

Random all-dielectric anti-reflective metasurfaces on the waveguide facet

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Multipole decomposition

Electric and magnetic fields can be described by scalar and vector potentials:

$$\Phi(\mathbf{R}, t) = \frac{1}{4\pi\epsilon\epsilon_0} \int_V \frac{\rho\left(\mathbf{r}, t - \frac{|\mathbf{R}-\mathbf{r}|}{v}\right)}{|\mathbf{R}-\mathbf{r}|} dV, \quad \mathbf{A}(\mathbf{R}, t) = \frac{\mu\mu_0}{4\pi} \int_V \frac{\mathbf{J}\left(\mathbf{r}, t - \frac{|\mathbf{R}-\mathbf{r}|}{v}\right)}{|\mathbf{R}-\mathbf{r}|} dV$$

Expansion of vector potential into Taylor series:

$$\mathbf{A}(\mathbf{R}, t) = \frac{\mu\mu_0}{4\pi R} \left[\int_V \mathbf{J} dV + \frac{R_i}{vR} \int_V \dot{\mathbf{J}} r_i dV + \frac{R_j R_k}{2v^2 R^2} \int_V \ddot{\mathbf{J}} r_j r_k dV + \frac{R_j R_k R_m}{6v^3 R^3} \int_V \dddot{\mathbf{J}} r_j r_k r_m dV + \dots \right]$$

Consider second term:
$$\begin{aligned} R_j \int_V \nabla (\mathbf{J} r_j) dV &= R_j \int_V (\dot{\rho} r_j + J_i R_j r_j + r_i R_j J_j) dV = \\ &= -\dot{Q}_{ij} R_j + 2 \int_V J_i (R_j r_j) dV + \int_V \underbrace{[r_i (R_j J_j) - J_i (R_j r_j)]}_{\mathbf{b}(\mathbf{a}\mathbf{c}) - \mathbf{c}(\mathbf{a}\mathbf{b}) = [\mathbf{a} \times \mathbf{b}] \times \mathbf{c}} dV = \\ &= -\dot{Q}_{ij} R_j + 2 \int_V J_i (R_j r_j) dV + \mathbf{R} \times \int_V [\mathbf{r} \times \mathbf{J}] dV \end{aligned}$$

Finally:
$$R_j \int_V J_i r_j dV = \frac{1}{2} \dot{Q}_{ij} R_j + [\mathbf{m} \times \mathbf{R}] + \frac{1}{2} U'_{ij} R_j$$

Multipole moments

Electric multipole moments:

$$\begin{aligned}d_i &= \int_V \rho(\mathbf{r}) r_i dV = \frac{1}{i\omega} \left(U_i + \int_V J_i dV \right) \\Q_{ij} &= \int_V \rho(\mathbf{r}) r_i r_j dV = \frac{1}{i\omega} \left(U'_{ij} + \int_V (J_i r_j + J_j r_i) dV \right) \\O_{ijk} &= \int_V \rho(\mathbf{r}) r_i r_j r_k dV = \frac{1}{i\omega} \left(U''_{ijk} + \int_V (J_i r_j r_k + r_i J_j r_k + r_i r_j J_k) dV \right)\end{aligned}$$

Magnetic multipole moments:

$$\begin{aligned}\mathbf{m} &= \frac{1}{2} \int_V [\mathbf{r} \times \mathbf{J}] dV \\M_{qm} &= \frac{2}{3} \int_V [\mathbf{r} \times \mathbf{J}]_q r_m dV\end{aligned}$$

Surface integrals:

$$\begin{aligned}U_i &= \oint_S (\mathbf{n}_S \cdot \mathbf{J}) r_i dS \\U'_{ij} &= \oint_S (\mathbf{n}_S \cdot \mathbf{J}) r_i r_j dS \\U''_{ijk} &= \oint_S (\mathbf{n}_S \cdot \mathbf{J}) r_i r_j r_k dS\end{aligned}$$

Multipole decomposition of electric and magnetic fields

Multipole decomposition of vector potential:

$$\mathbf{A}(\mathbf{R}, t) = \frac{\mu\mu_0}{4\pi R} \left[\dot{\mathbf{d}} + \mathbf{U} + \frac{1}{2v} \ddot{\mathbf{Q}}\mathbf{n} + \frac{1}{v} [\dot{\mathbf{m}} \times \mathbf{n}] + \frac{1}{2v} \dot{U}'\mathbf{n} + \frac{1}{6v^2} \ddot{\mathbf{O}}\mathbf{n}\mathbf{n} + \right. \\ \left. + \frac{1}{2v^2} [\mathbf{n} \times \ddot{\mathbf{M}}\mathbf{n}] + \frac{1}{6v^2} \ddot{U}''\mathbf{n}\mathbf{n} + \dots \right]$$

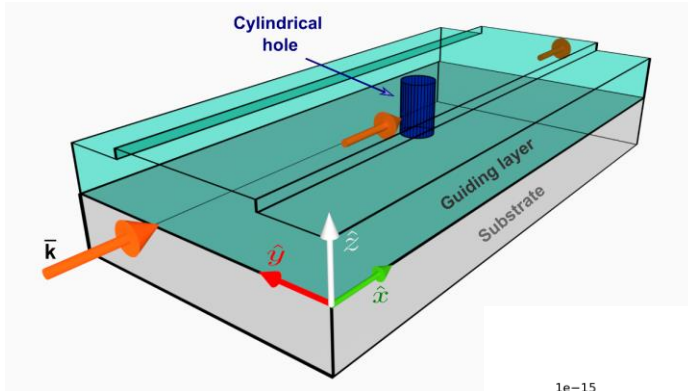
Multipole decomposition of electric and magnetic fields:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \text{grad } \varphi, \quad \mathbf{H} = \text{rot } \mathbf{A}$$

$$\mathbf{E} = \frac{k^2}{4\pi\epsilon\epsilon_0} \frac{e^{ikR}}{R} \left([\mathbf{n} \times [\mathbf{d} \times \mathbf{n}]] + \frac{i}{kv} [\mathbf{n} \times [\mathbf{U} \times \mathbf{n}]] + \frac{ik}{2} [\mathbf{n} \times [\mathbf{Q}\mathbf{n} \times \mathbf{n}]] + \frac{1}{v} [\mathbf{m} \times \mathbf{n}] + \right. \\ \left. + \frac{1}{2v} [\mathbf{n} \times [\mathbf{U}'\mathbf{n} \times \mathbf{n}]] + \frac{k^2}{6} [\mathbf{n} \times [\mathbf{n} \times \mathbf{O}\mathbf{n}\mathbf{n}]] + \frac{ik}{2v} [\mathbf{n} \times \mathbf{M}\mathbf{n}] + \frac{ik}{6v} [\mathbf{n} \times [\mathbf{n} \times \mathbf{U}''\mathbf{n}\mathbf{n}]] \right)$$

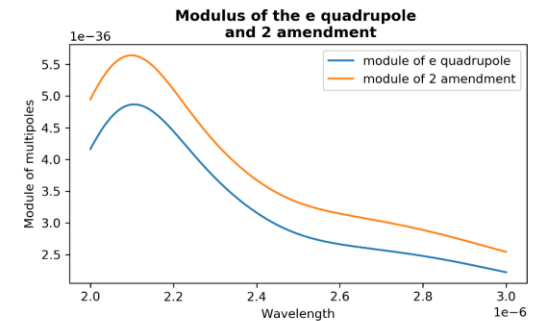
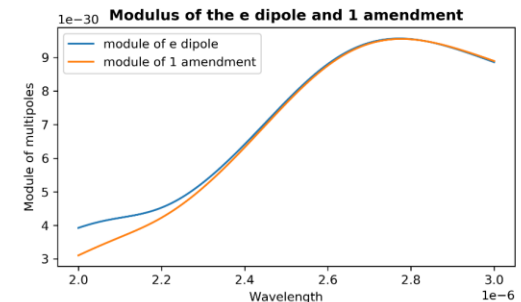
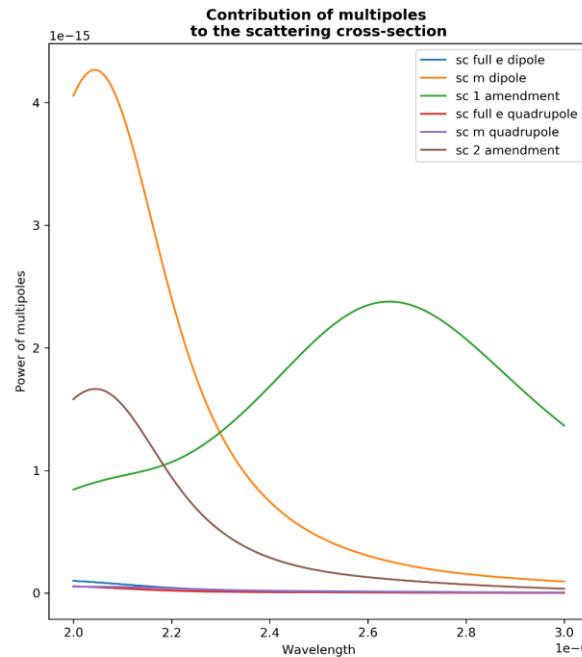
$$\mathbf{H} = \frac{\mu\mu_0}{4\pi R v} \frac{e^{ikR}}{R} \left([\ddot{\mathbf{d}} \times \mathbf{n}] + [\dot{\mathbf{U}} \times \mathbf{n}] + \frac{1}{2v} [\ddot{\mathbf{Q}}\mathbf{n} \times \mathbf{n}] + \frac{1}{v} [\mathbf{n} \times [\mathbf{n} \times \dot{\mathbf{m}}]] + \right. \\ \left. + \frac{1}{2v} [\ddot{U}'\mathbf{n} \times \mathbf{n}] + \frac{1}{6v^2} [\ddot{\mathbf{O}}\mathbf{n}\mathbf{n} \times \mathbf{n}] + \frac{1}{2v^2} [\mathbf{n} \times [\ddot{\mathbf{M}}\mathbf{n} \times \mathbf{n}]] + \frac{1}{6v^2} [\ddot{U}''\mathbf{n}\mathbf{n} \times \mathbf{n}] \right)$$

Multipole decomposition of the inclusion in the waveguide

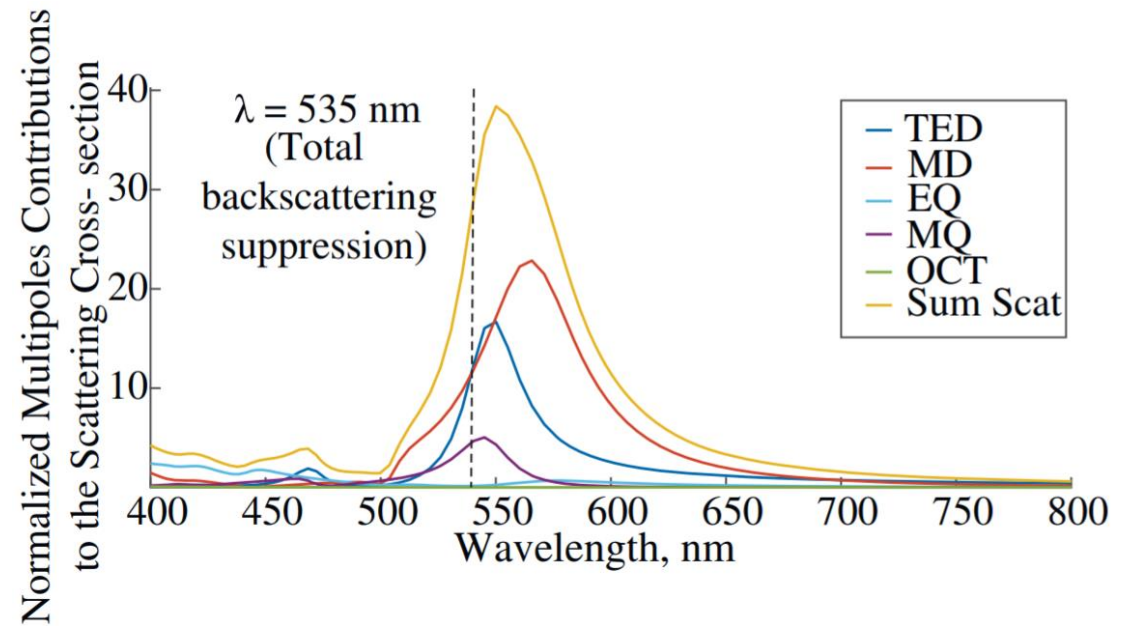
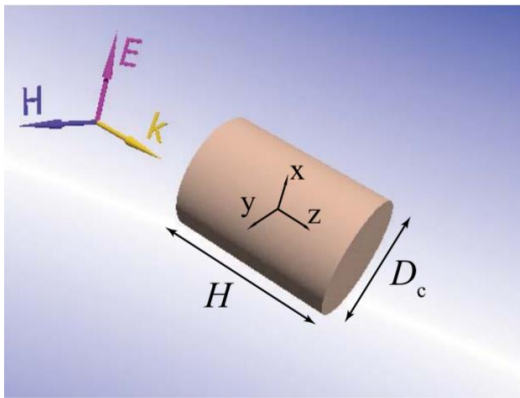


Cylindrical inclusion filled with air inside the waveguide.

Scattering cross-section:



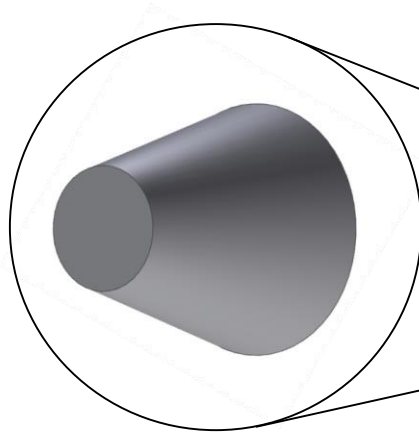
Kerker Effect



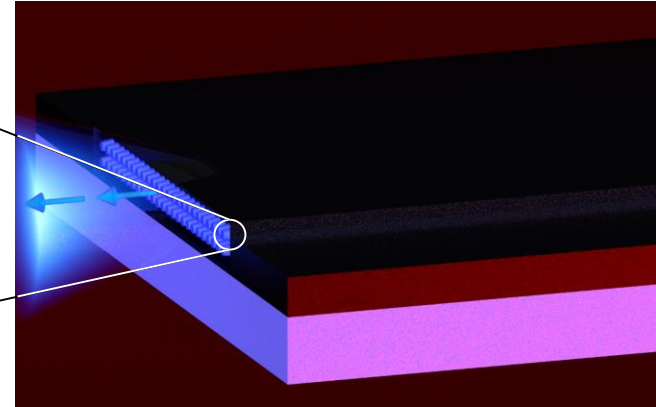
Kerker effect – is the suppression of the back scattering in case when electric and magnetic dipole moments are of the equal magnitude and opposite phase.

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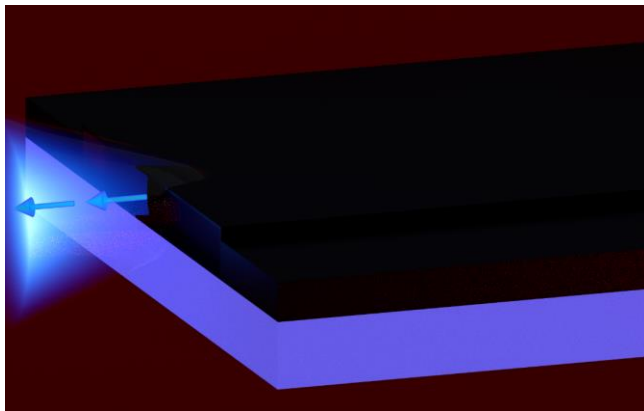
Various structures on the waveguide facets



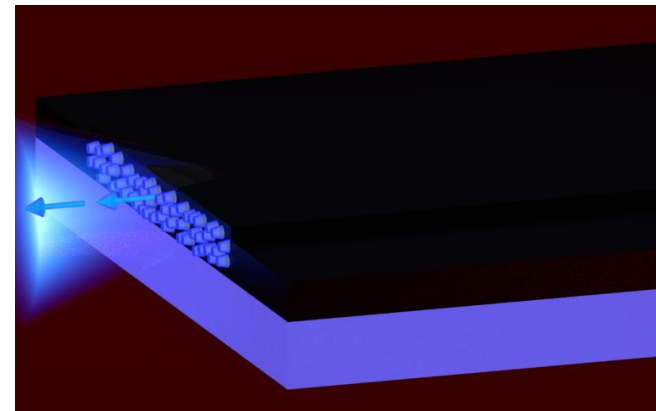
Silicon **Frustum**



Periodic structure on facet

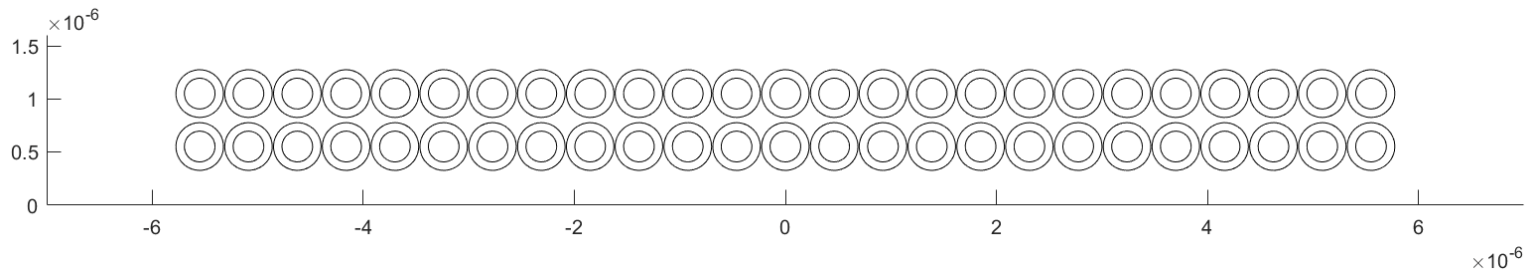


Flat facet

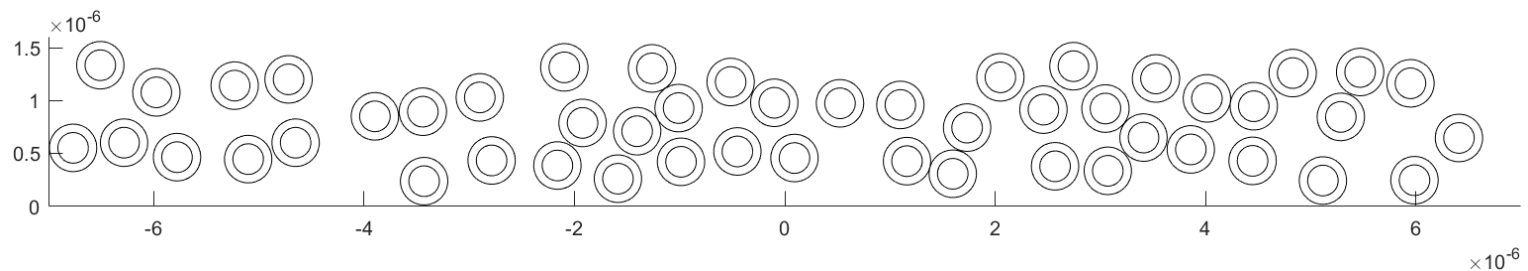


Random structure on facet

Periodic and random structures on waveguide facet



Periodic structure on the waveguide facet (Top view)

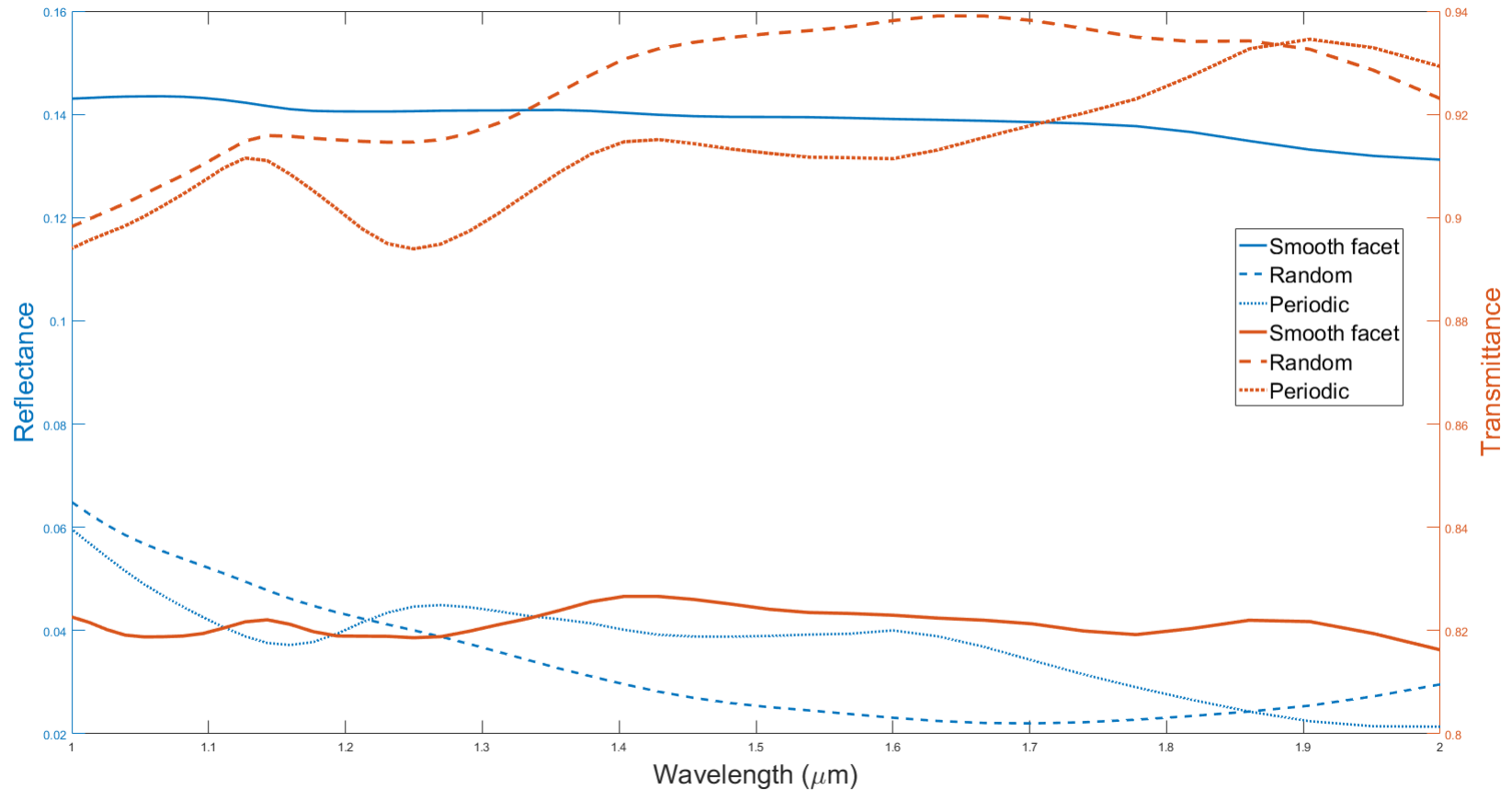


Random structure on the waveguide facet (Top view)

Frustum,
three-quarter view:



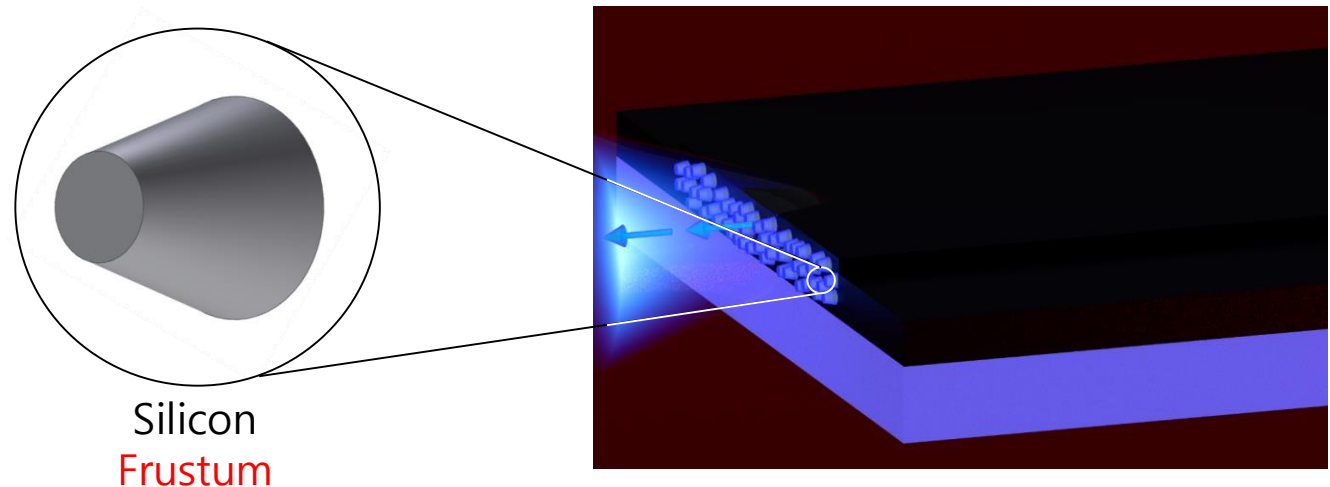
Transmittance and reflectance



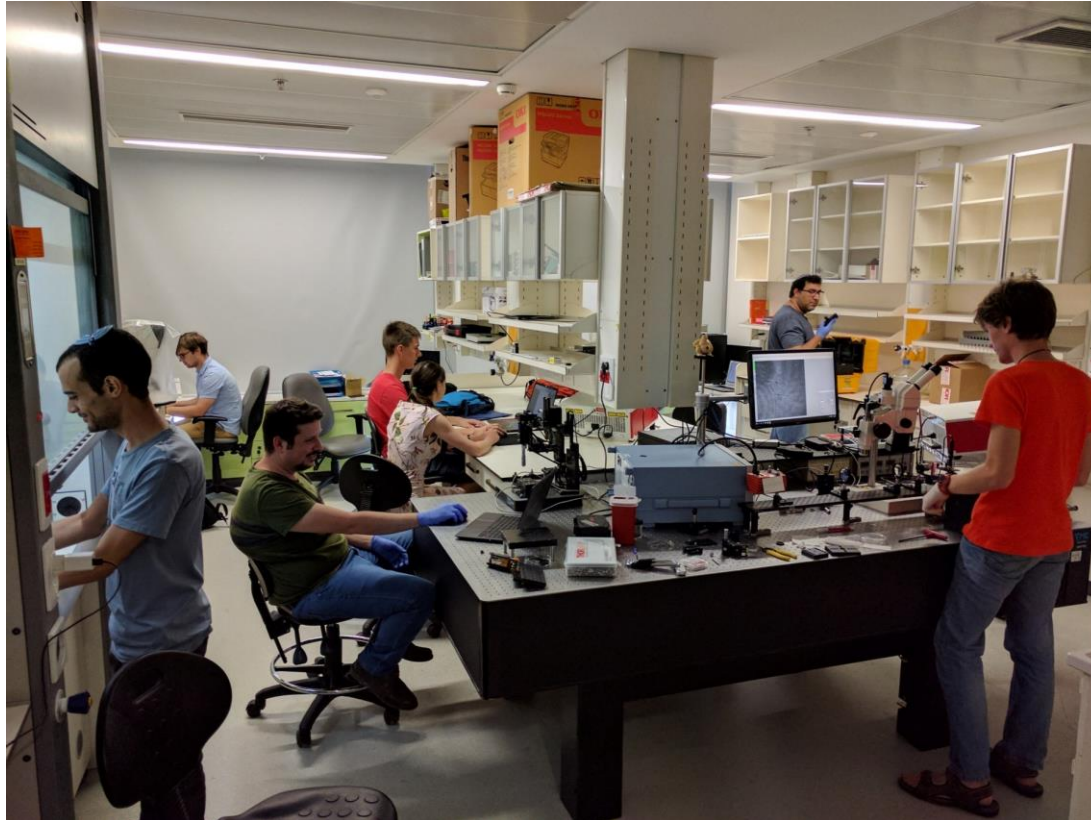
At $\lambda = 1,66 \mu\text{m}$ transmittance is up to 94%!

Conclusion

- **Amendments** were proposed to the classical multipole decomposition to allow for analysis of open systems in which charges and currents are not localised.
- In case of an **inclusion filled with** air on an optical waveguide, electric multipole moments are replaced by the surface integrals.
- **Random** metasurfaces engraved on Si waveguide facet appear to allow for superior anti-reflective properties (as high as 94% transmittance) compared to the periodic metasurface, (90% transmittance) compared to the smooth facet (82% transmittance) .



Thank you for your attention!



Our Light-on-a-Chip team at Ben-Gurion University, Israel