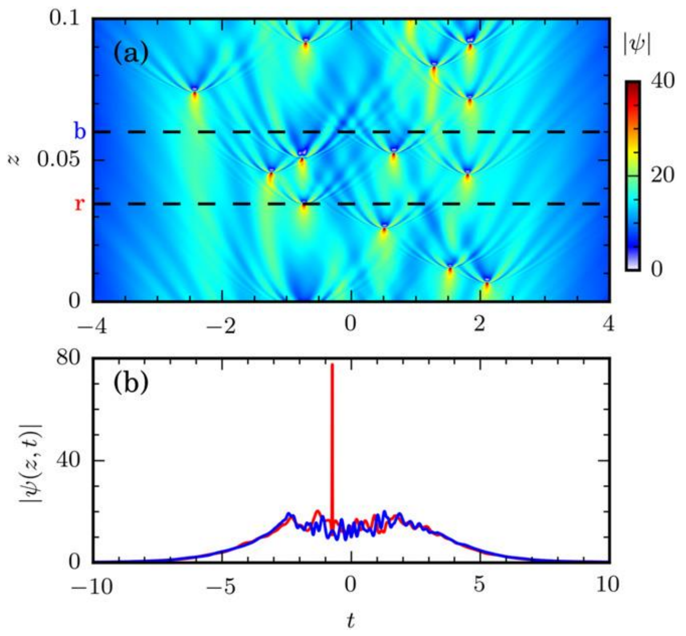


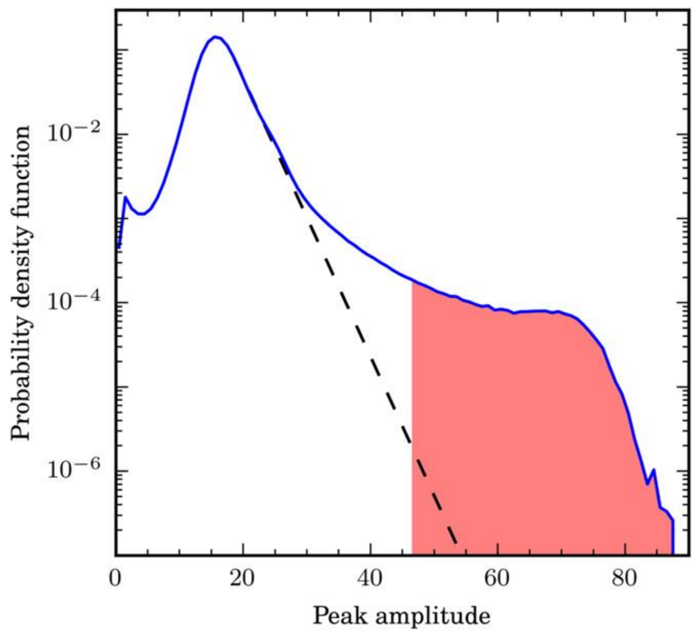


What is a rogue wave?

- Rogue waves are characterized by a maximum peak amplitude that is at least twice as large as the significant wave height (SWH).
- The significant wave height is defined as the average of the highest one-third of the waves nearby and represents a widely accepted parameter for the identification of rogue waves.
- This threshold criterion has been initially introduced in hydrodynamics, and is therefore formulated with reference to the wave amplitude.



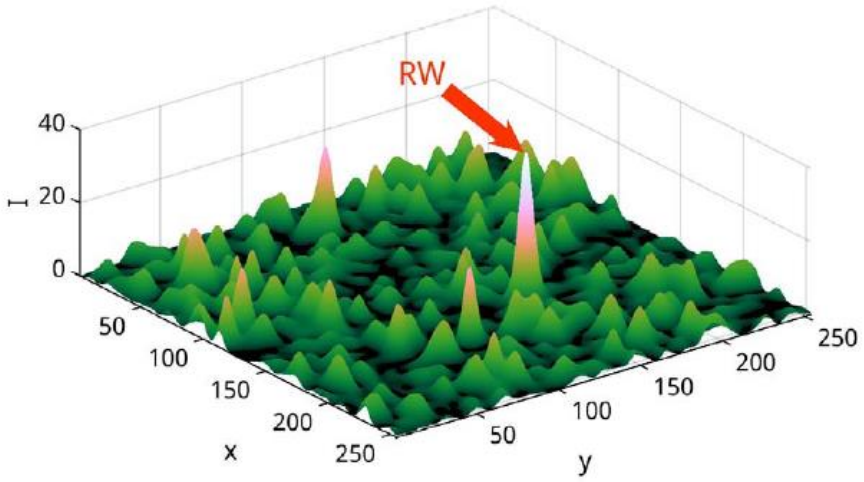


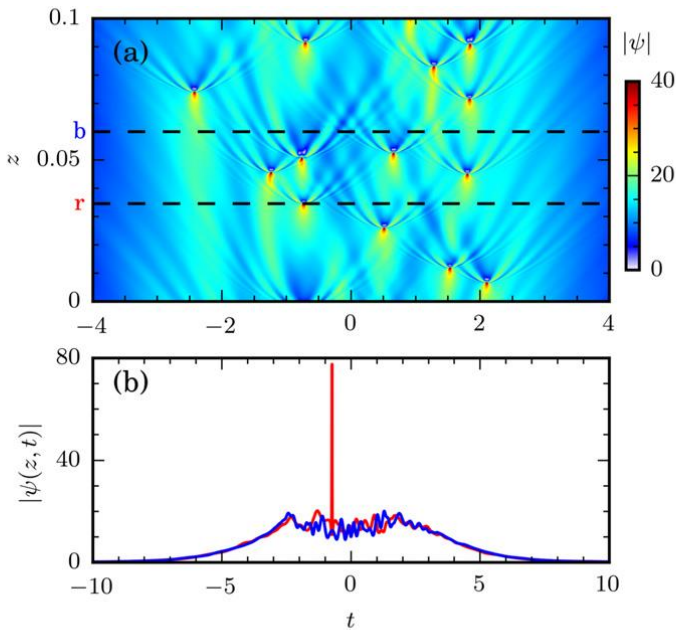


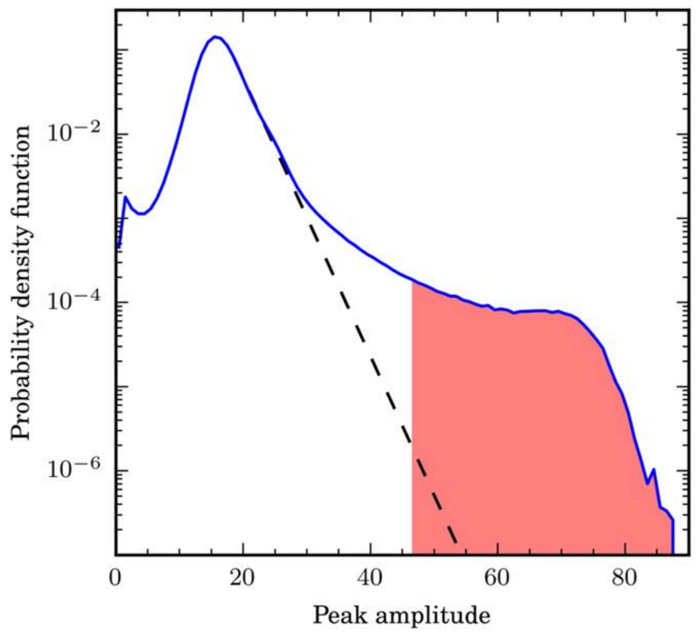
What is a rogue wave?

Rogue wave characteristics:

- Maximum amplitude at least twice as large as the significant wave height (SWH).
- ‘L-shaped’ statistics: most waves have small amplitudes, but extreme outliers also occur much more frequently than expected in ordinary (Gaussian) wave statistics.
- Form and disappear spontaneously with unpredictable lifetime.







Outline

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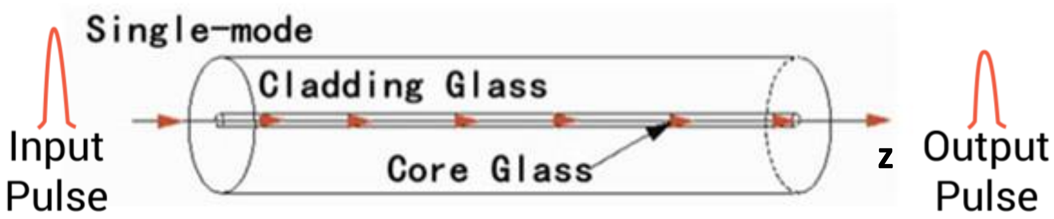
Rogue waves formation in fibers: NLSE

The generalized nonlinear Schrödinger equation (NLSE) describes the pulse propagation through a fiber in the presence of dispersion and nonlinearity [2]:

$$\frac{\partial A}{\partial z} + \frac{\alpha}{2} A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3} = i\gamma\left(|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial T}\right)$$

- *A(z,T)* is a slowly varying electric field envelope.
- The optical field is assumed to be quasi-monochromatic. The pulse spectrum, centered at ω_0, is assumed to have a spectral width $\Delta\omega$ such that $\Delta\omega/\omega_0 \ll 1$. For $\omega_0 \sim 10^{15}\ s^{-1}$, the last assumption is valid for pulses as short as 20 fs.



- The waveguide is single mode.
- The optical field is assumed to maintain its polarization along the fiber length so that a scalar approach is valid. This is not really the case, unless polarization-maintaining fibers are used, but the approximation works quite well in practice.
- T is measured in a frame of reference moving with the pulse at the group velocity v_g ($T = t - z/v_g$).

Rogue waves formation in fibers: NLSE

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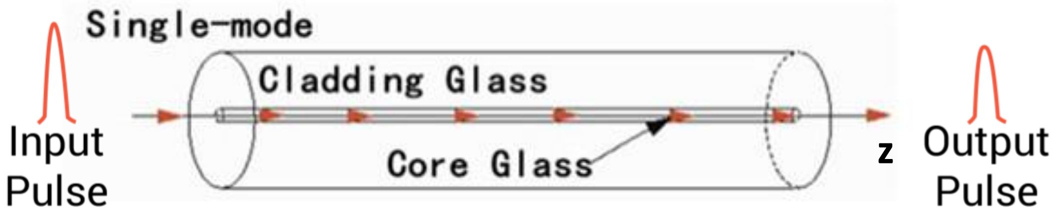
- β_2 and β_3 are values that characterize the 2nd and 3rd order dispersion (group velocity dispersion and dispersion slope), α is the loss coefficient, γ is the nonlinear coefficient of the fiber, and T_R is a parameter that characterizes the delayed nonlinear response of the fiber.



- The bracketed terms on the right-hand side of the equation describe the Kerr nonlinearity, self-steepening and the vibrational Raman response of the medium, respectively.

Rogue waves formation in fibers: NLSE

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Physical effects important to understand the rogue waves generation:

- Interplay of dispersion and nonlinearity
- Soliton formation
- Modulation instability
- Supercontinuum generation

Rogue waves formation in fibers: NLSE

NLSE accounting only for dispersion and nonlinearity:

$$\boldsymbol{\frac{\partial A}{\partial z}} + \frac{\alpha}{2}A + \boldsymbol{\frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2}} - \boldsymbol{\frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3}} = i\gamma\left(\boldsymbol{|A|^2 A} + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A\frac{\partial |A|^2}{\partial T}\right)$$

$$\Downarrow$$

$$\frac{\partial A}{\partial z} = i\gamma|A|^2 A - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3}$$

Normalized NLSE:

$$\frac{\partial U}{\partial z} = \frac{i}{L_{NL}}|U|^2 U - \frac{i sign(\beta_2)}{2L_D}\frac{\partial^2 U}{\partial \tau^2}$$

- $L_D = \frac{T_0^2}{|\beta_2|}$ - dispersion length scale
- $L_{NL} = \frac{1}{\gamma P_0}$ - nonlinear length scale

- $\tau = \frac{T}{T_0}$ - normalized T coordinate
- T_0 and P_0 - temporal width and power of the input pulse, L – the fiber length.
- $U(z,\tau) = \frac{1}{\sqrt{P_0}}A(z,\tau)$ – normalized pulse amplitude

Rogue waves formation in fibers: dispersion

Consider dispersion only:

$$\frac{\partial U}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 U}{\partial T^2}$$

Using Fourier transform, one can write this equation in frequency domain:

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{U}\,(z,\omega)\,e^{-i\omega T} d\omega$$

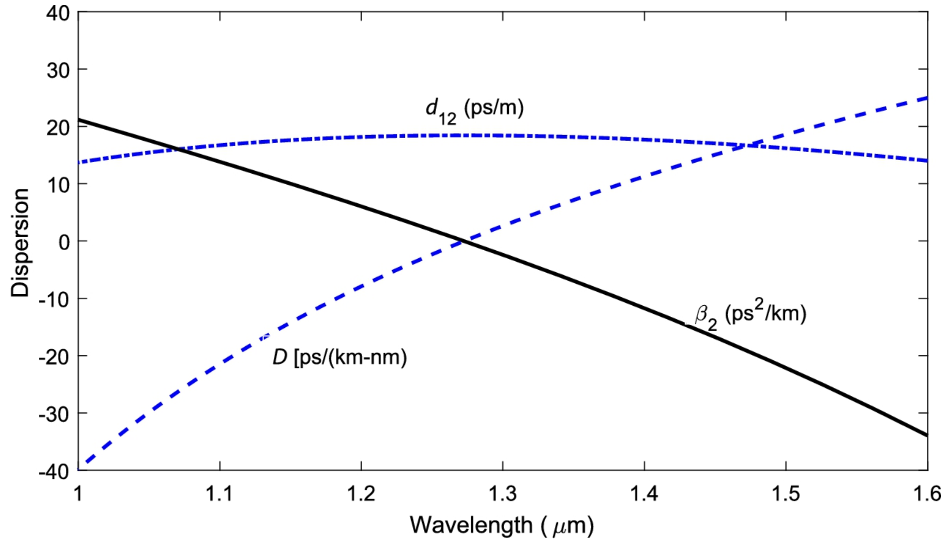
$$\Downarrow$$

$$\frac{\partial \tilde{U}}{\partial z} = \frac{i}{2}\beta_2\omega^2\tilde{U}$$

Solution:

$$\tilde{U}(z,\omega) = \tilde{U}(0,\omega)\,e^{\frac{i}{2}\beta_2\omega^2 z}$$

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \tilde{U}\,(0,\omega)\,e^{\frac{i}{2}\beta_2\omega^2 z - i\omega T} d\omega$$



- $\beta_2 = \frac{d}{d\omega}\left(\frac{L}{v_g}\right) = \frac{d^2\beta}{d\omega^2}$ - group velocity dispersion (GVD).
- Because of the GVD, different frequency components of a pulse travel at different speeds along the fiber.
- In the normal-GVD region (β_2>0), red components travel faster than blue components, while the opposite occurs in the anomalous-GVD region (β_2<0).
- In silica fibers, β_2<0 for λ≳1.3 um.

Rogue waves formation in fibers: dispersion

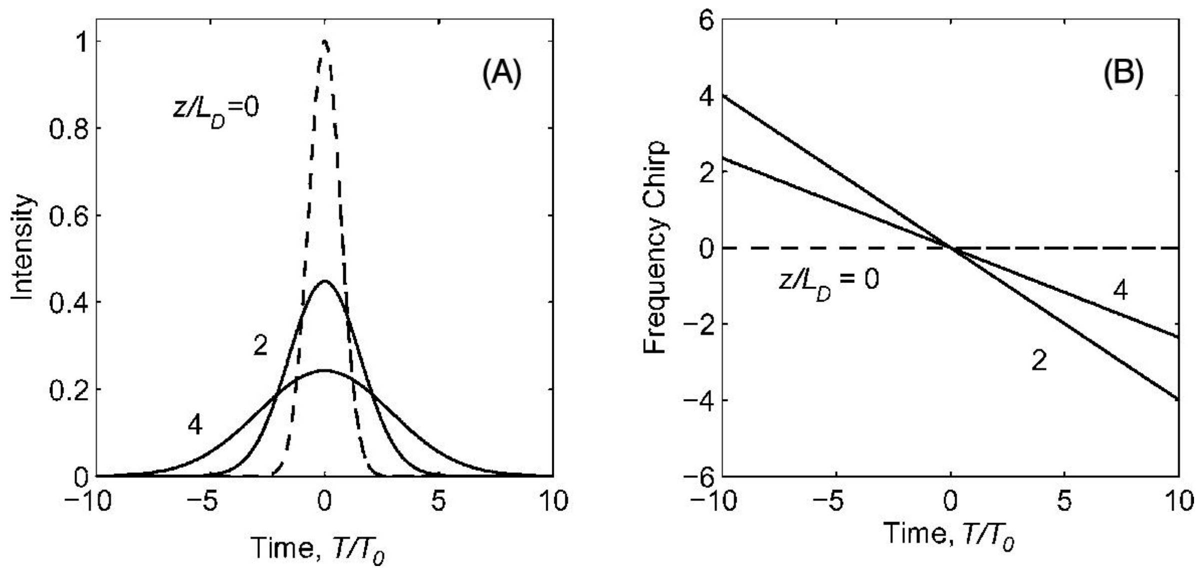
Consider dispersion only:

$$\frac{\partial U}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 U}{\partial T^2}$$

Solution:

$$\widetilde{U}(z,\omega) = \widetilde{U}(0,\omega)\, e^{\frac{i}{2}\beta_2\omega^2 z}$$

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \widetilde{U}\,(0,\omega)\, e^{\frac{i}{2}\beta_2\omega^2 z - i\omega T} d\omega$$



Normalized intensity (A) and frequency chirp $\delta\omega T_0$ (B) as functions of T/T_0 for a Gaussian pulse at $z = 2L_D$ and $4L_D$. Dashed lines show the input profiles at $z = 0$.

- GVD modifies the phase of each spectral component of the pulse by an amount that depends on its frequency and the propagated distance.
- Even though such phase changes do not affect the optical spectrum, they can modify the pulse shape.
- Dispersion-induced pulse broadening can be understood by recalling that different frequency components of a pulse travel at slightly different speeds along the fiber because of GVD.
- Any time delay in the arrival of different spectral components leads to pulse broadening.

Rogue waves formation in fibers: dispersion

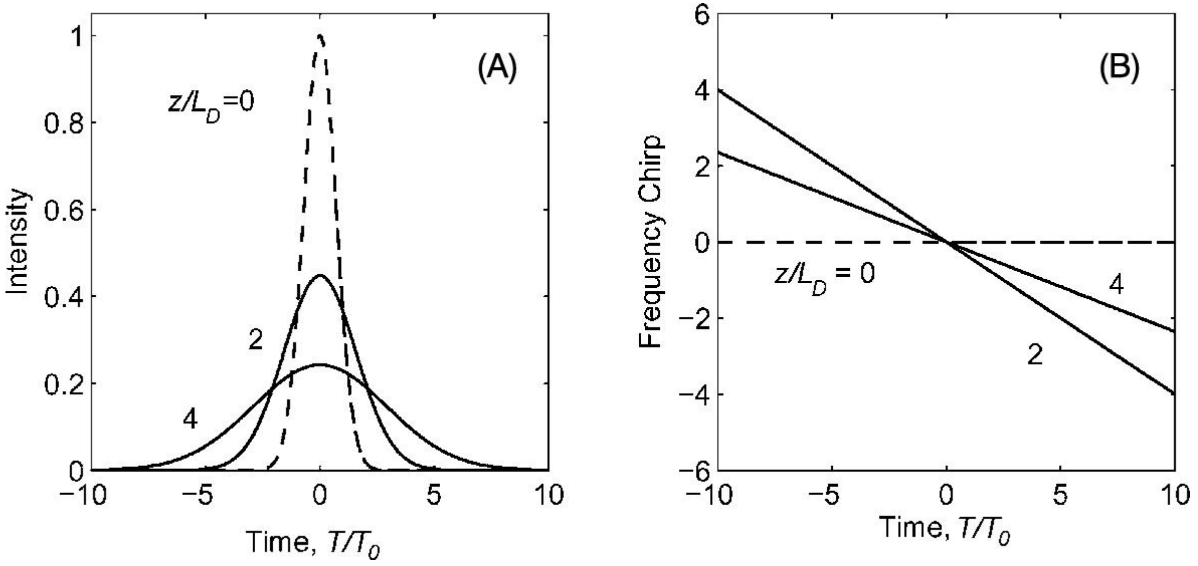
- Even if the incident pulse is unchirped (with no phase modulation), the transmitted pulse becomes chirped. For example, for the input Gaussian pulse

$$U(z,T) = |U(z,T)|\, e^{i\varphi(z,T)},$$

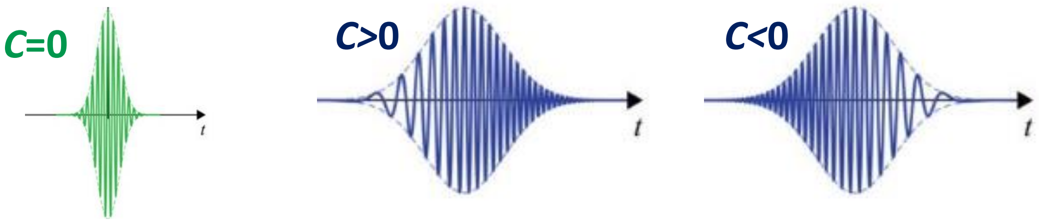
$$\varphi(z,T) = -\frac{sign(\beta_2)z/L_D}{1+(z/L_D)^2}\frac{T^2}{2T_0^2} + \frac{1}{2}tan^{-1}\left(sign(\beta_2)\frac{z}{L_D}\right)$$

- The time dependence of $\varphi(z,T)$ implies that the instantaneous frequency differs across the pulse from the central frequency ω_0. The frequency chirp corresponds to the difference $\delta\omega$ depending on time *T:*

$$\delta\omega(T) = \omega(T) - \omega_0 = -\frac{\partial\varphi(z,T)}{\partial T} = \frac{sign(\beta_2)z/L_D}{1+(z/L_D)^2}\frac{T}{T_0^2}$$



Normalized intensity (A) and frequency chirp $\delta\omega T_0$ (B) as functions of T/T_0 for a Gaussian pulse at $z = 2L_D$ and $4L_D$. Dashed lines show the input profiles at $z = 0$



- The frequency changes linearly with *T*, i.e., a fiber imposes linear frequency chirp across the pulse.

- The chirp $\delta\omega$ also depends on the sign of β_2. In the normal-GVD region (β_2>0), $\delta\omega$ is negative at the leading edge (*T*<0) and increases linearly across the pulse (positive chirp). The opposite occurs in the anomalous-GVD region (β_2<0) the chirp is negative (this case is shown in the figure).

Rogue waves formation in fibers: dispersion

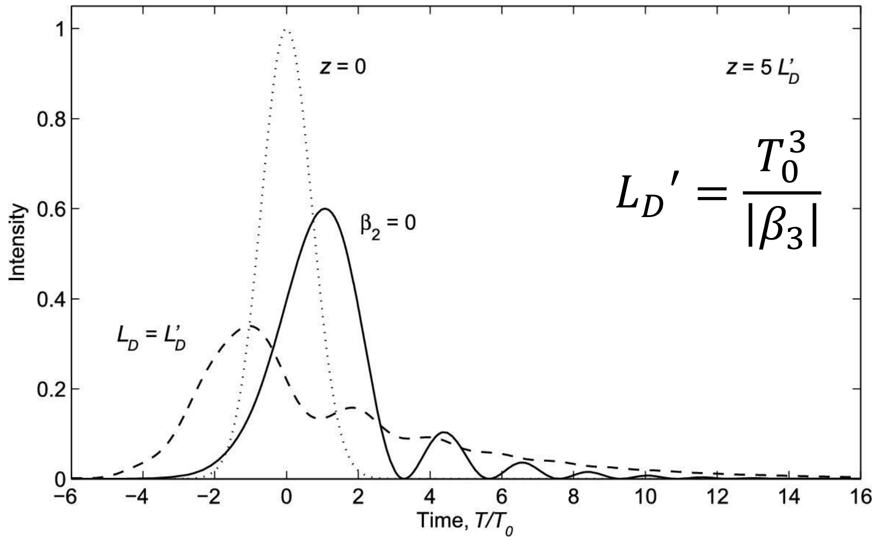
Consider 3rd order dispersion:

$$\frac{\partial U}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 U}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 U}{\partial T^3}$$

Solution:

$$\widetilde{U}(z,\omega) = \widetilde{U}(0,\omega)\, e^{\frac{i}{2}\beta_2\omega^2 z + \frac{i}{6}\beta_3\omega^3 z}$$

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \widetilde{U}\,(0,\omega)\, e^{\frac{i}{2}\beta_2\omega^2 z + \frac{i}{6}\beta_3\omega^3 z - i\omega T}\, d\omega$$



$$L_D{}' = \frac{T_0^3}{|\beta_3|}$$

Pulse shapes at z=5L_D' of an initially Gaussian pulse at z = 0 (dotted curve) in the presence of higher-order dispersion. Solid curve is for β_2=0. Dashed curve shows the effect of finite β_2 in the case of L_D=L_D'.

- If the pulse wavelength nearly coincides with the zero-dispersion wavelength of the fiber and β_2≈0, the β_3 term provides the dominant contribution to the GVD effects. For ultrashort pulses (with width T_0<1 ps), it is necessary to include the β_3 term even when β_2≠0. The TOD effects play a significant role only if L_D'≤L_D or $T_0|\beta_2/\beta_3| \leq 1$.

- The TOD distorts the pulse such that it becomes asymmetric and develops an oscillatory structure near one of its edges. In the case of positive β_3 (shown in the figure), oscillations appear near the trailing edge of the pulse. When β_3 is negative, it is the leading edge of the pulse that develops oscillations.

Rogue waves formation in fibers: dispersion

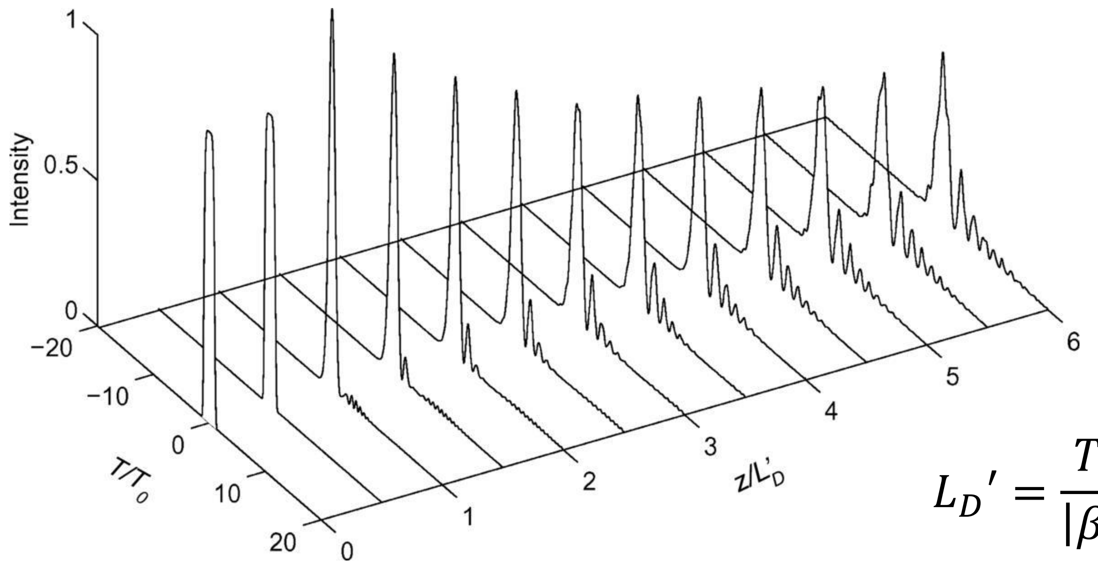
Consider 3rd order dispersion:

$$\frac{\partial U}{\partial z} = -\frac{i\beta_2}{2}\frac{\partial^2 U}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3 U}{\partial T^3}$$

Solution:

$$\widetilde{U}(z,\omega) = \widetilde{U}(0,\omega)\, e^{\frac{i}{2}\beta_2\omega^2 z + \frac{i}{6}\beta_3\omega^3 z}$$

$$U(z,T) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \widetilde{U}\,(0,\omega)\, e^{\frac{i}{2}\beta_2\omega^2 z + \frac{i}{6}\beta_3\omega^3 z - i\omega T}\, d\omega$$



$$L_D{}' = \frac{T_0^3}{|\beta_3|}$$

Evolution of a super-Gaussian pulse with m = 3 along the fiber length for the case of β_2=0 and β_3>0. Third-order dispersion is responsible for the oscillatory structure near the trailing edge of the pulse.

- If the pulse wavelength nearly coincides with the zero-dispersion wavelength of the fiber and $\beta_2 \approx 0$, the β_3 term provides the dominant contribution to the GVD effects. For ultrashort pulses (with width T_0<1 ps), it is necessary to include the β_3 term even when $\beta_2 \neq 0$. The TOD effects play a significant role only if $L_D' \leq L_D$ or $T_0|\beta_2/\beta_3| \leq 1$.

- The TOD distorts the pulse such that it becomes asymmetric and develops an oscillatory structure near one of its edges. In the case of positive β_3 (shown in the figure), oscillations appear near the trailing edge of the pulse. When β_3 is negative, it is the leading edge of the pulse that develops oscillations.

Rogue waves formation in fibers: nonlinearity

Consider nonlinearity only:

$$\frac{\partial U}{\partial z} = i\gamma P_0 |U|^2 U = \frac{i}{L_{NL}} |U|^2 U$$

This equation can be solved using $U = Ve^{i\varphi_{NL}}$ and evaluating *Re* and *Im* parts:

$$\frac{\partial V}{\partial z} = 0, \qquad \frac{\partial \varphi_{NL}}{\partial z} = \frac{V^2}{L_{NL}}$$

$$\Downarrow$$

Solution:

$$U(L,T) = U(0,T)\, e^{i\varphi_{NL}(L,T)}$$

$$\varphi_{NL}(L,T) = |U(0,T)|^2 \frac{L}{L_{NL}}$$

- $\gamma = \gamma(\omega_0) = \frac{\omega_0 n_2}{c A_{eff}}$ - the nonlinear parameter [W-1/m];
- $n_2 = \frac{2\overline{n_2}}{\varepsilon_0 n c}$ and $\overline{n_2}$ is the Kerr coefficient ($\tilde{n} = n + \overline{n_2}|E|^2$);
- $A_{eff} = \frac{\left(\iint_{-\infty}^{+\infty} |F(x,y)|^2 dxdy\right)^2}{\iint_{-\infty}^{+\infty} |F(x,y)|^4 dxdy}$ - effective mode area in the fiber.
- In the so-called highly nonlinear fibers, A_{eff} is reduced intentionally to enhance the nonlinear effects inside the fibers.

Rogue waves formation in fibers: nonlinearity

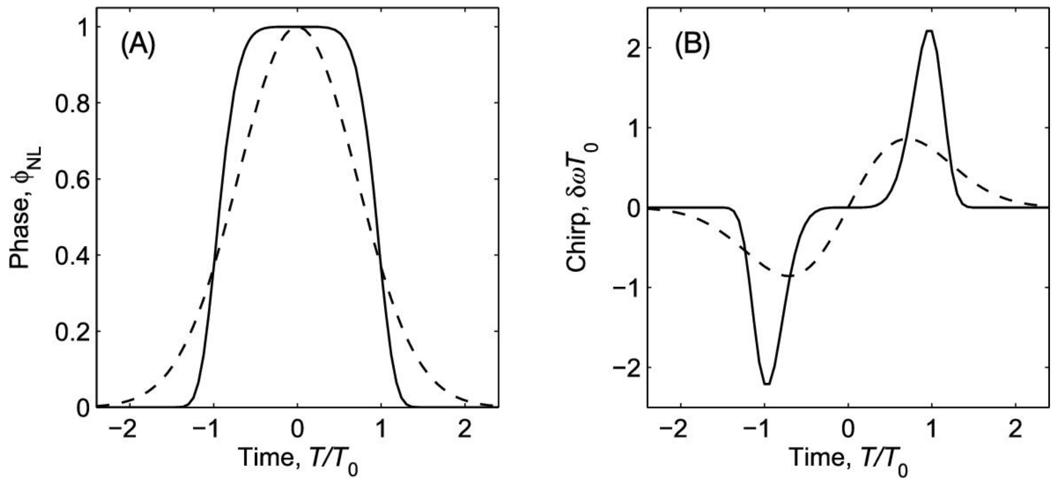
Consider nonlinearity only:

$$\frac{\partial U}{\partial z} = i\gamma P_0 |U|^2 U = \frac{i}{L_{NL}} |U|^2 U$$

Solution:

$$U(L,T) = U(0,T)\, e^{i\varphi_{NL}(L,T)}$$

$$\varphi_{NL}(L,T) = |U(0,T)|^2 \frac{L}{L_{NL}}$$



Temporal variation of SPM-induced (A) phase shift φ_{NL} and (B) frequency chirp $\delta\omega$ for Gaussian (dashed curve) and super-Gaussian (solid curve) pulses.

- Self phase modulation (SPM) gives rise to an intensity-dependent phase shift but the pulse shape remains unaffected. The nonlinear phase shift φ_{NL} increases with fiber length *L*.

- The maximum phase shift φ_{max} occurs at the peak of the pulse, assumed to be located at *T*=0. With *U* normalized such that $|U(0,0)| = 1$, it is given by $\varphi_{max} = \frac{L}{L_{NL}} = \gamma P_0 L$

- The physical meaning of the nonlinear length L_{NL}: it is the effective fiber length over which $\varphi_{max} = 1$. If we use a typical value γ=2 W^{-1}/km, the nonlinear length is 50 km for P_0=10 mW but is reduced to 100 m for P_0=5 W.

Rogue waves formation in fibers: nonlinearity

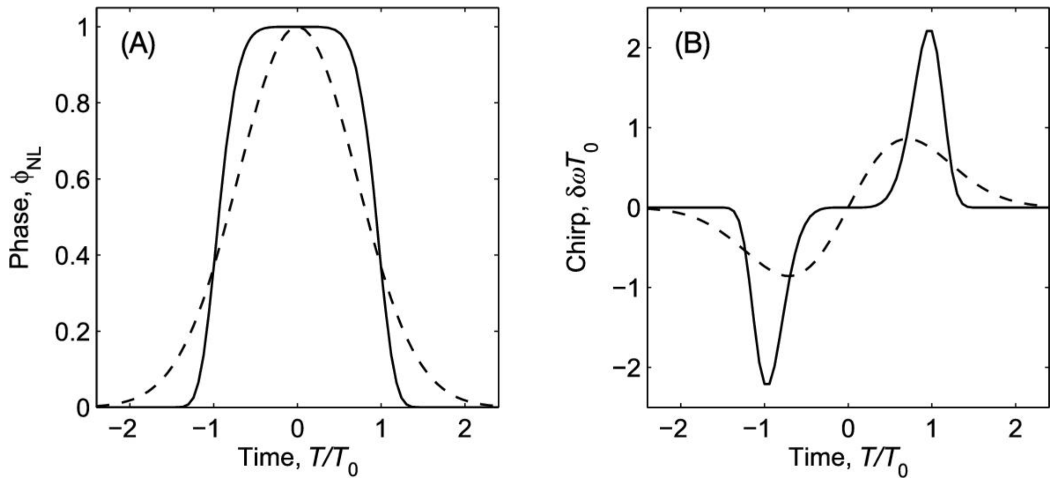
Consider nonlinearity only:

$$\frac{\partial U}{\partial z} = i\gamma P_0 |U|^2 U = \frac{i}{L_{NL}} |U|^2 U$$

Solution:

$$U(L,T) = U(0,T)\, e^{i\varphi_{NL}(L,T)}$$

$$\varphi_{NL}(L,T) = |U(0,T)|^2 \frac{L}{L_{NL}}$$



Temporal variation of SPM-induced (A) phase shift φ_{NL} and (B) frequency chirp $\delta\omega$ for Gaussian (dashed curve) and super-Gaussian (solid curve) pulses.

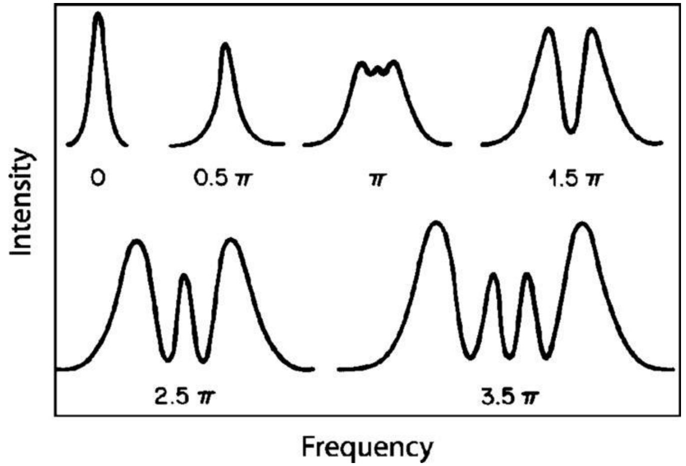
- The SPM effect induces frequency chirp (time dependence of instantaneous optical frequency) in the pulse, which increases in magnitude with the propagated distance:

$$\delta\omega(T) = \omega(T) - \omega_0 = -\frac{\partial \varphi_{NL}}{\partial T} = -\frac{L}{L_{NL}} \frac{\partial |U(0,T)|^2}{\partial T}$$

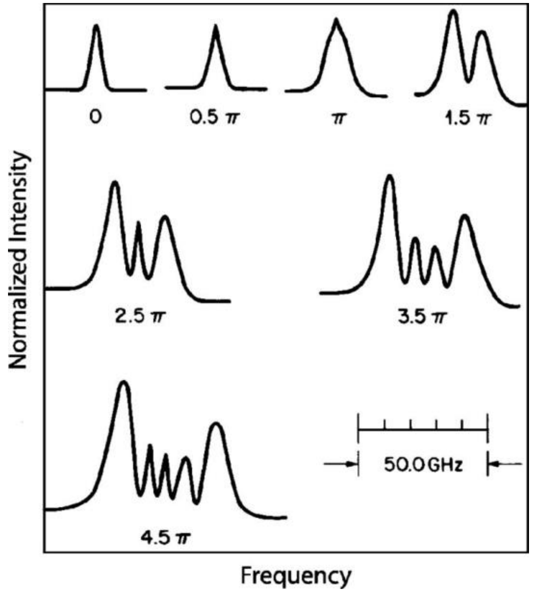
- In other words, new frequency components are generated continuously as the pulse propagates down the fiber. These SPM-generated frequency components broaden the spectrum over its initial width at *z*=0 for initially unchirped pulses.

- The qualitative features of frequency chirp depend on the pulse shape.

Rogue waves formation in fibers: nonlinearity



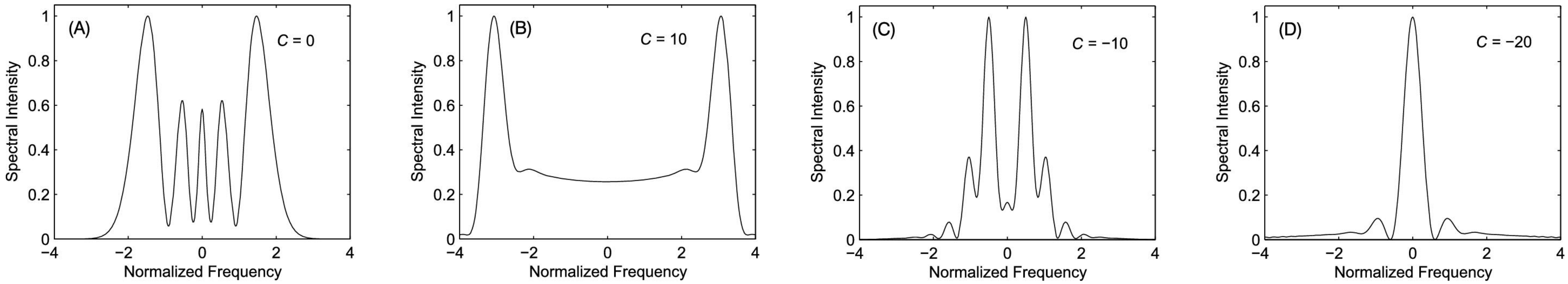
Predicted optical spectra for several values of φ_{max} when unchirped Gaussian pulses are launched into a fiber.



Experimentally observed spectra at the output of a 99-m long fiber labeled by φ_{max}. Input pulses are nearly Gaussian.

- The SPM-induced chirp can produce spectral broadening or narrowing depending on how the input pulse is chirped.
- In the case of unchirped input pulses, SPM always leads to spectral broadening.

Rogue waves formation in fibers: nonlinearity



Comparison of output spectra for Gaussian pulses for four values of chirp parameter C when fiber length and peak powers are chosen such that $\phi_{max} = 4.5\pi$. Spectrum broadens for $C > 0$ but becomes narrower for $C < 0$ when compared with that of the input pulse.

- An initial frequency chirp can also lead to drastic changes in the SPM-broadened pulse spectrum.
- For the input pulse with positive chirp C>0, spectral broadening increases and the oscillatory structure becomes less pronounced. However, a negatively chirped pulse undergoes spectral narrowing through SPM.
- This behavior can be understood by noting that the SPM-induced chirp is linear and positive (frequency increases with increasing T) over the central portion of a Gaussian pulse. Thus, it adds to the initial chirp for C>0, resulting in a broader spectrum. In the case of C<0 , the two chirp contributions are of opposite signs (except near the pulse edges), and the pulse becomes less chirped.

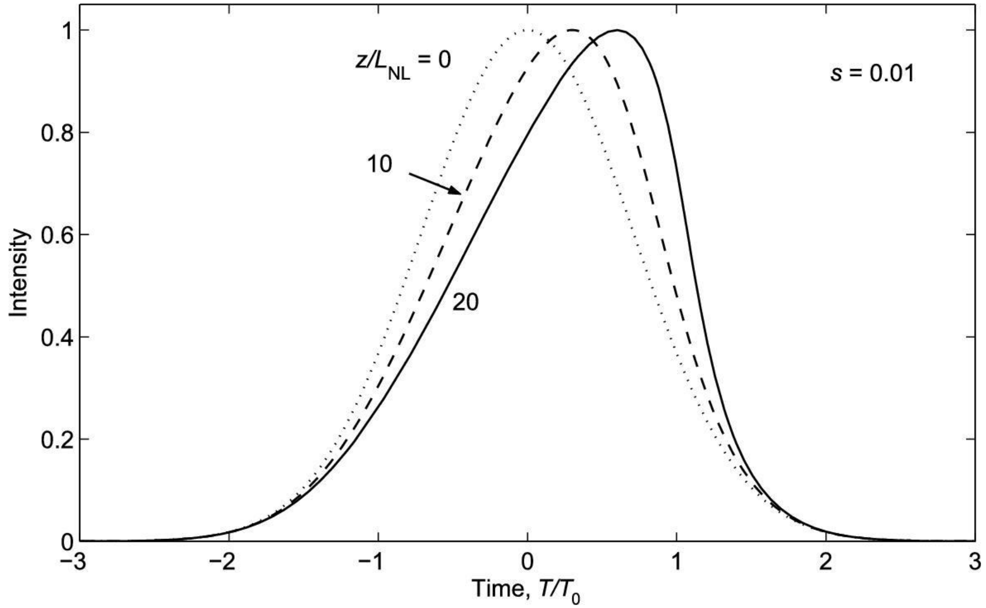
Rogue waves formation in fibers: self-steepening

Consider nonlinearity only:

$$\frac{\partial A}{\partial z} = i\gamma\left(|A|^2A + \frac{\boldsymbol{i}}{\boldsymbol{\omega_0}}\frac{\boldsymbol{\partial}}{\boldsymbol{\partial T}}(|\boldsymbol{A}|^2\boldsymbol{A})\right)$$

The normalized equation:

$$\frac{\partial U}{\partial Z} = i|U|^2U - s\frac{\partial}{\partial T}(|U|^2U), \quad s = \frac{1}{\omega_0 T_0}, \quad Z = \frac{z}{L_{NL}}$$



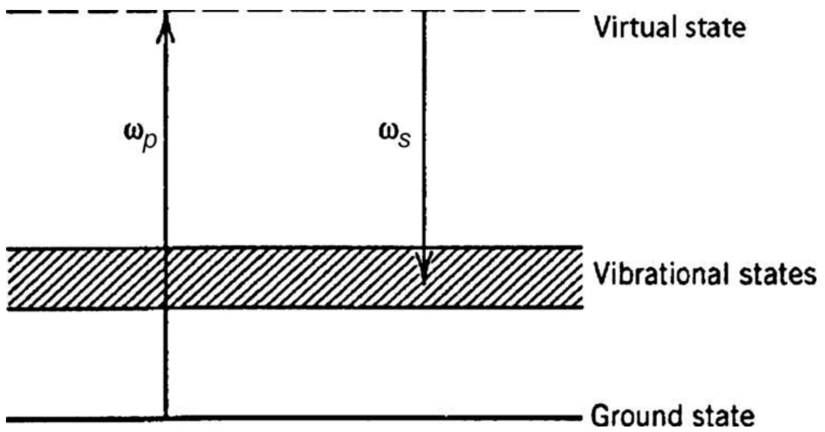
Self-steepening of a Gaussian pulse in the dispersionless case. The dashed curve shows the pulse shape at z = 0.

- Self-steepening results from the intensity dependence of the group velocity.
- This effect comes from the frequency dependence of the nonlinear parameter $\gamma = \gamma(\omega_0) + \gamma_1(\omega - \omega_0), \gamma_1 \approx \frac{\gamma}{\omega_0}$.
- As the pulse propagates inside the fiber, it becomes asymmetric, with its peak shifting toward the trailing edge. As a result, the trailing edge becomes steeper and steeper with increasing *Z*. Physically, the group velocity of the pulse is intensity dependent such that its peak moves at a lower speed than its wings.
- Self-steepening of the pulse eventually creates an optical shock, analogous to the development of an acoustic shock on the leading edge of a sound wave.

Rogue waves formation in fibers: Raman scattering

Raman scattering:

$$\frac{\partial A}{\partial z} = i\gamma\left(|A|^2 A - \boldsymbol{T_R A \frac{\partial |A|^2}{\partial T}}\right)$$



A photon of reduced energy $\hbar\omega_s$ is created spontaneously after a pump photon of energy $\hbar\omega_p$ excites the molecule to a virtual state.

- In any molecular medium, spontaneous Raman scattering can transfer a small fraction (typically $\sim 10^{-6}$) of power from one optical field to another field, whose frequency is downshifted by an amount determined by the vibrational modes of the medium.

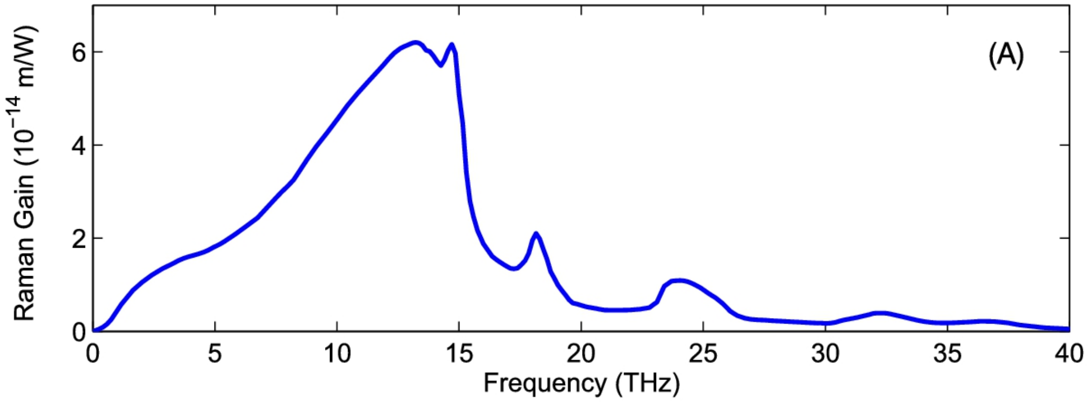
- Raman effect: a photon of energy $\hbar\omega_p$ is converted by a molecule to a lower-frequency photon with energy $\hbar\omega_s$, as the molecule makes transition to a vibrational excited state. Incident light acts as a pump and generates the frequency-shifted radiation, the so-called Stokes wave.

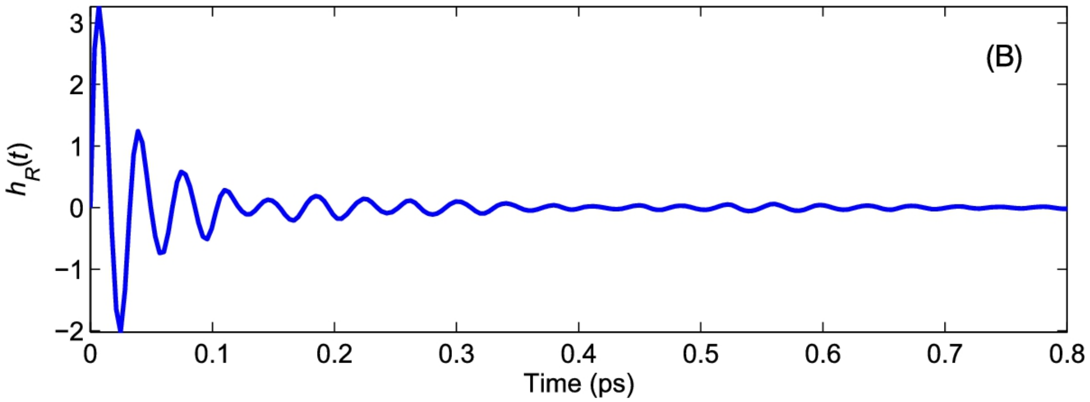
- For intense pump fields, the nonlinear phenomenon of stimulated Raman scattering can occur in which the Stokes wave grows rapidly inside the medium such that most of the pump energy is transferred to it.

Rogue waves formation in fibers: Raman scattering

Raman scattering:

$$\frac{\partial A}{\partial z} = i\gamma\left(|A|^2 A - \boldsymbol{T_R A}\frac{\boldsymbol{\partial |A|^2}}{\boldsymbol{\partial T}}\right)$$





(A) Measured Raman gain spectrum of silica fibers; (B) temporal form of the Raman response function deduced from the gain data.

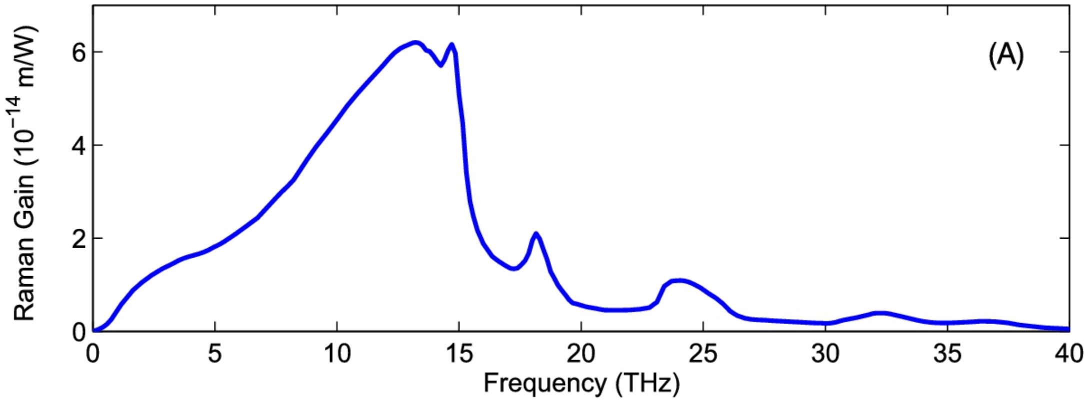
- The Raman gain in silica fibers extends over a large frequency range (~40 THz) with a broad peak located near 13 THz. This behavior is due to the noncrystalline nature of silica glass: in amorphous materials such as fused silica, molecular vibrational frequencies spread out into bands that overlap and create a continuum.

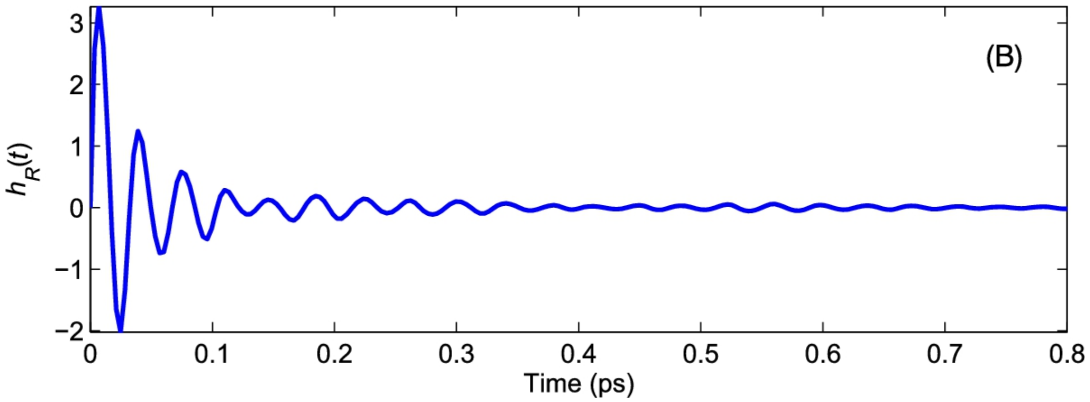
- For pulses with a wide spectrum (T_0<1 ps), the Raman gain can amplify the low-frequency components of a pulse by transferring energy from the high-frequency components of the same pulse. This phenomenon is called intrapulse Raman scattering.

Rogue waves formation in fibers: Raman scattering

Raman scattering:

$$\frac{\partial A}{\partial z} = i\gamma\left(|A|^2 A - \boldsymbol{T_R A}\frac{\boldsymbol{\partial |A|^2}}{\boldsymbol{\partial T}}\right), \qquad T_R = \int_{-\infty}^{\infty} tR(t)dt \approx f_R \int_0^{\infty} h_R(t)tdt\,, \qquad R(t) = (1 - f_R)\delta(t) + f_R h_R(t)$$





(A) Measured Raman gain spectrum of silica fibers; (B) temporal form of the Raman response function deduced from the gain data.

- The physical origin of this Raman-induced effect is related to the delayed nonlinear response of molecules to an optical field.
- The form of the Raman response function $h_R(t)$ is set by vibrations of silica molecules. The nonlinear response function $R(t)$ includes both the Kerr and Raman contributions.
- f_R represents the fractional contribution of the delayed Raman response to nonlinear polarization; T_R~2-3 fs.

Rogue waves formation in fibers: solitons

Consider both dispersion and nonlinearity:

$$\frac{\partial A}{\partial z} + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} = i\gamma|A|^2 A$$

The normalized equation:

$$i\frac{\partial U}{\partial \xi} = \frac{sign(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2} - N^2|U|^2 U, \quad \xi = \frac{z}{L_D}, \quad N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

- $L_D = \frac{T_0^2}{|\beta_2|}$ - dispersion length scale, $L_{NL} = \frac{1}{\gamma P_0}$ - nonlinear length scale. These two parameters govern the length scales when the dispersive or nonlinear effects become important.

The effects of dispersion and nonlinearity can be described in terms of the frequency chirp induced to a pulse:

- The GVD-induced chirp depends on the sign of β_2: for β_2>0, the pulse exhibits positive chirp; for β_2<0, the chirp is negative.

- The SPM-induced chirp is positive (frequency increases with increasing *T*).

- What would happen if one adjusts the fiber and pulse parameters so that the total chirp becomes zero?

Rogue waves formation in fibers: solitons

Consider both dispersion and nonlinearity:

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- The interplay between the dispersive and nonlinear effects results in formation of optical solitons.
- The word **soliton** refers to special kinds of wave packets that can propagate undistorted over long distances.
- Optical solitons correspond to specific solutions of the NLSE in the region where β_2<0, that either do not change along the fiber or follow a periodic evolution.
- The solitonic solutions can be found directly by assuming that a shape-preserving solution of the NLSE exists and has the form

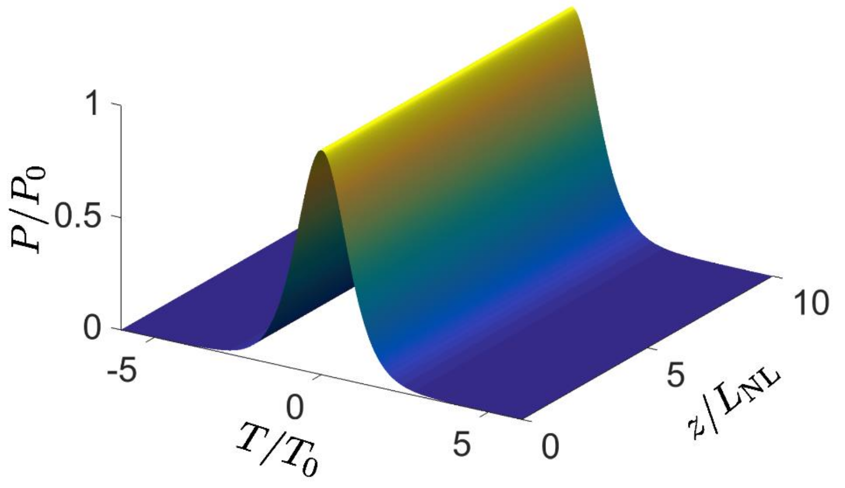
$$U(\xi, \tau) = NV(\tau)e^{i\varphi(\xi,\tau)}$$

Rogue waves formation in fibers: solitons

The equation with account of $sign(\beta_2) = -1, u(\xi,\tau) = NU(\xi,\tau)$:

$$i\frac{\partial u}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 u}{\partial \tau^2} - |u|^2 u, \quad \xi = \frac{z}{L_D}, \quad N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

$$u(\xi,\tau) = V(\tau)e^{i\varphi(\xi,\tau)}$$



Solution – a fundamental soliton:

$$u(\xi,\tau) = sech(\tau)e^{i\xi/2}$$

- This solution indicates that if a hyperbolic secant pulse, whose width T_0 and the peak power P_0 are chosen such that N=1, is launched inside an ideal lossless fiber, the pulse will propagate undistorted for arbitrarily long distances, without change in its shape.

- The peak power P_0 required to support a fundamental soliton is obtained from the condition N=1 and is given by

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \approx \frac{3.11|\beta_2|}{\gamma T_{FWHM}^2}$$

- Using typical parameter values, β_2=−1 ps^2/km and γ=3 W^{-1}/km, for dispersion-shifted fibers, the required P_0 is ~1 W for T_0=1 ps but reduces to only 10 mW when T_0=10 ps.
Thus, fundamental solitons can form in optical fibers at power levels available from semiconductor lasers.

Rogue waves formation in fibers: solitons

Characteristics of optical solitons:

- The peak power P_0 required to support a fundamental soliton is obtained from the condition N=1 and is given by

$$P_0 = \frac{|\beta_2|}{\gamma T_0^2} \approx \frac{3.11|\beta_2|}{\gamma T_{FWHM}^2}$$

- The soliton width scales inversely with its amplitude. This inverse relationship between the amplitude and the width of a soliton is the most crucial property of solitons.

Rogue waves formation in fibers: solitons

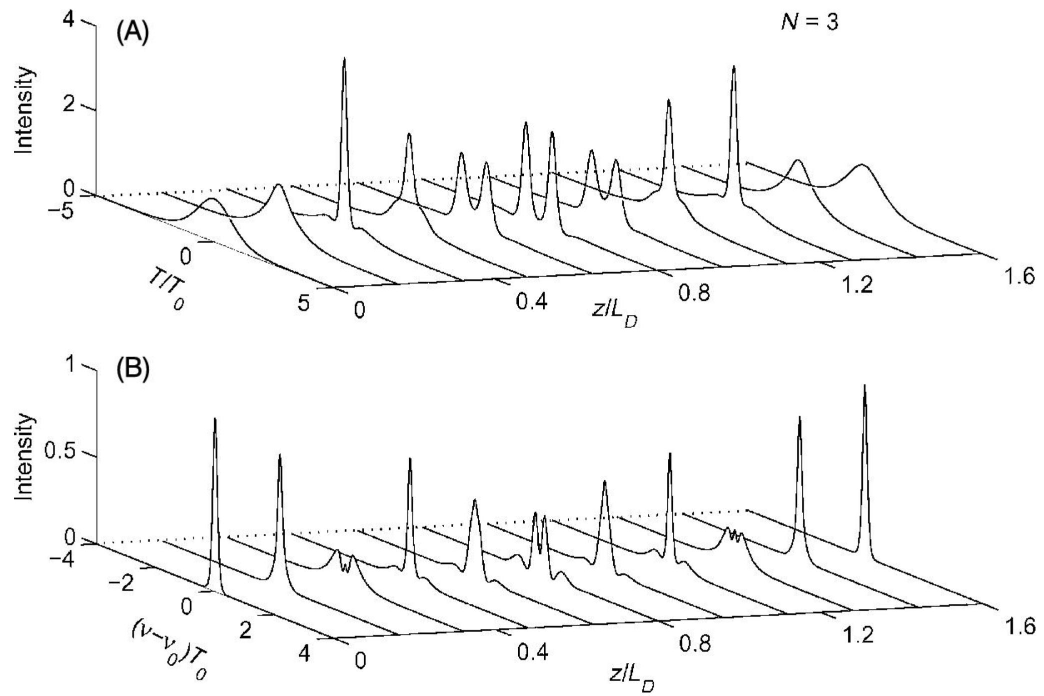
Higher-order solitons:

- Among the subset of all possible solitonic solutions, a special role is played by solitons whose initial shape at ξ=0 is given by

$$u(0,\tau) = N sech(\tau)$$

- The parameter *N* defines soliton order (an integer number).
- The peak power necessary to launch the Nth-order soliton is N^2 times that required for the fundamental soliton.
- An interesting property of higher-order solitons is that $u(\xi,\tau)$ is periodic in ξ with the period $\xi_0 = \pi/2$. In fact, this periodicity occurs for all solitons with *N*≥2.
- Using the definition $\xi = \frac{z}{L_D}$, the soliton period z_0 in real units becomes

$$z_0 = \frac{\pi}{2} L_D = \frac{\pi}{2} \frac{T_0^2}{|\beta_2|}$$

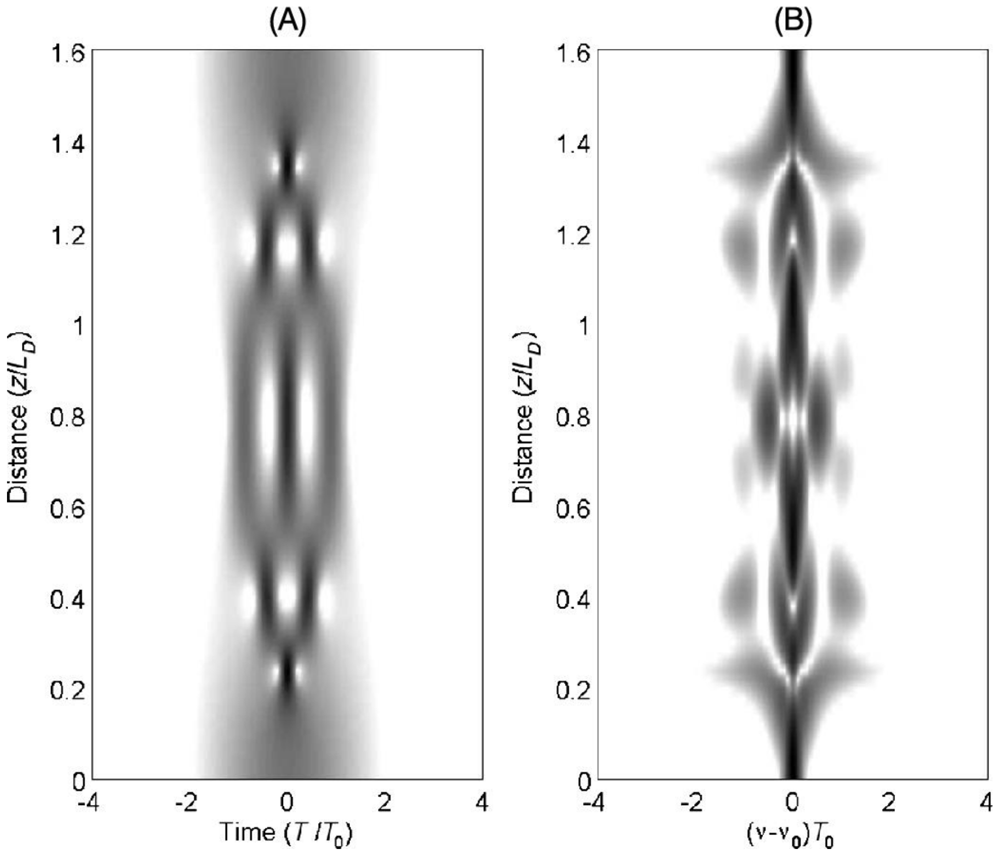


(A) Temporal and (B) spectral evolution of the third-order soliton over one soliton period. Note splitting of the pulse near z/L D = 0.5 and its recovery beyond that.

Rogue waves formation in fibers: solitons

Higher-order solitons:

- It is this mutual interaction between the GVD and SPM effects that is responsible for the periodic evolution pattern.
- In the case of a fundamental soliton (*N*=1), GVD and SPM balance each other in such a way that neither the pulse shape nor the pulse spectrum changes along the fiber length.
- In the case of higher-order solitons, SPM dominates initially but GVD soon catches up and leads to pulse contraction.



(A) Temporal and (B) spectral evolution of a fourth-order soliton over one soliton period. The gray intensity scale is over a 20-dB range.

Rogue waves formation in fibers: solitons

Soliton stability:

- What happens if the initial pulse shape or the peak power is not matched to that required by the soliton equation, and the input pulse does not correspond to an optical soliton?
- Similarly, one may ask how the soliton is affected if it is perturbed during its propagation inside the fiber.

Rogue waves formation in fibers: solitons

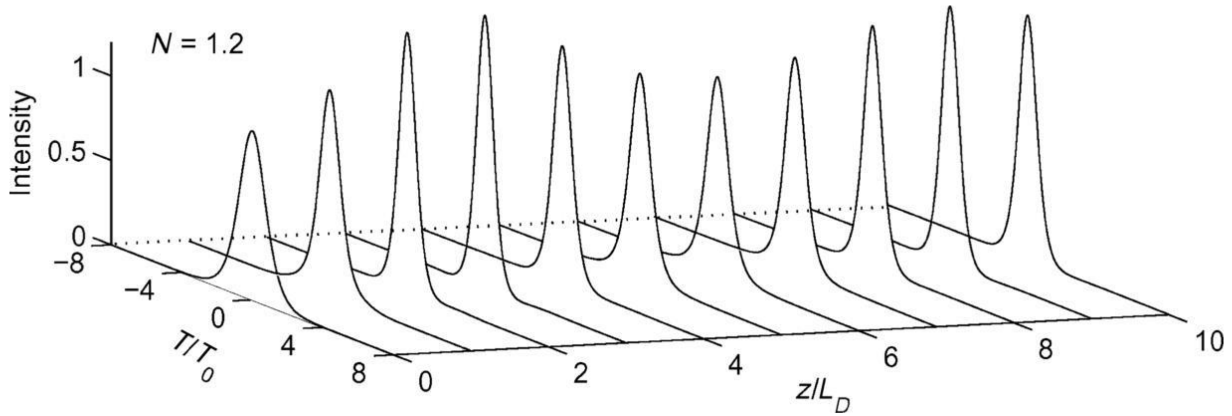
Soliton stability:

- Figure shows the evolution of a "sech" pulse launched with N=1.2 by solving the NLSE numerically. Even though pulse width and peak power change initially, the pulse eventually evolves toward a fundamental soliton of narrower width for which N=1 asymptotically.

- The pulse adjusts its shape and width as it propagates along the fiber and evolves into a soliton.

- A part of the pulse energy is dispersed away in the process.

- This part is known as the continuum radiation. It separates from the soliton as ξ increases and its amplitude decays as $\xi^{-1/2}$.



Temporal evolution over 10 dispersion lengths when N = 1.2 at z = 0. The pulse evolves into a fundamental soliton of narrower width for which N approaches 1 asymptotically.

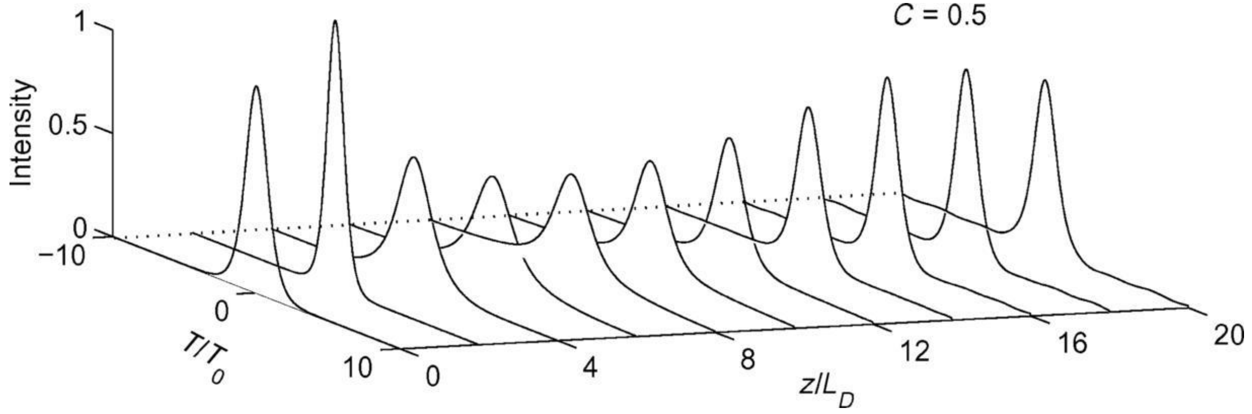
- For $\xi \gg 1$, the pulse evolves asymptotically into a soliton whose order is an integer $\widetilde{N}$ closest to the launched value of $N = \widetilde{N} + \epsilon$.

- The pulse broadens if $\epsilon<0$ and narrows if $\epsilon>0$. No soliton is formed when $N\leq1/2$.

Rogue waves formation in fibers: solitons

Soliton stability:

- Chirp of the input pulse also affects the soliton formation.
- Formation of a soliton is expected for small values of |C| because solitons are generally stable under weak perturbations.
- However, a soliton is destroyed if |C| exceeds some critical value. The critical value depends on *N* and is found to be about 1.64 for *N*=1.



Soliton formation in the presence of an initial linear chirp for the case N = 1 and C = 0.5.
The pulse compresses initially mainly because of the positive chirp; initial compression occurs even in the absence of nonlinear effects. The pulse then broadens but is eventually compressed a second time with the tail separating from the main peak gradually. The main peak evolves into a soliton over a propagation distance ξ>15 .
A similar behavior occurs for negative values of C.

Rogue waves formation in fibers: solitons

Soliton stability:

- The exact shape of the input pulse used to launch a fundamental (N=1) soliton is not critical.
- Moreover, as solitons can form for values of N in the range 0.5<N<1.5, even the width and peak power of the input pulse can vary over a wide range.
- When input parameters deviate substantially from their ideal values, a part of the pulse energy is shed away in the form of dispersive waves as the pulse evolves to form a fundamental soliton.
- Such dispersive waves can interfere with the soliton itself and modify its characteristics.
- In a practical situation, solitons can be subjected to many types of perturbations as they propagate inside an optical fiber. Examples of perturbations include fiber losses, amplifier noise (if amplifiers are used to compensate fiber losses), third-order dispersion, and intrapulse Raman scattering.

Rogue waves formation in fibers: modulation instability

Consider continuous-wave (CW) input, so that $A(0,T) = const$:

$$\frac{\partial A}{\partial z} = i\gamma|A|^2 A - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} \quad \Rightarrow \quad \frac{\partial A}{\partial z} = i\gamma|A|^2 A$$

Solution:

$$A(z,T) = \sqrt{P_0}e^{i\varphi_{NL}}, \quad \varphi_{NL} = \gamma P_0 z$$

- This solution implies that CW light should propagate through the fiber unchanged except for acquiring a power-dependent phase shift.

- But is this solution stable against small perturbations? What will happen, for example, if one adds some noise to the ideal input signal constant in time?

- To answer this question, one can perturb the steady state slightly such that

$$A(z,T) = \left(\sqrt{P_0} + a\right)e^{i\varphi_{NL}}$$

- Evolution of the perturbation *a(z,T)* is investigated using a linear stability analysis by substituting *A(z,T)* in NLSE and linearizing in *a.*

Rogue waves formation in fibers: modulation instability

The equation for the perturbation *a(z,T)*:

$$\frac{\partial a}{\partial z} = i\gamma P_0(a + a^*) - \frac{i\beta_2}{2}\frac{\partial^2 a}{\partial T^2}$$

- This linear equation can be solved in the frequency domain.

- However, because of the a^* term, the Fourier components at frequencies Ω and −Ω are coupled. Thus, we should consider its solution in the form

$$a(z,T) = a_1 e^{i(Kz-\Omega T)} + a_2 e^{-i(Kz-\Omega T)},$$

where *K* and Ω are the wave number and the frequency of perturbation, respectively.

- A nontrivial solution of the NLSE exists only when *K* and Ω satisfy the following dispersion relation:

$$K = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + sign(\beta_2)\Omega_c^2},$$

$$\Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|}$$

Rogue waves formation in fibers: modulation instability

Solution:

$$a(z,T) = a_1 e^{i(Kz-\Omega T)} + a_2 e^{-i(Kz-\Omega T)}$$

Dispersion relation:

$$K = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + sign(\beta_2)\Omega_c^2}, \quad \Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|}$$

- The dispersion relation shows that steady-state stability depends critically on whether light experiences normal or anomalous GVD inside the fiber.
- In the case of normal GVD ($\beta_2 > 0$) , the wave number K is real for all Ω, and the steady state is stable against small perturbations.
- By contrast, K becomes imaginary for $|\Omega| < \Omega_c$ in the case of anomalous GVD ($\beta_2 < 0$) , and the perturbation $a(z,T)$ grows exponentially with z. As a result, the CW solution is inherently unstable for $\beta_2 < 0$.
- This instability is called the **modulation instability** because it leads to a spontaneous temporal modulation of the CW beam and transforms it into a pulse train.

Rogue waves formation in fibers: modulation instability

Solution:

$$a(z,T) = a_1 e^{i(Kz-\Omega T)} + a_2 e^{-i(Kz-\Omega T)}$$

Dispersion relation:

$$K = \pm\frac{1}{2}|\beta_2 \Omega|\sqrt{\Omega^2 + sign(\beta_2)\Omega_c^2}, \quad \Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|}$$

- Because of the factor $\exp[i(\beta_0 z - \omega_0 t)]$ that has been factored out, the actual wave number and the frequency of perturbation are $\beta_0 \pm K$ and $\omega_0 \pm \Omega$, respectively.
- With this factor in mind, the two terms in $a(z,T)$ represent two different frequency components, $\omega_0 + \Omega$ and $\omega_0 - \Omega$, that are present simultaneously.
- These frequency components correspond to the two spectral sidebands that are generated when modulation instability occurs.

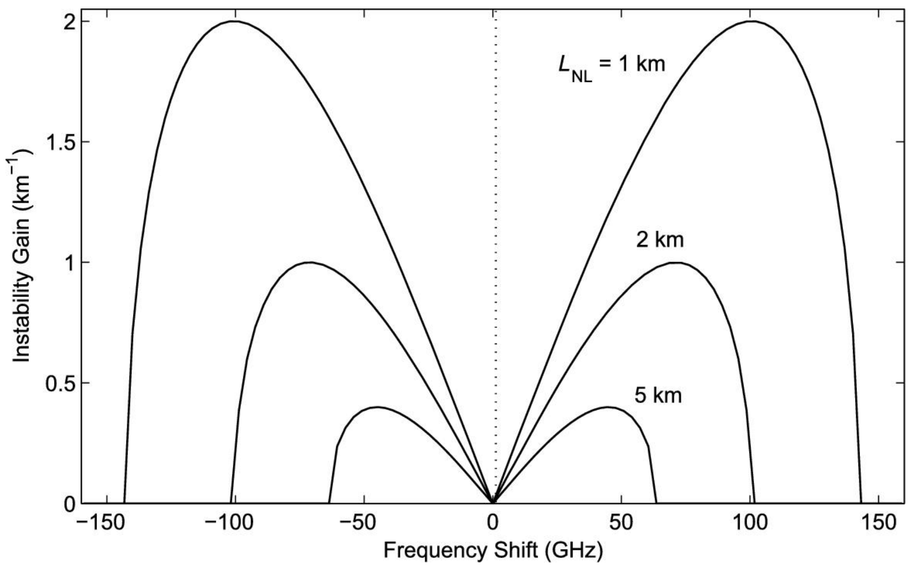
Rogue waves formation in fibers: modulation instability

Solution:

$$a(z,T) = a_1 e^{i(Kz-\Omega T)} + a_2 e^{-i(Kz-\Omega T)}$$

Dispersion relation:

$$K = \pm\frac{1}{2}|\beta_2\Omega|\sqrt{\Omega^2 + sign(\beta_2)\Omega_c^2}, \quad \Omega_c^2 = \frac{4\gamma P_0}{|\beta_2|}$$



Gain spectra of modulation instability for three values of the nonlinear length L_{NL} when a CW beam is launched into a fiber with β_2=−5 ps^2/km.

- The gain spectrum of modulation instability is obtained by setting $sign(\beta_2) = -1$.
- The gain exists only if $|\Omega| < \Omega_c$ and is given by

$$g(\Omega) = 2Im(K) = |\beta_2\Omega|\sqrt{\Omega_c^2 - \Omega^2},$$

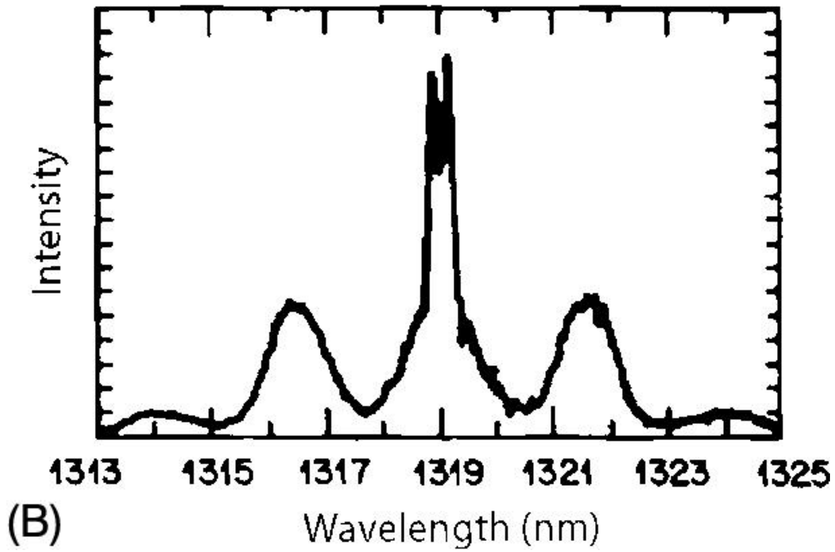
- The gain spectrum is symmetric with respect to Ω=0 such that g(Ω) vanishes at Ω=0. The gain becomes maximum at two frequencies given by

$$\Omega_{max} = \pm\frac{\Omega_c}{\sqrt{2}} = \pm\sqrt{\frac{2\gamma P_0}{|\beta_2|}}, \qquad g(\Omega_{max}) = \frac{1}{2}|\beta_2|\Omega_c^2 = 2\gamma P_0$$

Rogue waves formation in fibers: modulation instability

Modulation instability:

- Modulation instability can lead to spontaneous breakup of a CW beam into a periodic pulse train.
- Noise photons act as a probe in this situation and are amplified by the gain provided by modulation instability.
- As the largest gain occurs for frequencies $\omega_0 \pm \Omega_{max}$, these frequency components are amplified most.
- Thus, a clear-cut evidence of spontaneous modulation instability is provided by the appearance of two spectral sidebands at the fiber output, shifted symmetrically by $\pm\Omega_{max}$ on each side of the pump located at ω_0.

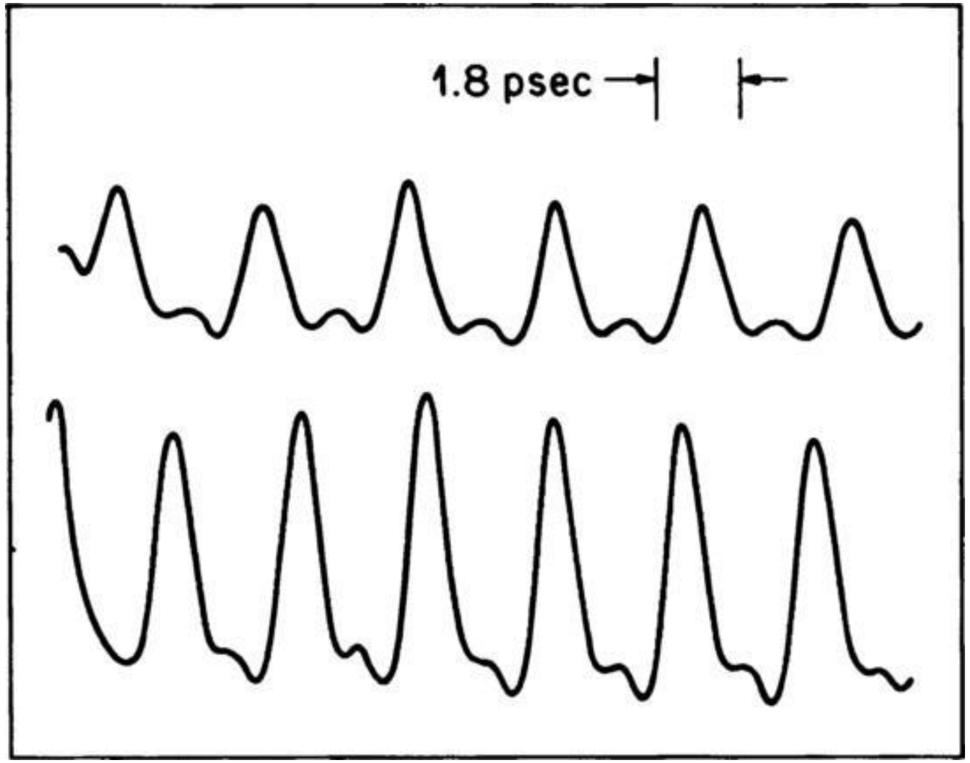


100-ps (FWHM) pulses at a wavelength of 1.319 μm were transmitted through a 1-km-long fiber with $\beta_2 \approx -3$ ps^2/km The optical spectrum shows the evidence of modulation instability at a peak power of 7.1 W. The location of spectral sidebands is in agreement with the prediction of theory.

Rogue waves formation in fibers: modulation instability

Modulation instability:

- In the time domain, the CW beam is converted into a periodic pulse train with a period $T_m = 2\pi/\Omega_{max}$.
- As the observed pulse width is <1 ps, this technique is useful for generating subpicosecond pulses whose repetition rate can be controlled by tuning the probe wavelength.

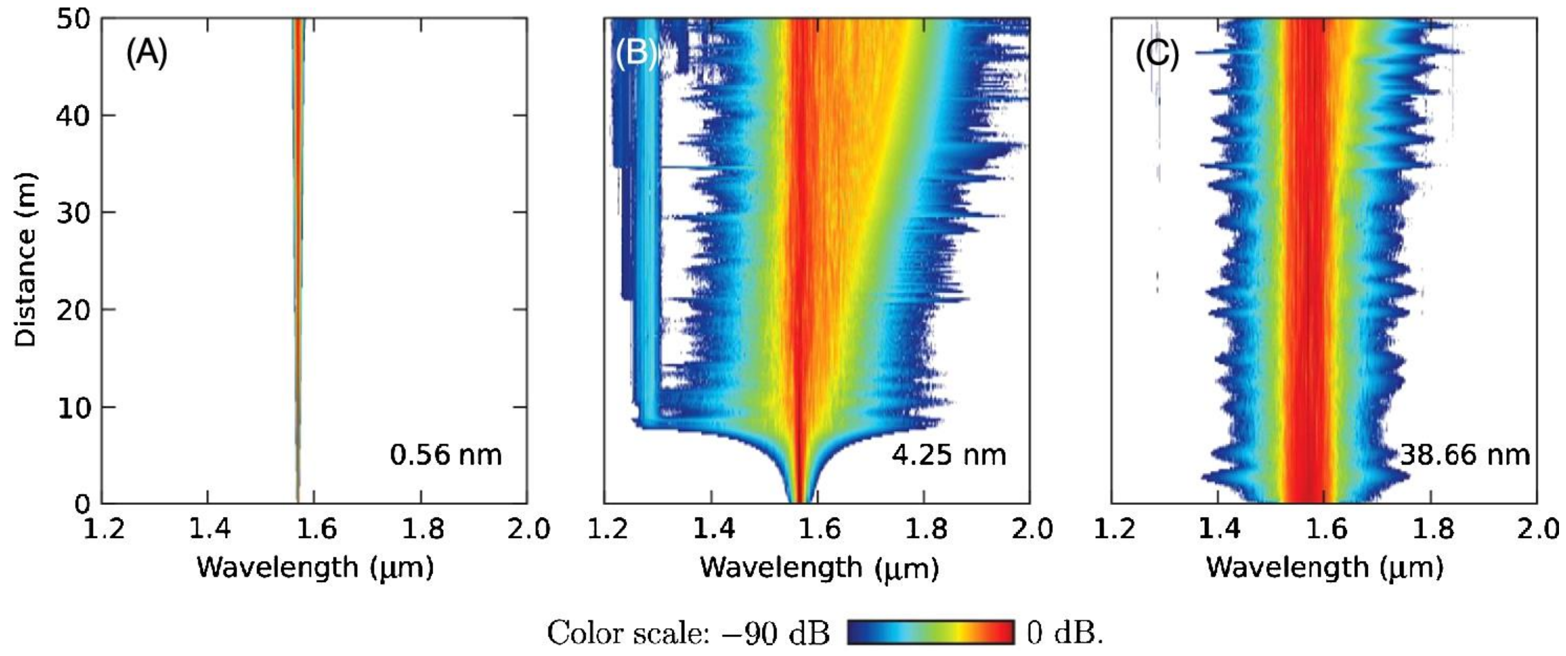


Autocorrelation traces showing induced modulation instability at two different probe wavelengths. The modulation period can be adjusted by tuning the semiconductor laser acting as a probe.

Rogue waves formation in fibers: supercontinuum generation

Supercontinuum generation:

- When short optical pulses propagate through a highly nonlinear fiber, their temporal and spectral features are affected by a multitude of the nonlinear effects (SPM, FWM, SRS) and by the dispersive properties of this fiber.
- As most nonlinear processes are capable of generating new frequencies within the pulse spectrum, for sufficiently intense pulses, the pulse spectrum can become so broad that it extends over a frequency range exceeding 100 THz.
- Such extreme spectral broadening is referred to as **supercontinuum generation.**



Numerically simulated spectral evolution along a 50 m fiber for three CW sources of different spectral bandwidths.
In all cases, the pump power is 6.3 W at a wavelength of 1550 nm.

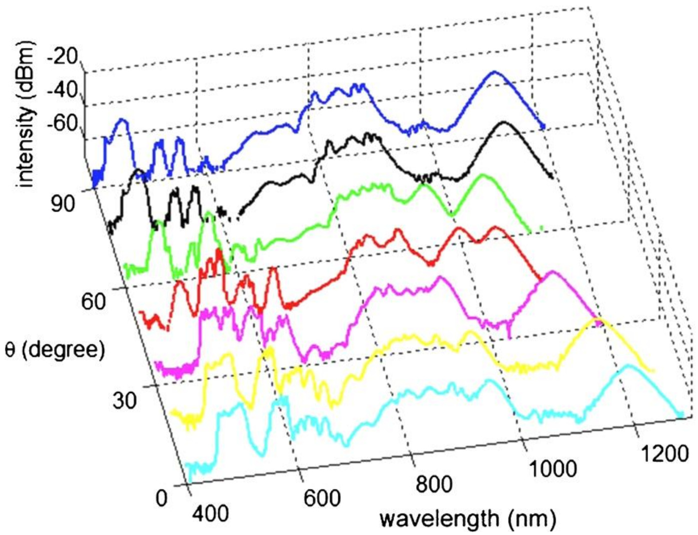
Rogue waves formation in fibers: supercontinuum generation

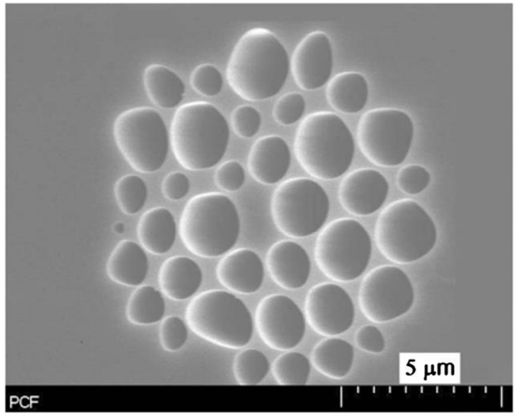
Fiber properties for supercontinuum generation:

- Flattened dispersion: supercontinuum generation is the most effective near zero-dispersion wavelength.

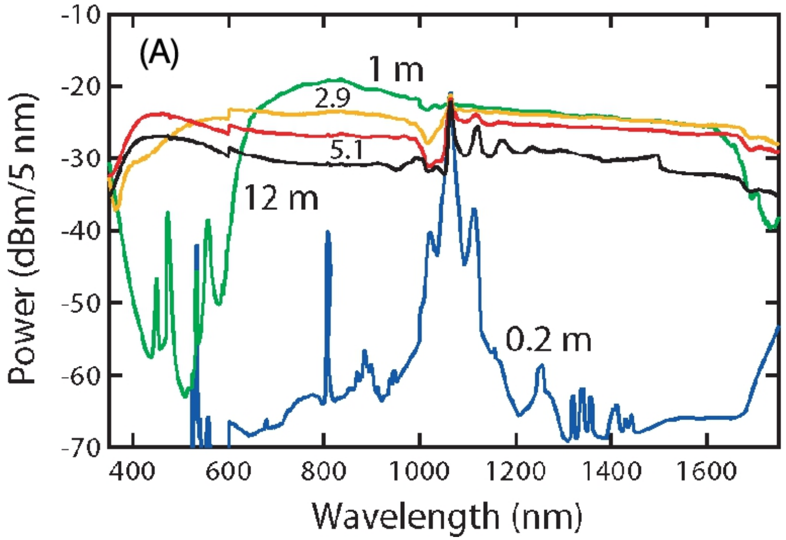
- High nonlinear coefficient (obtained for smaller core radius of the fiber): allows to use lower input power and lower fiber length.

- It is possible do generate supercontinuum using special microstructured fibers (photonic crystal fibers), which are designed to have the required properties. For example, it became possible to generate a supercontinuum extending over the entire visible region using such fibers.





Supercontinuum spectra at the output of a highly birefringent fiber (micrograph on the right) for several different polarization angles.



Measured spectra for several PCF lengths when pumped with 600-ps.

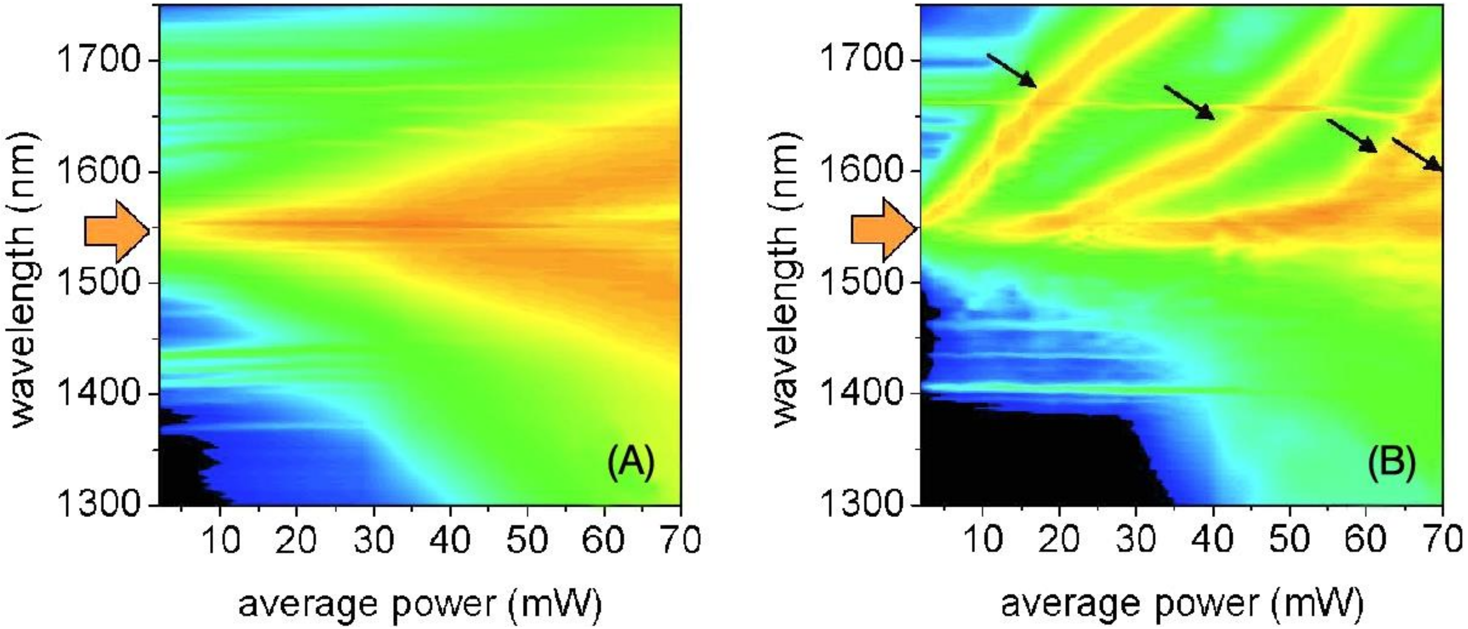
Rogue waves formation in fibers: supercontinuum generation

Supercontinuum generation through soliton fission:

- For the high peak power levels and anomalous GVD used in the experiments, input pulses can form a relatively high-order soliton inside the fiber (the order of a soliton is governed by the parameter *N*).

- High-order solitons are perturbed considerably by third-order dispersion and intrapulse Raman scattering and they breakup into multiple, much narrower, fundamental solitons through soliton fission.

- It turns out that if *N* is relatively large (more than 10), the phenomenon of soliton fission can produce a supercontinuum.

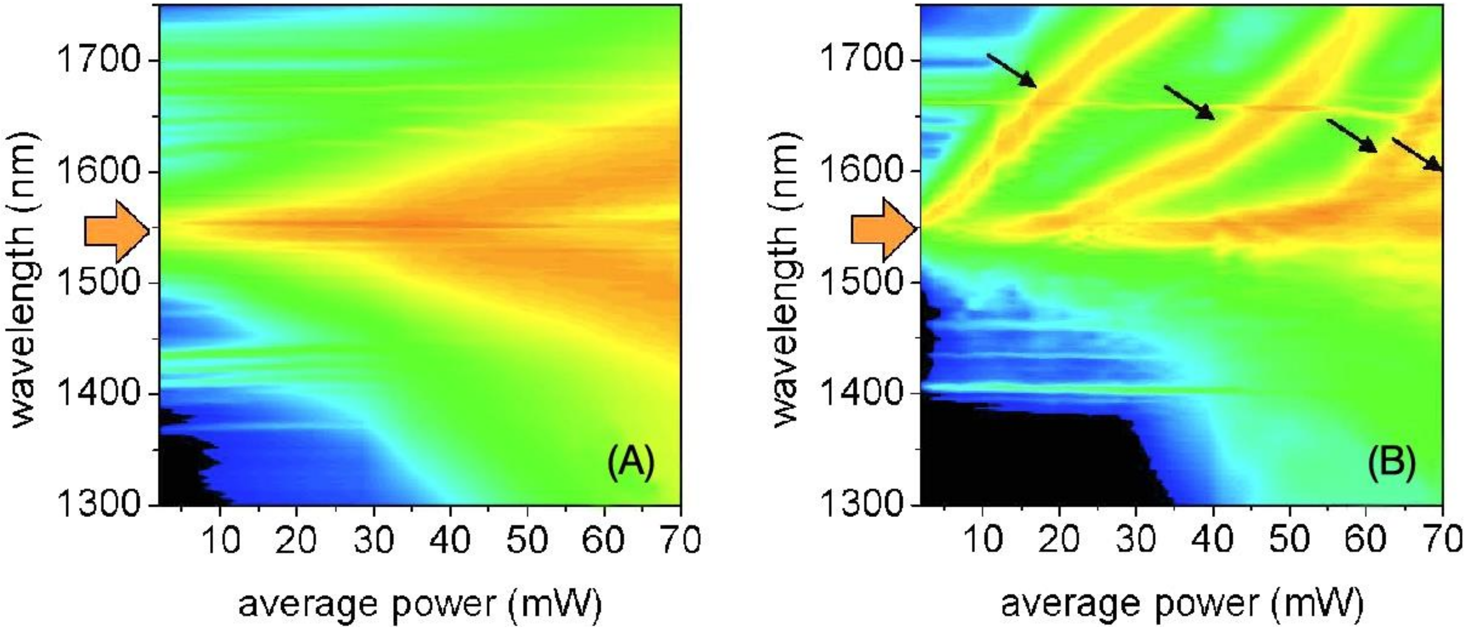


Recorded spectra as a function of average power at the output of a microstructured fiber when 110-fs pulses are propagated over a fiber length of (A) 0.57 and (B) 70 cm.
Arrows indicate solitons created after the fission.

Rogue waves formation in fibers: supercontinuum generation

Supercontinuum generation through soliton fission:

- Fission of a Nth-order soliton produces *N* fundamental solitons which are shorter than the original input pulse, the shortest one being narrower by a factor of *2N−1*.
- The choice of the pump wavelength relative to the ZDWL of the fiber was found to be a critical factor for supercontinuum generation.
- When the input wavelength fell in the normal-GVD region of the fiber, spectral broadening was reduced considerably because higher-order solitons could not form in this situation.

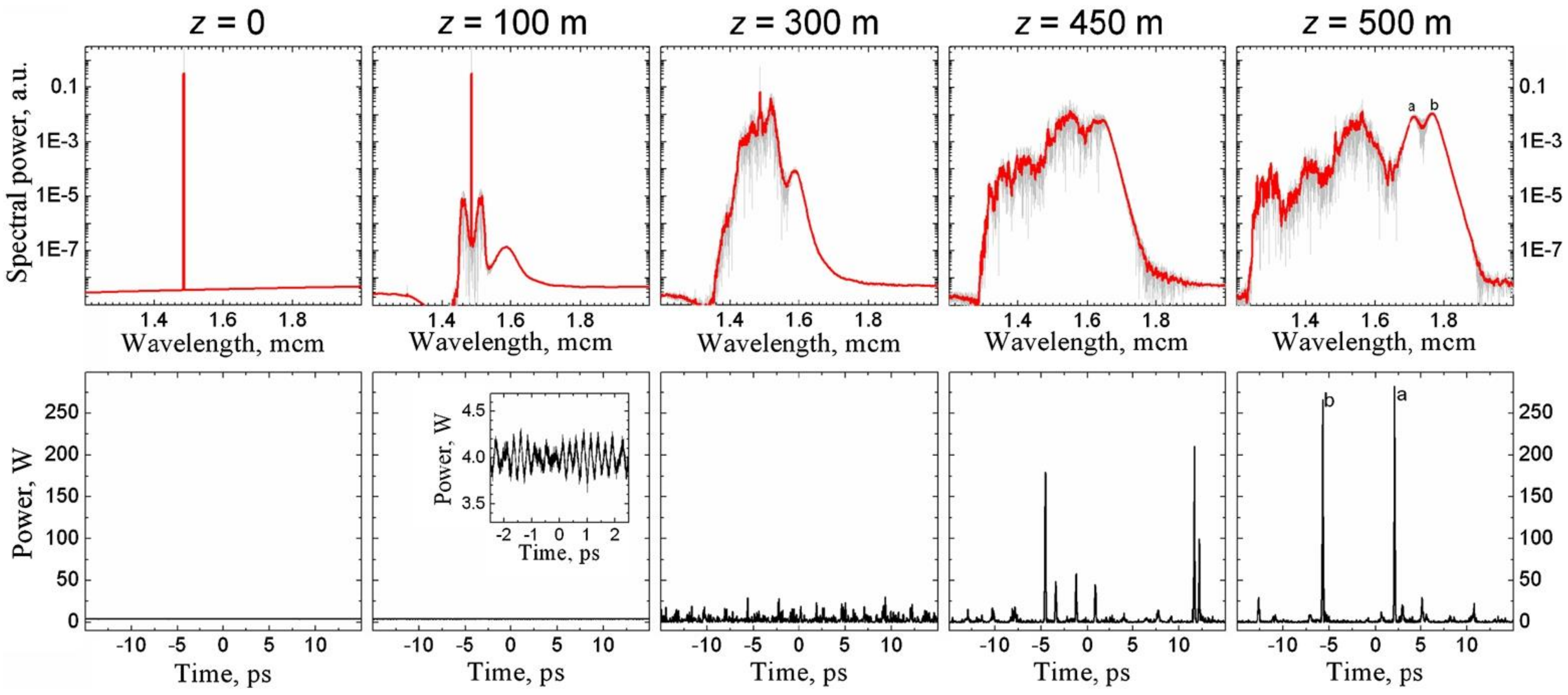


Recorded spectra as a function of average power at the output of a microstructured fiber when 110-fs pulses are propagated over a fiber length of (A) 0.57 and (B) 70 cm.
Arrows indicate solitons created after the fission.

Rogue waves formation in fibers: supercontinuum generation

Supercontinuum generation trough modulation instability:

- Supercontinuum generation can occur in the case of CW or quasi-CW pumping without soliton fission when modulation instability creates a multitude of fundamental solitons of different widths.

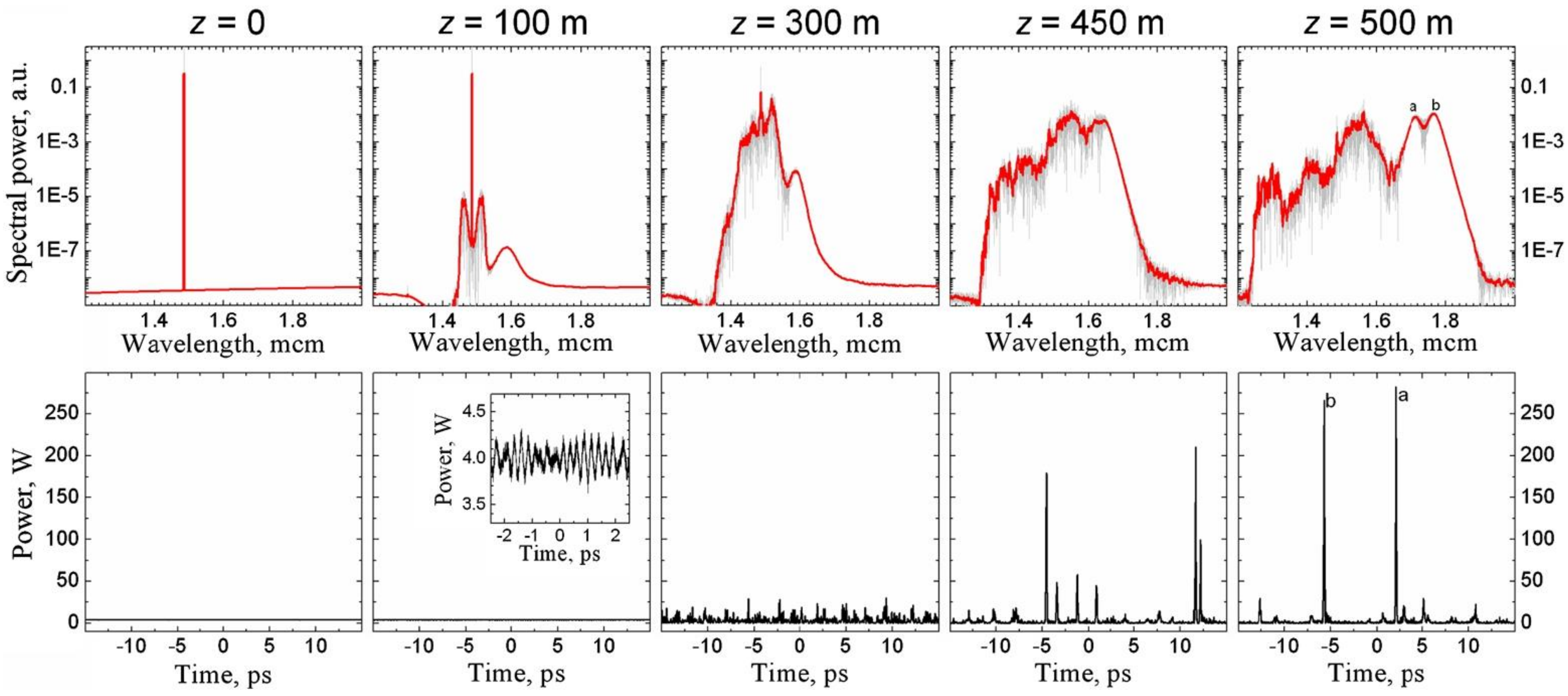


Temporal (bottom) and spectral (top) evolutions over 500 m of fiber when a CW beam is launched with 4 W of power. Dark-gray curves represent an average over 50 light-gray spectra.

Rogue waves formation in fibers: supercontinuum generation

Supercontinuum generation trough modulation instability:

- As the nonlinear length shortens, modulations become sharper and sharper and take the form of a train of short optical pulses of different widths that propagate as solitons in the anomalous-GVD regime of the fiber.



Temporal (bottom) and spectral (top) evolutions over 500 m of fiber when a CW beam is launched with 4 W of power. Dark-gray curves represent an average over 50 light-gray spectra.

Outline

1. What is a rogue wave?
    - definition, properties and examples
2. Rogue waves formation in fibers: conditions and process
    - interplay of dispersion and nonlinearity in fibers
    - formation of solitons
    - modulation instability
    - supercontinuum generation and generation of rogue waves
3. **Optical rogue waves in fibers: first report**
    - **experimental observation**
    - **numerical simulations**
4. Numerical simulation of rogue waves in fibers
    - solving NLSE with the use of split-step Fourier method

Optical rogue waves: first report

First experimental and theoretical investigation of rogue waves in optical systems was performed by **Solli D.R., Ropers C., Koonath P. and Jalali B.** in 2007 [1].

The authors asked a question: **what conditions are needed to generate an optical rogue wave?**

- So far, the study of rogue waves in the scientific literature has focused on hydrodynamic studies and experiments. Intriguingly, there are other physical systems that possess similar nonlinear characteristics and may also support rogue waves.

- Observations indicate that RWs have unusually steep, solitary or tightly grouped profiles, which appear like ‘‘walls of water’’. These features imply that rogue waves have relatively broadband frequency content compared with normal waves, and also suggest a possible connection with solitons.

- Possible mechanisms that have been suggested: nonlinear focusing via modulation instability, nonlinear spectral instability, focusing with caustic currents and anomalous wind excitation.
  Nonlinear mechanisms have attracted particular attention because they possess the requisite extreme sensitivity to initial conditions.

[1] Solli D.R., Ropers C., Koonath P. and Jalali B., Optical rogue waves, Nature 450 1054, 2007

Optical rogue waves: first report

The authors' idea: optical rogue waves can be observed in a system based on probabilistic supercontinuum generation in a highly nonlinear microstructured optical fibre.

- An extremely broadband supercontinuum source can be created by launching intense seed pulses into a nonlinear fibre at or near its zero-dispersion wavelength.
- In this situation, supercontinuum production involves generation of high-order solitons which fission into red-shifted solitonic and blueshifted non-solitonic components at different frequencies.
- The solitonic pulses shift further towards the red as they propagate through the nonlinear medium because of the Raman-induced self-frequency shift.
- The nonlinear processes responsible for supercontinuum generation amplify the noise present in the initial laser pulse. Especially for long pulses and continuous-wave input radiation, modulation instability — an incoherent nonlinear wave-mixing process — broadens the spectrum from seed noise in the initial stages of propagation and, as a result, the output spectrum is highly sensitive to the initial conditions.

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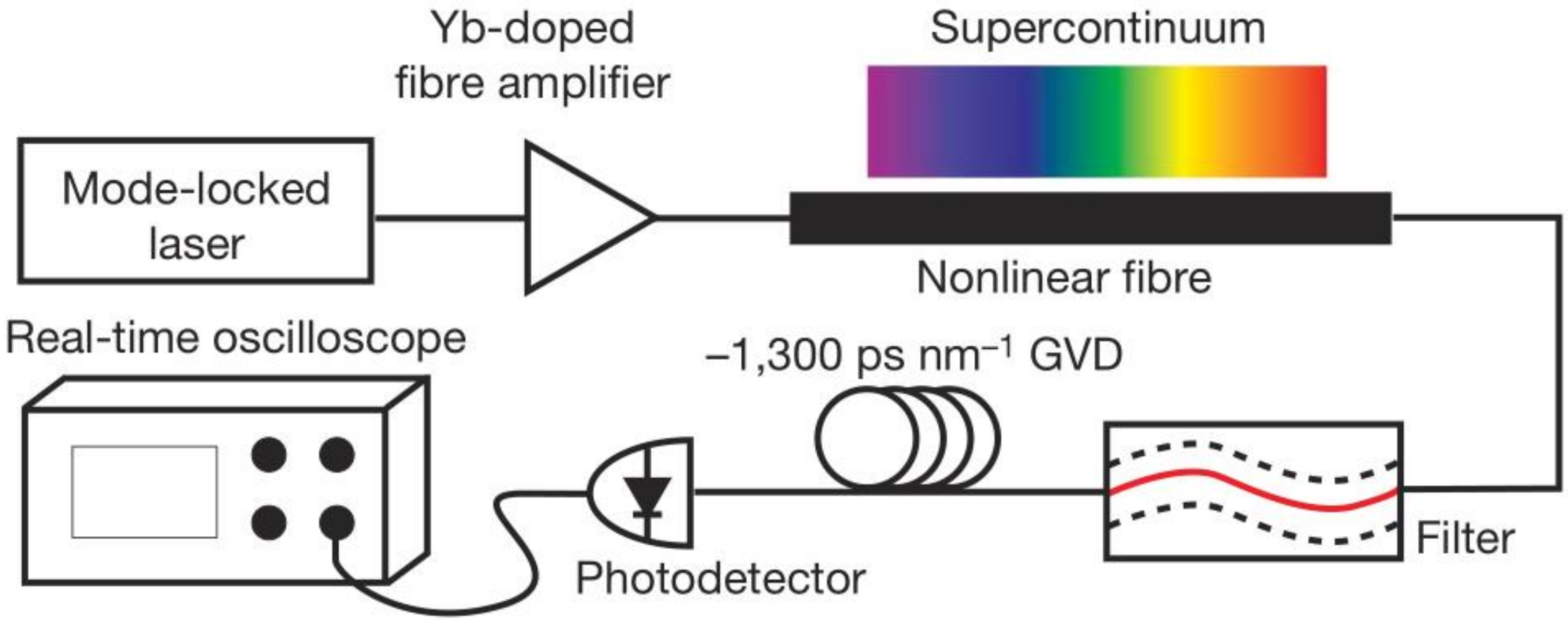
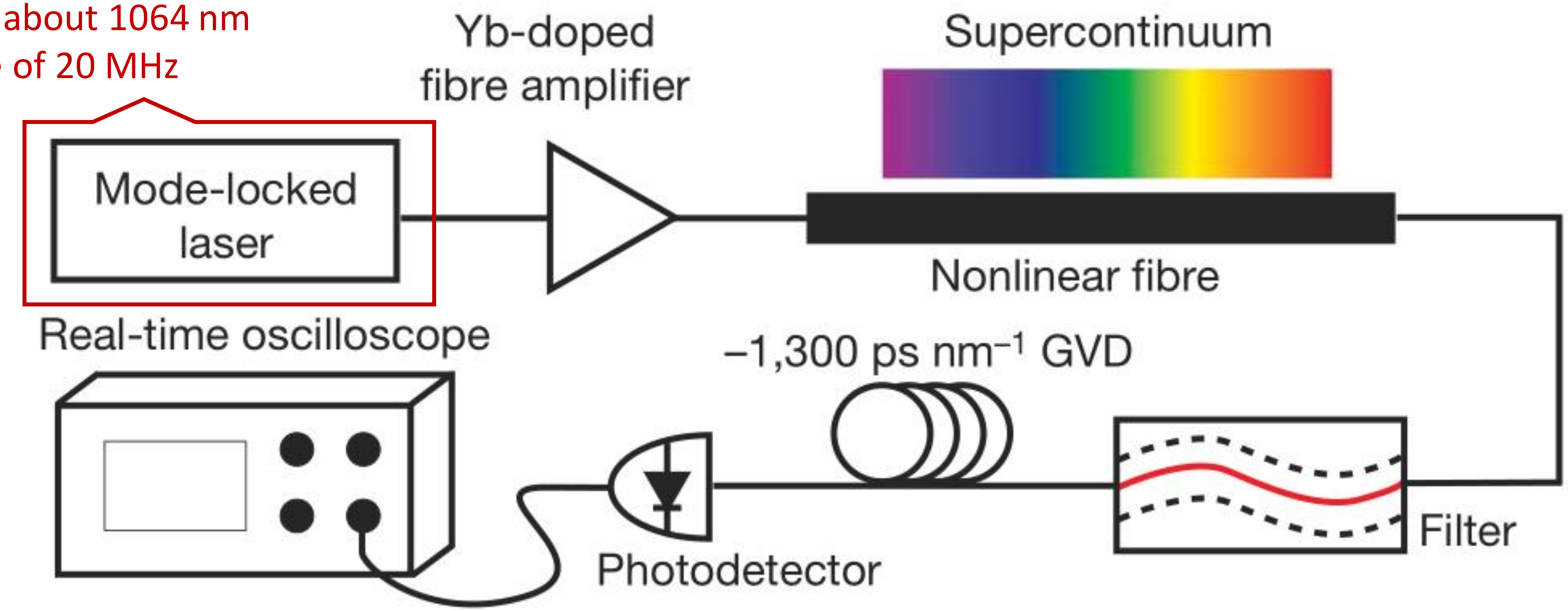
Optical rogue waves: experimental setup



Optical rogue waves: experimental setup

The master oscillator: a mode-locked Yb-doped fibre laser producing picosecond pulses at about 1064 nm with a repetition rate of 20 MHz



Optical rogue waves: experimental setup

Large-mode-area Yb-doped-fibre amplifier: outputs chirped pulses of ~5-nm bandwidth and a few picoseconds temporal width

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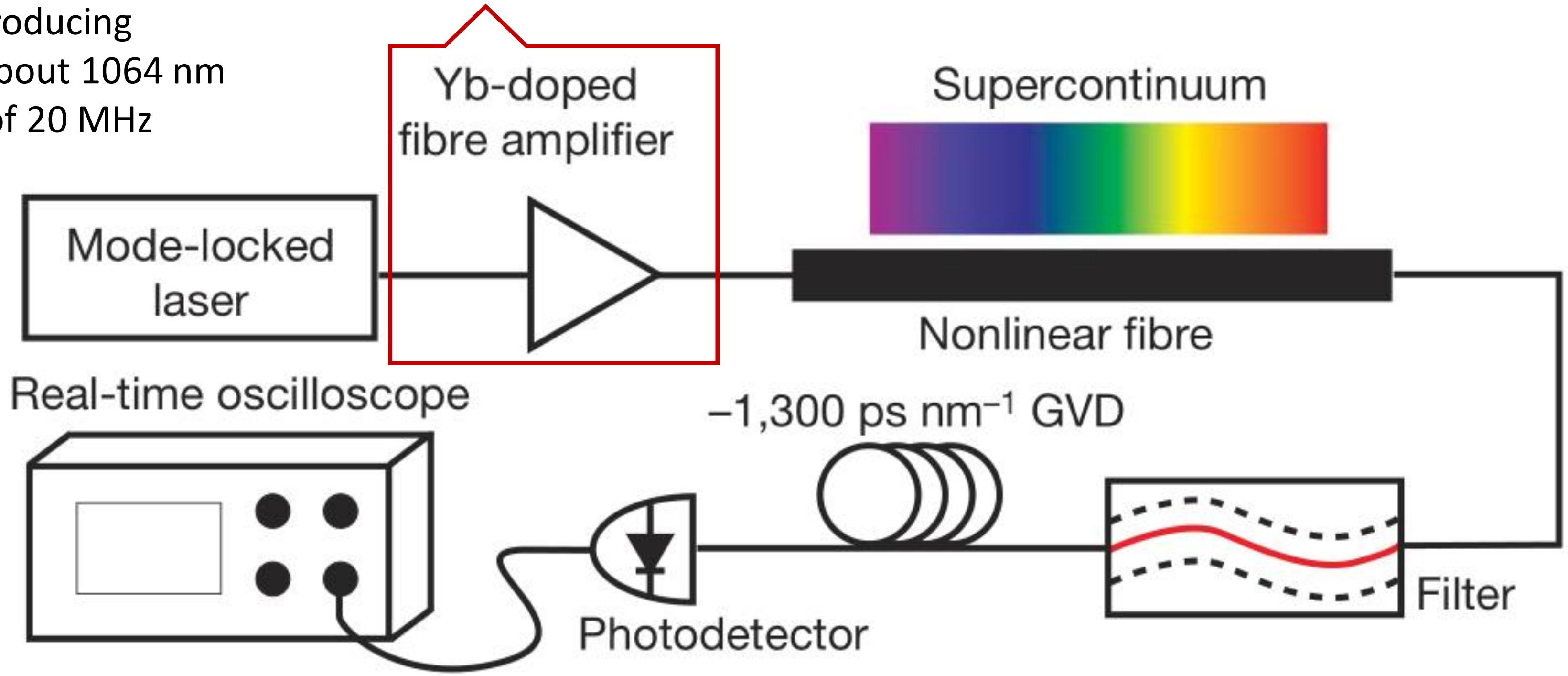

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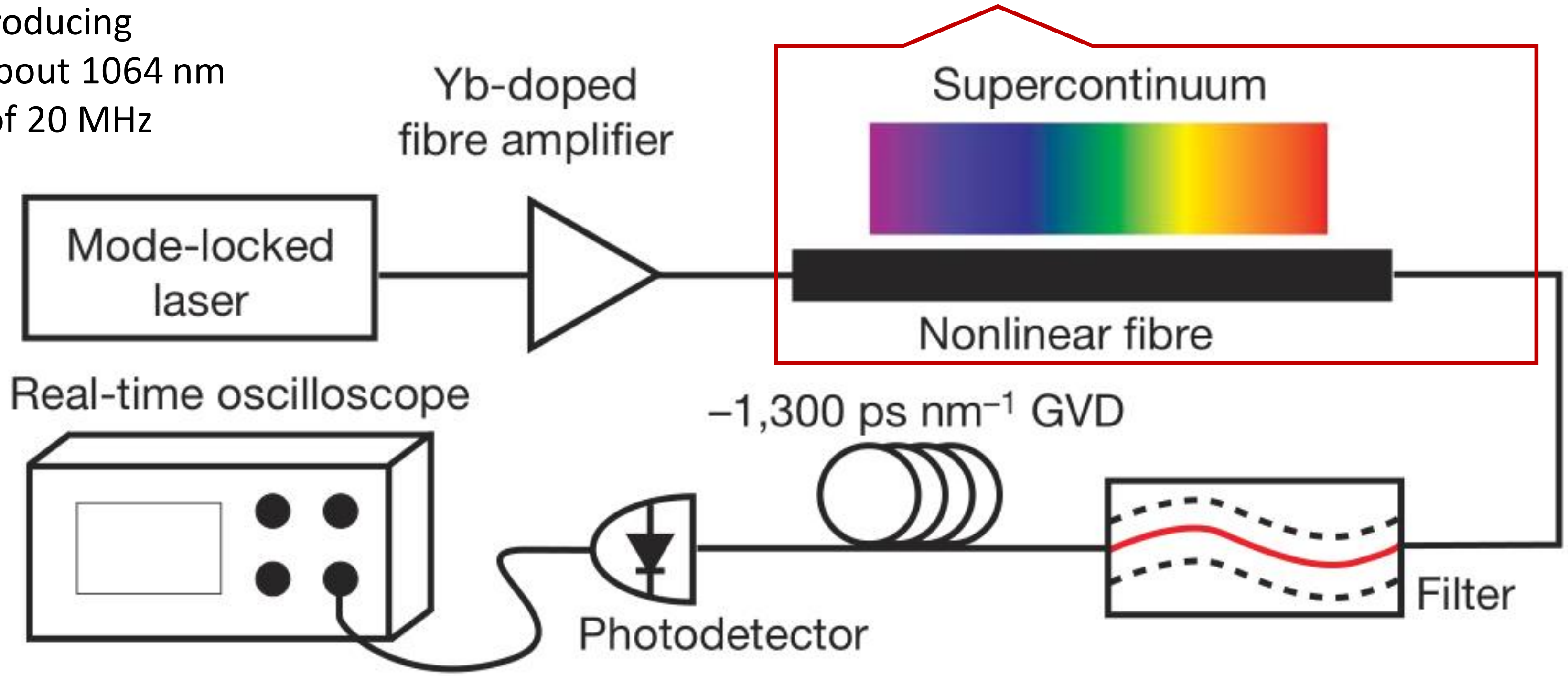
A 15m-length of highly nonlinear microstructured optical fibre with zero-dispersion point matching the seed wavelength



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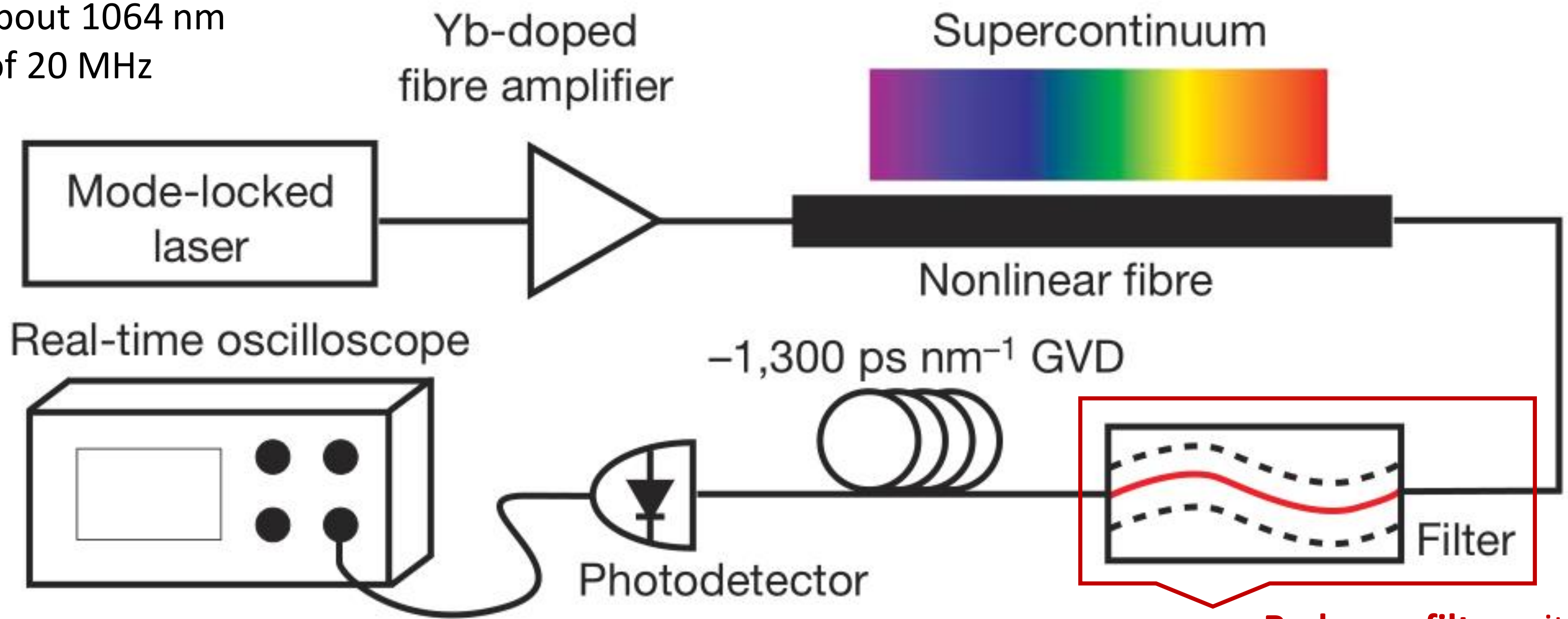

Red-pass filter with a cut-on wavelength of 1450 nm

Optical rogue waves: experimental setup

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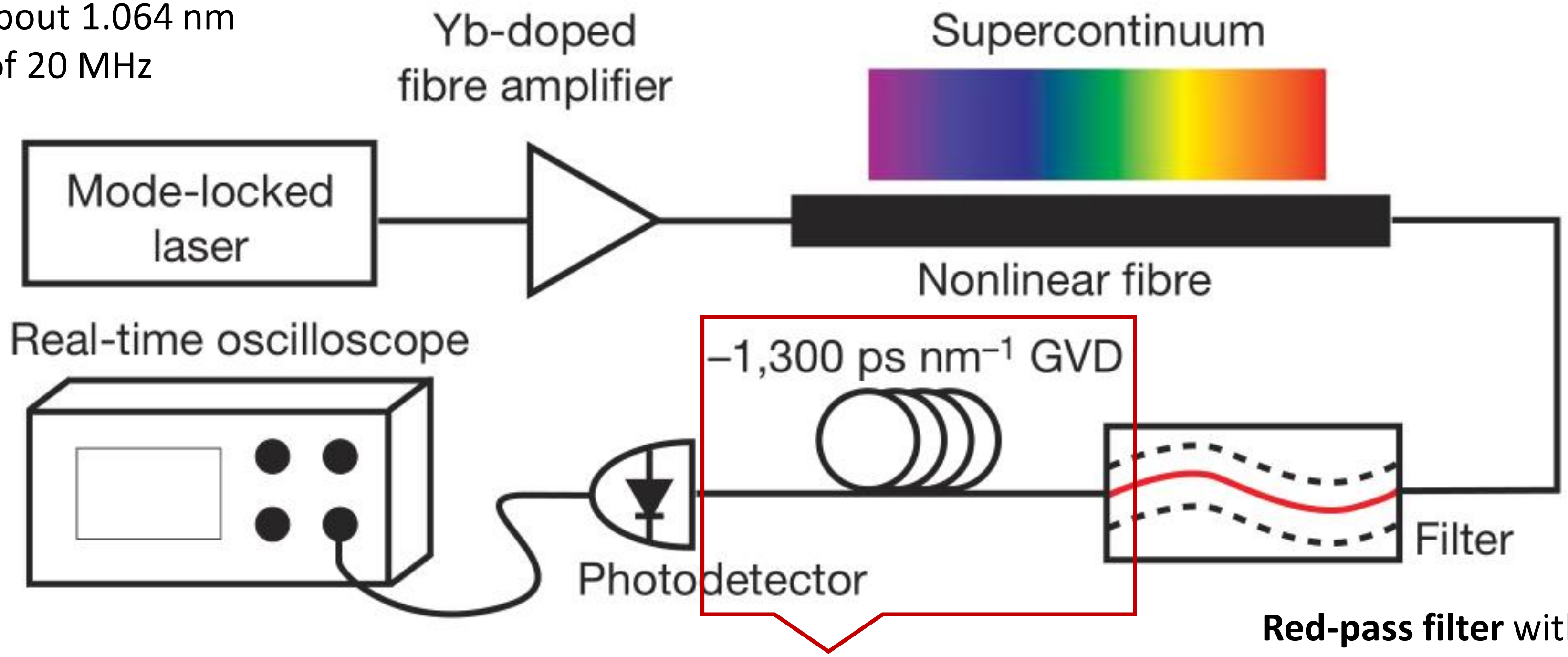

Red-pass filter with a cut-on wavelength of 1.450 nm

The wavelength-to-time transformation for real-time detection: a highly dispersive optical fibre with GVD of -1.300 ps/nm over the wavelength range of interest

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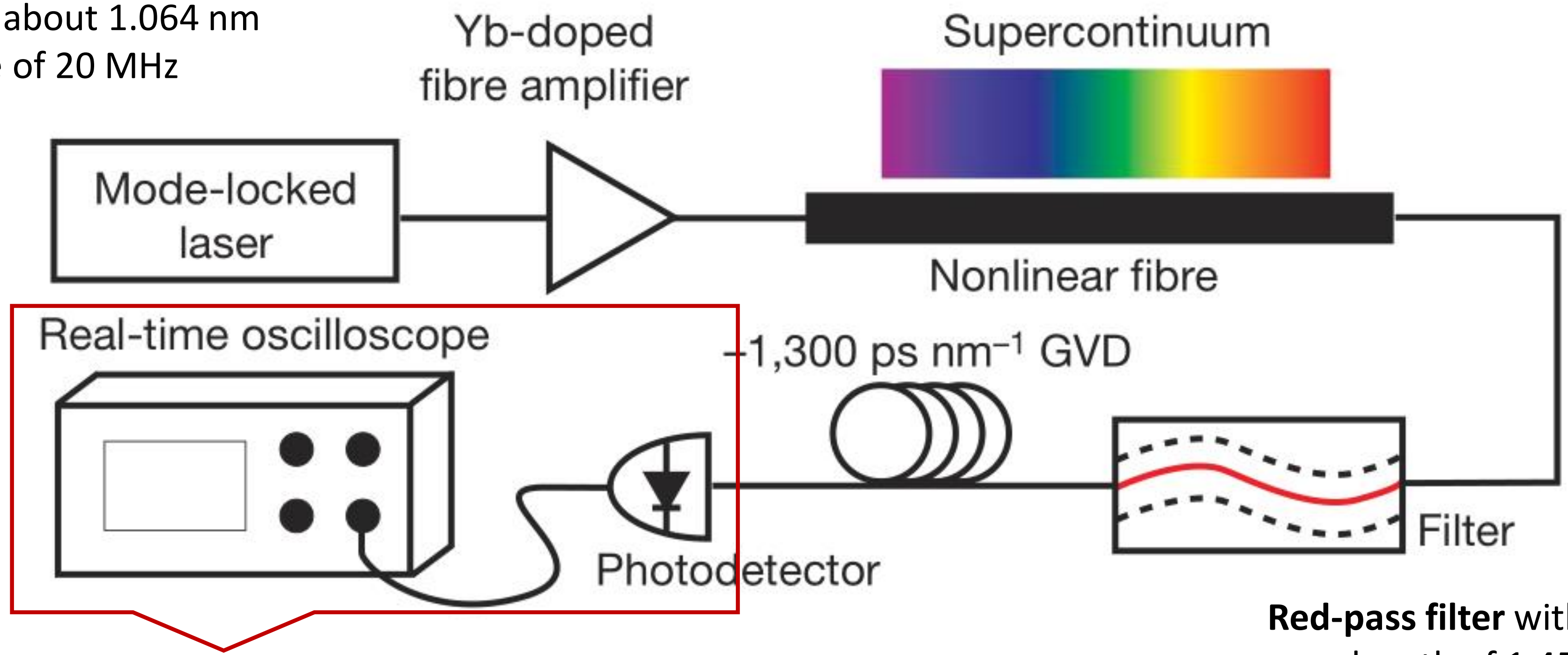

Red-pass filter with a cut-on wavelength of 1.450 nm

Fast photodetector and a real-time 20-gigasample-per-second oscilloscope: records sequences of ~15,000 pulses with high temporal resolution in a single-shot measurement

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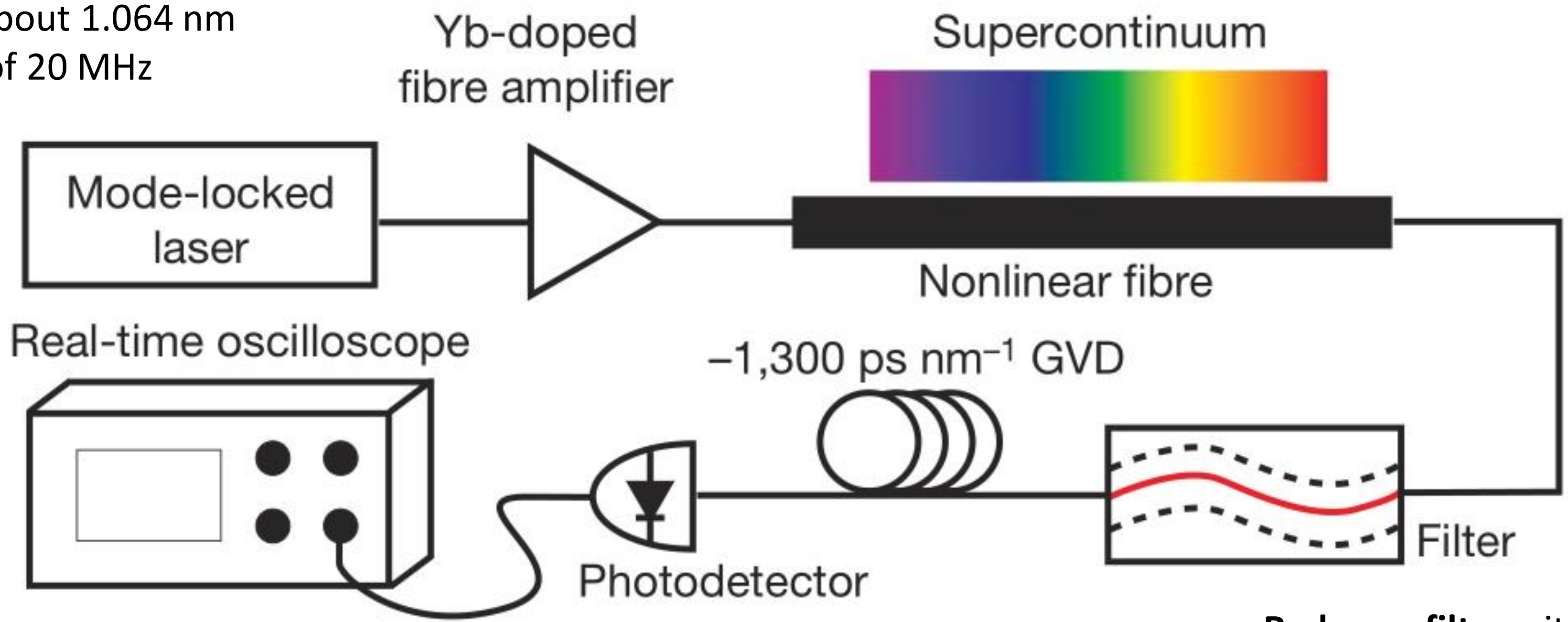

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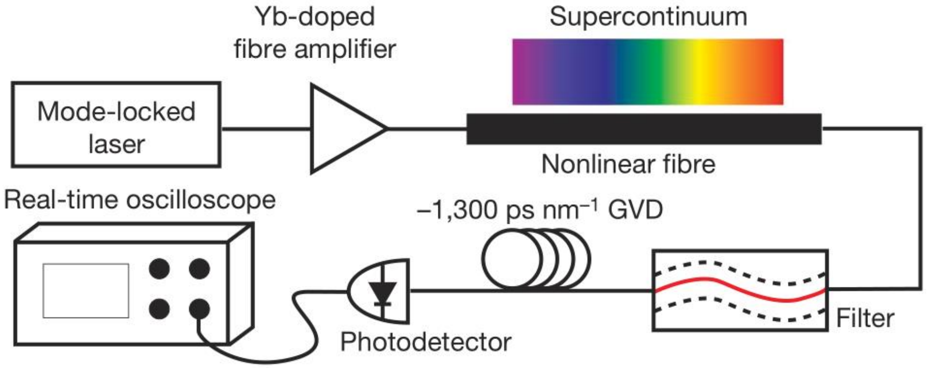
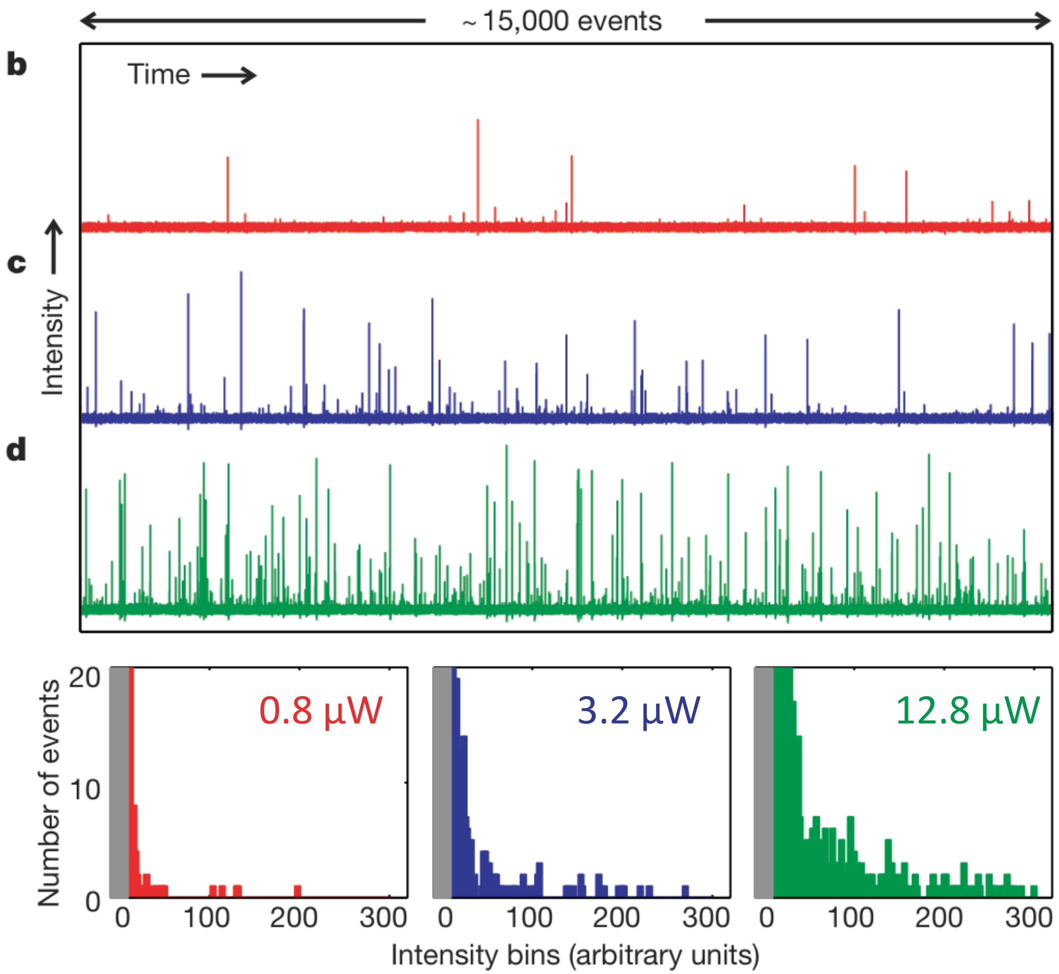
Optical rogue waves: experimental results

- Using this setup, large sets of pulses were acquired in real time for very low seed pulse power levels — below the threshold required to produce appreciable supercontinuum.
- The pulse-height distributions were sharply peaked with a well-defined mean, but contrary to expectation, rare events with far greater intensities also appeared.
- The histograms display a clear L-shaped profile, with extreme events occurring rarely, yet much more frequently than expected based on the relatively narrow distribution of typical events.





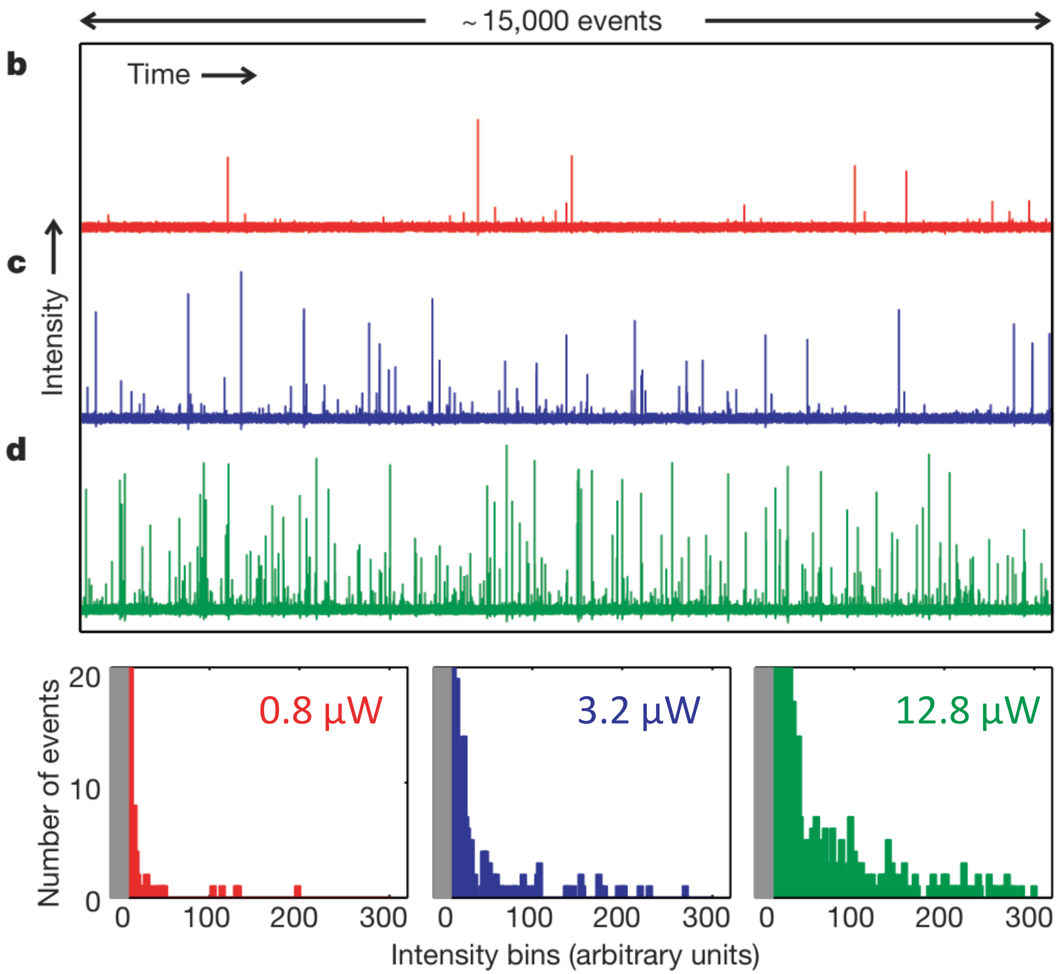
The grey shaded area corresponds to the noise floor of the measurement process. The vast majority of events (~99.5% for the lowest power) are buried in this low intensity range, and the rogue events reach intensities of at least 30–40 times the average value.

Optical rogue waves: experimental results

- Because the red-pass filter transmits only a spectral region that is nearly dark in the vast majority of events, the rare events clearly have extremely broadband, frequency-downshifted spectral content.

- The data also show that the frequency of occurrence of the rogue events increases with the average power, but the maximum height of a freak pulse remains relatively constant.

- These features indicate that the extreme events are sporadic, single solitons.



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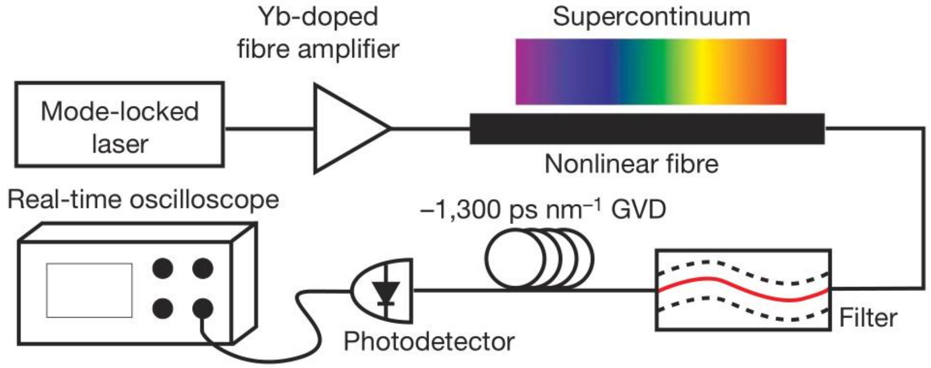

Optical rogue waves: theoretical investigation

The generalized nonlinear Schrödinger equation (NLSE) describes the evolution of the slowly varying electric field envelope, *A(z,t)*, in the presence of temporal dispersion and nonlinearity:

$$\frac{\partial A}{\partial z} - i \sum_{m=2} \frac{i^m \beta_m}{m!} \frac{\partial^m A}{\partial t^m} = i\gamma \left(|A|^2 A + \frac{i}{\omega_0} \frac{\partial}{\partial t} (|A|^2 A) - T_R A \frac{\partial |A|^2}{\partial t} \right)$$

- Here, β_m are values that characterize the fibre dispersion, γ is the nonlinear coefficient of the fibre, ω_0 is the central carrier frequency of the field, and T_R is a parameter that characterizes the delayed nonlinear response of silica fibre. Dispersion up to sixth order was included in the nonlinear fibre.

- The bracketed terms on the right-hand side of the equation describe the Kerr nonlinearity, self-steepening and the vibrational Raman response of the medium, respectively. The Kerr term produces self-phase modulation, and the Raman term causes frequency downshifting of the carrier wave. For completeness, the self-steepening term was also included in the simulations, but it was found that it is not required for rogue wave generation.

- This equation has been successfully used to model supercontinuum generation in the presence of noise and can be used to predict optical rogue waves (it was capable of qualitatively explaining the experimental results).

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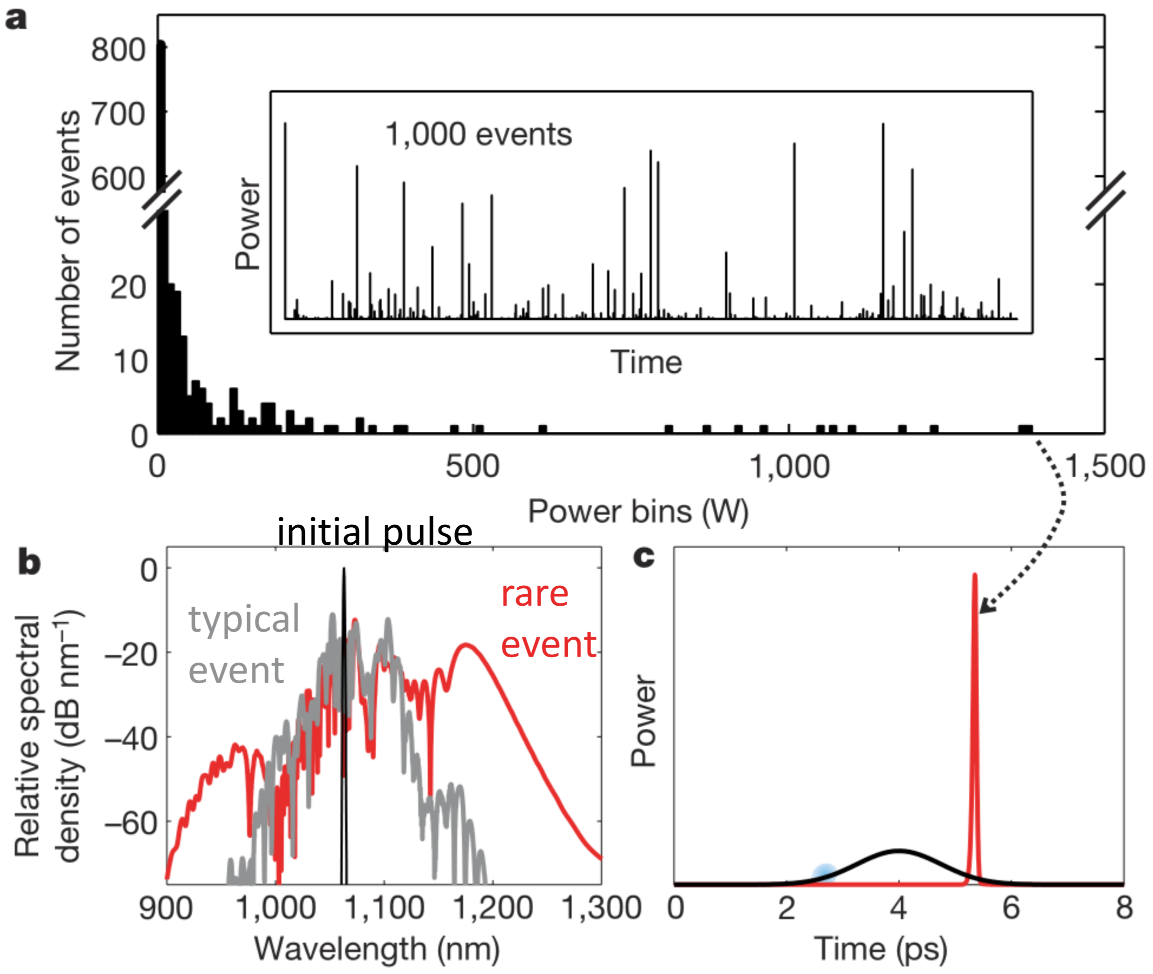
- To generate rogue waves, the input pulse was perturbed by adding a very small amount of amplitude noise directly to its temporal envelope. The peak power of the unperturbed pulse was chosen to be small enough that the pulse will not break up without the noise perturbation.

Optical rogue waves: theoretical investigation

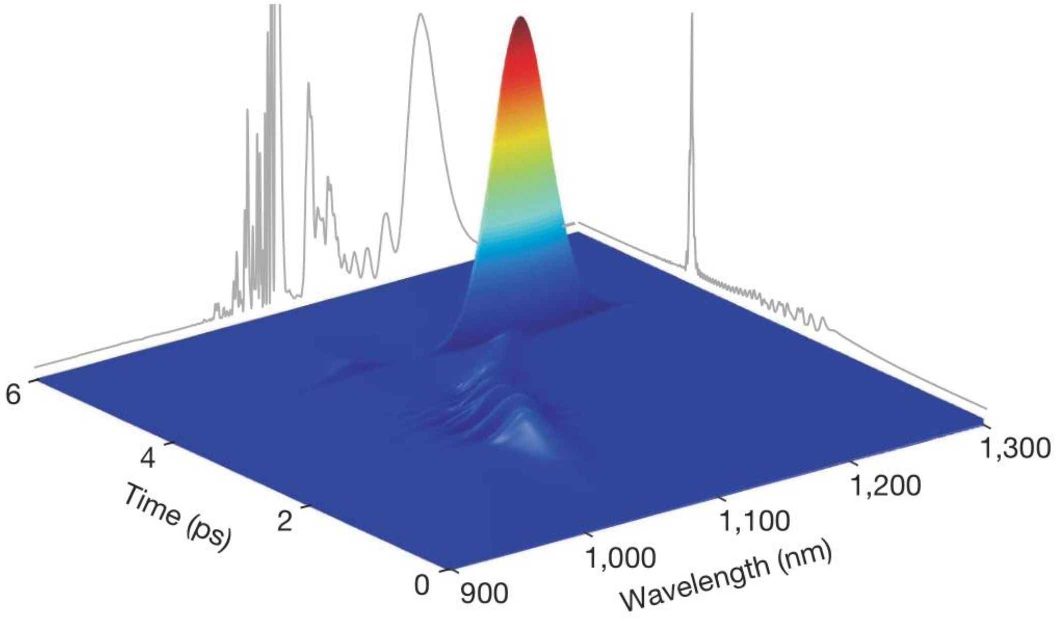
- The present model shows that a high-power, smooth input pulse ejects multiple redshifted solitons and blueshifted non-solitonic components, and a tiny amount of input noise varies their spectral content.
- For a small fraction of events, the spectrum becomes exceptionally broad with a clear redshifted solitonic shoulder.
- The rogue pulses have exceptionally steep leading and trailing edges compared with the initial pulses and the typical events.
- The histogram of heights is sharply peaked but has extended tails, and the distribution contains rogue events more than 50 times as large as the mean.



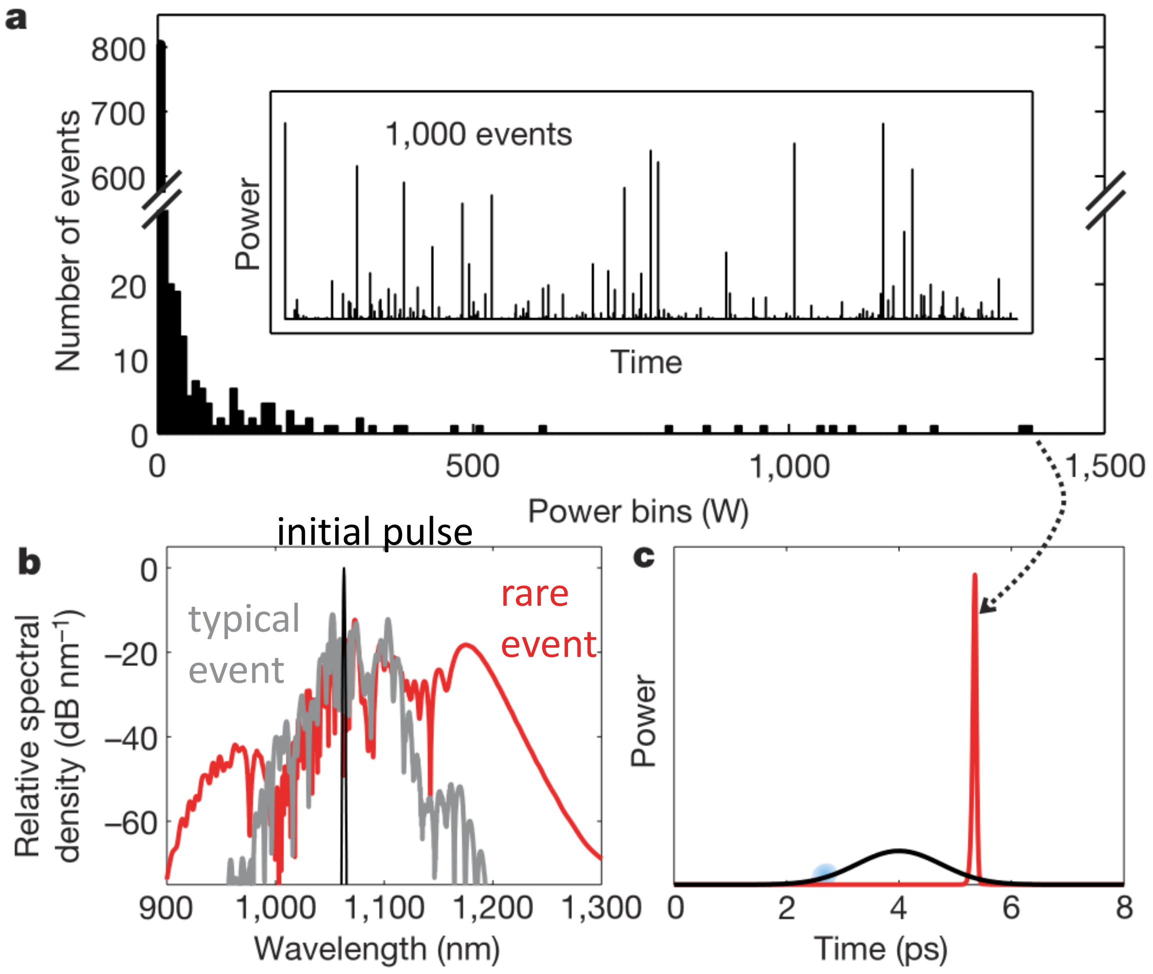
The initial (seed) pulses have width 3 ps, peak power 150 W, fractional noise 0.1%, and noise bandwidth 50 THz; red-pass filtering from 1,155 nm is used. The shaded blue region on the seed pulse delineates the time window that is highly sensitive to perturbation.

Optical rogue waves: theoretical investigation

- Time-wavelength profile of an optical rogue wave obtained from a short-time Fourier transform:



The optical wave has broad bandwidth and has extremely steep slopes in the time domain compared with the typical events. The grey traces show the full time structure and spectrum of the rogue wave.



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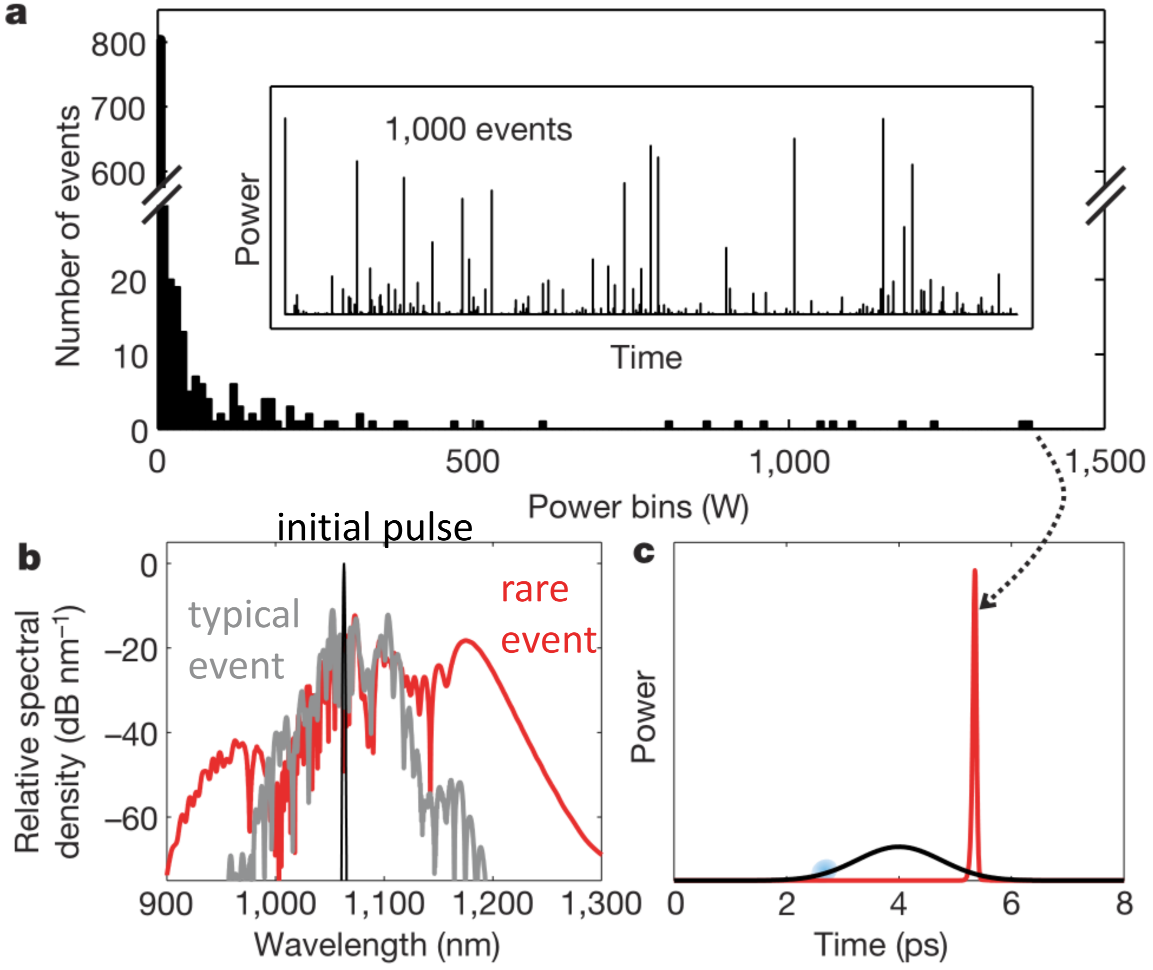
Optical rogue waves: theoretical investigation

- Because there are no apparent features in the perturbations that lead to the development of the rogue events, their appearance seems unpredictable.
- Examining the correlations between the initial conditions and their respective output waveforms, the authors found that if the random noise happens to contain energy with a frequency shift of about 8 THz within a 0.5-ps window centered about 1.4 ps before the pulse peak, a rogue wave is born.
- Noise at this particular frequency shift and on a leading portion of the pulse envelope efficiently seeds modulation instability, reshaping the pulse to hasten its breakup. The output wave height correlates in a highly nonlinear way with this specific aspect of the initial conditions.



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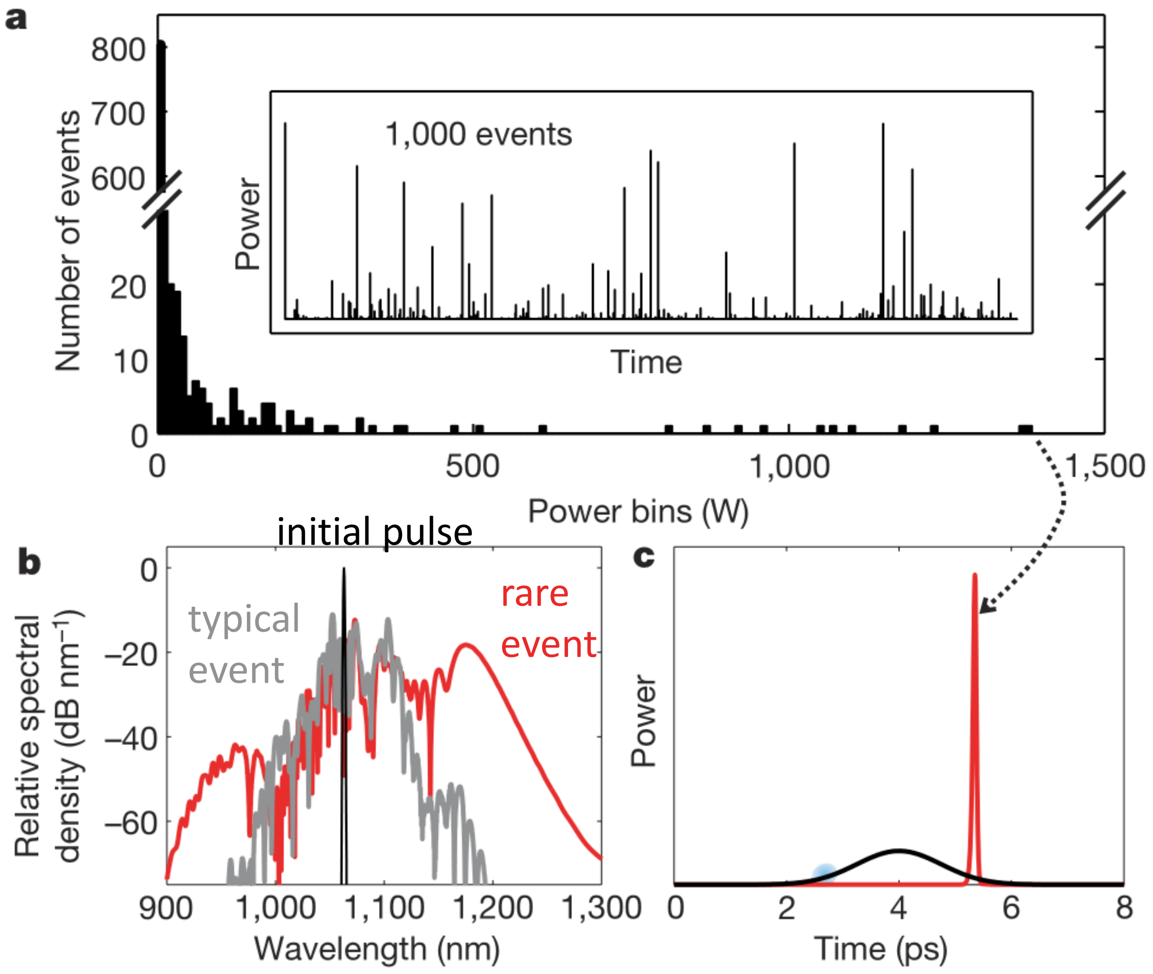
Optical rogue waves: theoretical investigation

The rogue waves have a number of other intriguing properties warranting further study:

- They propagate without noticeable broadening for some time, but have a finite, seemingly unpredictable lifetime before they suddenly collapse owing to cumulative effects of Raman scattering.
- This scattering seeded by noise dissipates energy or otherwise perturbs the soliton pulse beyond the critical threshold for its survival. The decay parallels the unpredictable lifetimes of oceanic rogue waves.
- The rogue optical solitons are also able to absorb energy from other wavepackets they pass through, which causes them to grow in amplitude, but appears to reduce their lifetime.



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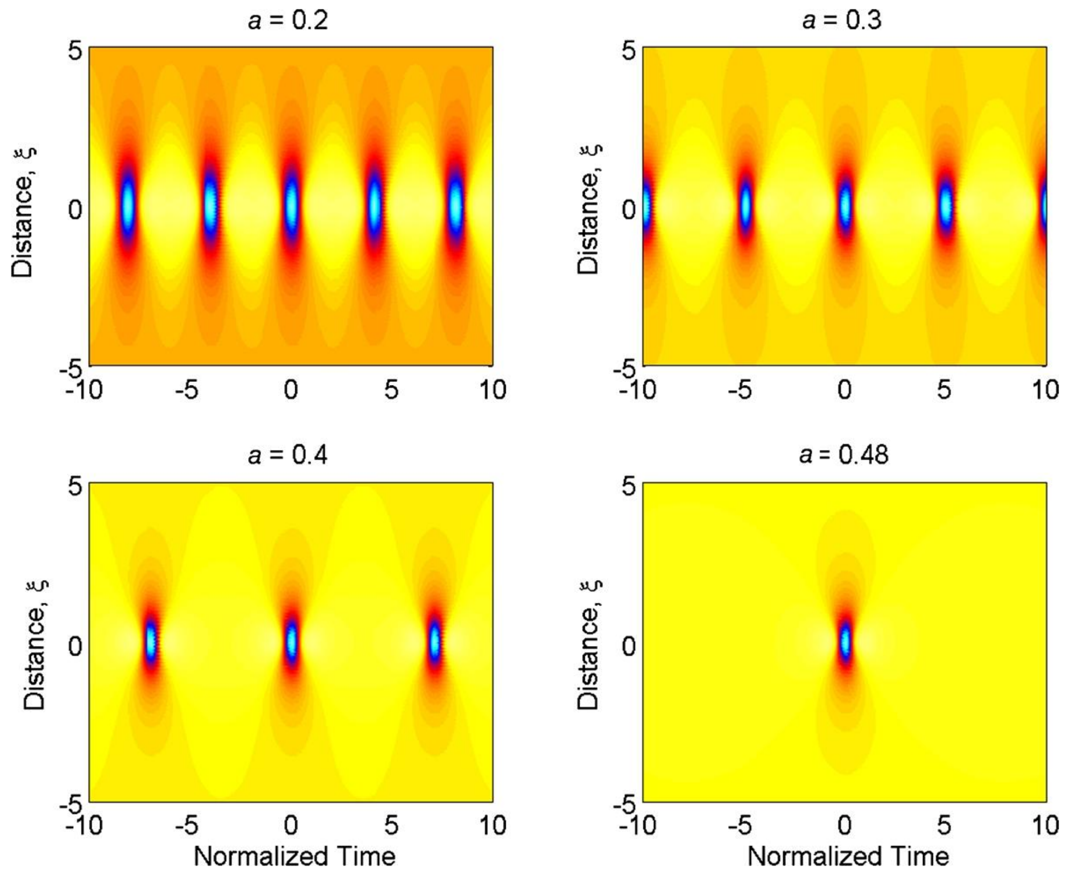
Optical rogue waves: conclusion

- The authors have observed extreme soliton-like pulses that are the optical equivalent of oceanic rogue waves.
- These rare optical events possess the hallmark phenomenological features of oceanic rogue waves — they are extremely large and seemingly unpredictable, follow unusual L-shaped statistics, occur in a nonlinear medium, and are broadband and temporally steep compared with typical events.
- On a physical level, the similarities also abound, with modulation instability, solitons, frequency downshifting and higher-order dispersion as striking points of connection.
- Intriguingly, the rogue waves of both systems can be modelled with the nonlinear Schrödinger equation.
- Although the parameters that characterize this optical system are of course very different from those describing waves on the open ocean, the rogue waves generated in the two cases bear some remarkable similarities.

Rogue waves formation in fibers: rogue waves generation

Rogue waves generation:

- The CW solution of the NLSE is unstable in the region of anomalous dispersion (β_2<0) because of modulation instability.
- The NLSE has a specific family of periodic solutions. Some of them are referred to as the Akhmediev breathers (“breathing” solitons that are periodic in time, and also confined spatially).
- In the limit *a*=1/2 , the pulse train reduces to a single pulse that is localized in both the *z* and *T* dimensions.



Surface plots of the periodic solution $|A|^2$ as a function of ξ and $\Omega_{max}T$ for four values of a (soliton parameter). Only a single pulse is within the plotting region for a=0.48.

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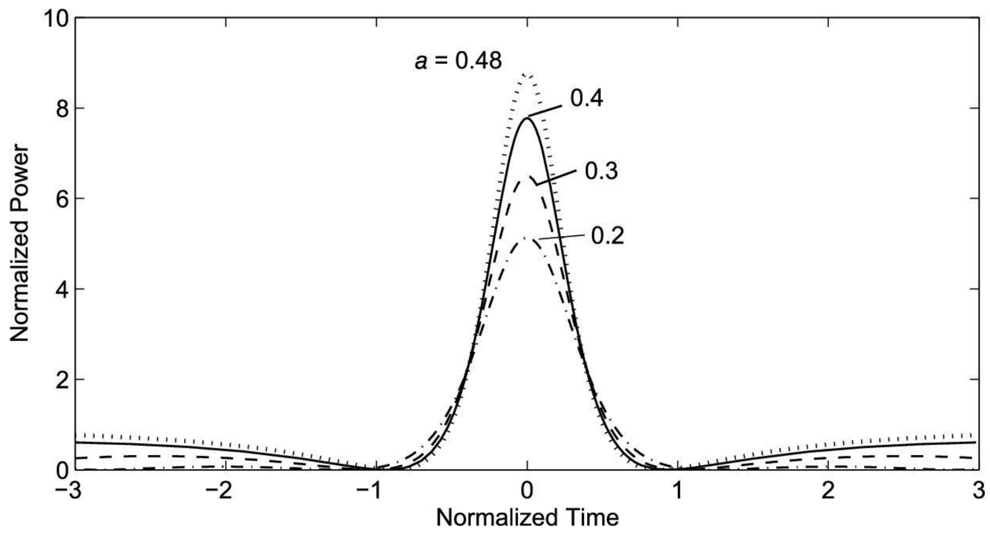
- The NLSE has a specific family of periodic solutions. Some of them are referred to as the Akhmediev breathers ("breathing" solitons that are periodic in time, and also confined spatially).
- In the limit *a*=1/2 , the pulse train reduces to a single pulse that is localized in both the *z* and *T* dimensions.
- This solution was discovered in 1983 in the form of an algebraic solution of the NLSE and is known as the Peregrine soliton.
- It was realized around 2009 that such periodic solutions were related to optical rogue waves.

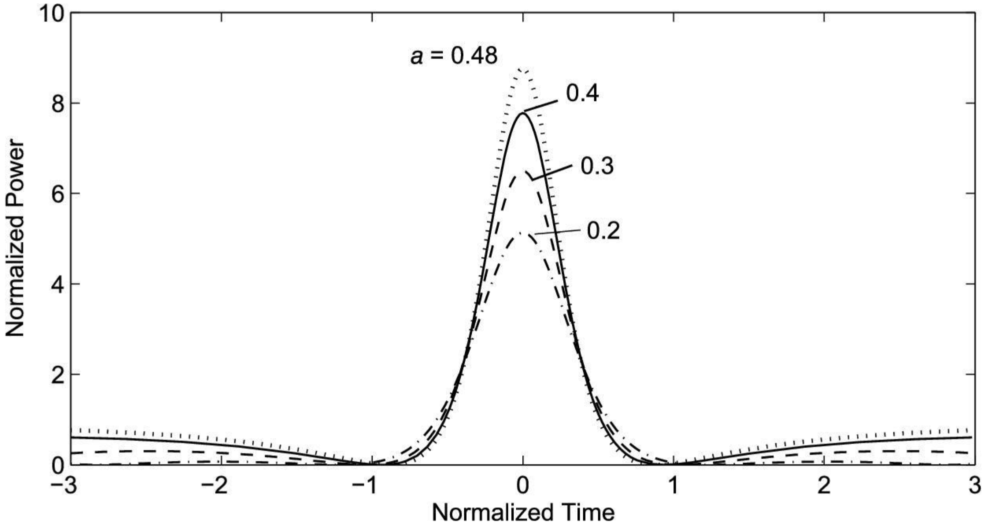


Pulse shapes for the same four values of a. The dotted curve for a = 0.48 is close to the shape of the Peregrine soliton.

Rogue waves formation in fibers: rogue waves generation

Rogue waves generation:

- The existence of the Peregrine soliton can be used to understand the origin of optical rogue waves when modulation instability is seeded with noise (spontaneous modulation instability).
- As a noise component at any frequency falling within the gain spectrum of modulation instability can be amplified, the parameter a can take any value in its range $0<a<1/2$.
- The shortest and most intense pulses are formed when a is close to 1/2, but the probability of the formation such pulses is relatively small because the instability gain is reduced considerably as $a \rightarrow 1/2$.
- The peak power of the pulse is nearly 9 times of the input CW power in this limit.



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Rogue waves formation in fibers: outlook

Outlook:

- The emergence, dynamics and prediction of rogue waves has been in the focus of interest in diverse fields of science (oceanography, physics of fluids, optics, matter waves physics, sociology, bio-sciences) over the last fifteen years.

- Optical RWs were observed both in time (1D) and space (2D).

- RWs were investigated in different nonlinear systems (optical fibers with dispersion and nonlinearity, mode-locked fiber and Ti:Sapphire lasers, semiconductor lasers, etc.) and even in linear systems (multimode fibers, photonic crystal chips).

- The open questions in understanding the physics of rogue waves are:
  the conditions of their emergence; the role of linear and nonlinear effects in this process; influence of higher-order nonlinear effects; formation of RWs in linear systems; possibility of prediction, control and artificial generation of RWs.

- Prediction and control of optical RWs involves the use of machine learning techniques to detect patterns and build models based on analysis of large data sets.

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    - supercontinuum generation and generation of rogue waves
3. Optical rogue waves in fibers: first report
    - experimental observation
    - numerical simulations
4. **Numerical simulation of rogue waves in fibers**
    - **solving NLSE with the use of split-step Fourier method**

Numerical simulation of rogue waves in fibers

- The NLSE is a nonlinear partial differential equation that does not generally lend itself to analytic solutions, except for some specific cases.

- A numerical approach is often necessary for an understanding of the nonlinear effects in optical fibers.

- Several numerical methods were developed for solving the NLSE.
  They can be classified into two broad categories: the finite-difference and pseudospectral methods.

- One method that has been used extensively for solving pulse-propagation problems in nonlinear dispersive media is the **split-step Fourier method**. The relative speed of this method compared with most finite-difference schemes can be attributed in part to the use of the fast Fourier transform (FFT) algorithm.

Numerical simulation of rogue waves in fibers

Consider the NLSE in the general form, taking into account dispersion, nonlinearity, losses, Raman scattering, and self-steepening effects:

$$\frac{\partial A}{\partial z} + \underbrace{\frac{\alpha}{2}A + \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6}\frac{\partial^3 A}{\partial T^3}}_{-\widehat{D}A} = \underbrace{i\gamma\left(|A|^2 A + \frac{i}{\omega_0}\frac{\partial}{\partial T}(|A|^2 A) - T_R A\frac{\partial |A|^2}{\partial T}\right)}_{\widehat{N}A}$$

This equation can be rewritten in the operator form:

$$\frac{\partial A}{\partial z} = (\widehat{D} + \widehat{N})A,$$

where $\widehat{D}$ is a differential operator that accounts for dispersion and losses, and $\widehat{N}$ is a nonlinear operator that governs the effect of fiber nonlinearities on pulse propagation:

$$\widehat{D} = -\frac{i\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial T^3} - \frac{\alpha}{2},$$

$$\widehat{N} = i\gamma\left(|A|^2 + \frac{i}{\omega_0}\frac{1}{A}\frac{\partial}{\partial T}(|A|^2 A) - T_R\frac{\partial |A|^2}{\partial T}\right)$$

Numerical simulation of rogue waves in fibers

- In general, dispersion and nonlinearity act together along the length of a fiber.
- The **split-step Fourier method** obtains an approximate solution by assuming that the dispersive and nonlinear effects can be assumed to act independently for propagating the optical field over a small distance *h*.
- More specifically, propagation from *z* to *z+h* is carried out in two steps:
  In the first step, the nonlinearity acts alone, and $\widehat{D} = 0$.
  In the second step, dispersion acts alone, and $\widehat{N} = 0$.

$$A(z+h,T) \approx exp\{h\widehat{D}\}exp\{h\widehat{N}\}A(z,T)$$

- The dispersion operator can be evaluated in the frequency domain, as we can use the properties of Fourier transform and replace the operator $\frac{\partial}{\partial T}$ in time domain by $-i\omega$ in frequency domain:

$$\widehat{D}(T) = -\frac{i\beta_2}{2}\frac{\partial^2}{\partial T^2} + \frac{\beta_3}{6}\frac{\partial^3}{\partial T^3} - \frac{\alpha}{2} \rightarrow \widehat{D}(\omega) = \frac{i\beta_2}{2}\omega^2 + \frac{\beta_3}{6}i\omega^3 - \frac{\alpha}{2}$$

Numerical simulation of rogue waves in fibers

- So, we can evaluate the dispersion operator in frequency domain, and nonlinear operator in time domain, and use fast Fourier transform to switch between the domains:

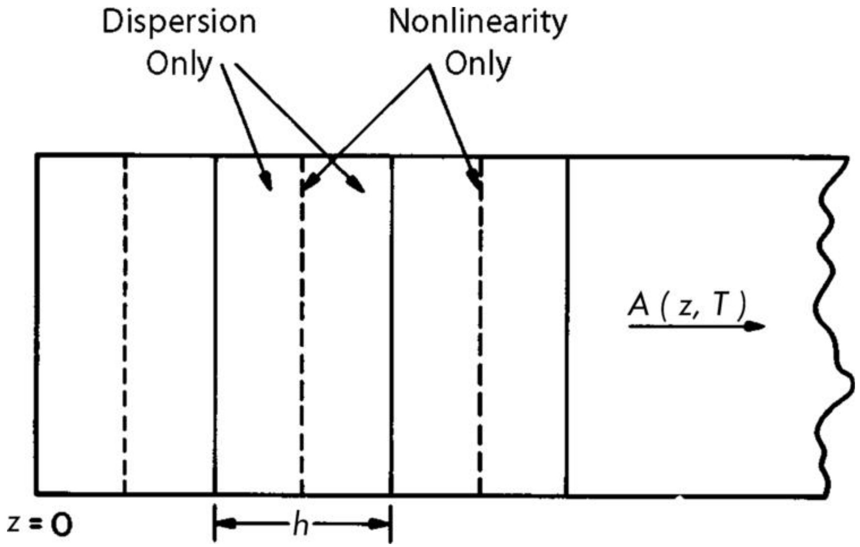
$$exp\{h\hat{D}\}A(z,T) = IFFT\left[exp\{h\hat{D}(\omega)\}FFT[A(z,T)]\right]$$

- The accuracy of the split-step Fourier method can be improved, if nonlinearity is included in the middle of the segment rather than at the segment boundary (this scheme is known as the symmetrized split-step Fourier method):

$$A(L,T) \approx exp\left\{-\frac{1}{2}h\hat{D}\right\}\left(\prod_{m=1}^{M} exp\{h\hat{D}\}exp\{h\hat{N}\}\right)exp\left\{\frac{1}{2}h\hat{D}\right\}A(0,T)$$

or

$$A(L,T) \approx exp\left\{-\frac{1}{2}h\hat{N}\right\}\left(\prod_{m=1}^{M} exp\{h\hat{N}\}exp\{h\hat{D}\}\right)exp\left\{\frac{1}{2}h\hat{N}\right\}A(0,T)$$



Numerical simulation of rogue waves in fibers

Consider the simplest NLSE, accounting for the 2nd order dispersion and nonlinearity only:

$$i\frac{\partial U}{\partial \xi} = -\frac{sign(\beta_2)}{2}\frac{\partial^2 U}{\partial \tau^2} - N^2|U|^2U,$$

$$N^2 = \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

$$\widehat{D} = \frac{isign(\beta_2)}{2}\omega^2, \quad \widehat{N} = iN^2|U|^2.$$

- To simulate solitons and rogue waves generation in the fiber, one need to use negative value of β_2 (anomalous dispersion).
- Also, small noise is added to the smooth input signal $A(0, T)$ to trigger the modulation instability effect required for the rogue waves generation.

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