Simulation task

Write in details your solution and the development of equations.

Given a planar waveguide with infinite layers. Wave is propagating in the waveguide with wavelength of 1.5 µm while $n_2 >> n_3$ as shown in Figure 1:

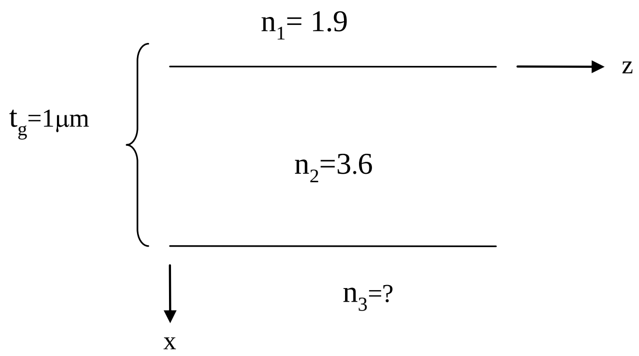

Figure 1 – planar waveguide with infinite layer illuminated by λ=1.5 µm

a. Find a refractive index n_3 that allows for propagation of three TE modes in the waveguide. Find the propagation constants (β) in the z direction for these modes.
b. Give an explicit expression for $E_y(x)$ at each layer for the modes you found in (a) (including the amplitude and the propagation constant in the z direction).
c. Draw $E_y(x)$ for each mode.
d. By using n_3 from section a, find the propagation constants (β) in the z direction for possible TM modes $H_y(x)$. Did you obtain the same propagation constants for the two polarizations? If not explain qualitatively.
e. Give an explicit expression for $H_y(x)$ at each layer for the modes you found (including the amplitude and the propagation constant in the z direction).
f. Draw $H_y(x)$ for each mode.
g. Draw $E_y(x)$ and $H_y(x)$ for the different modes on the same graph.
h. Draw $E_y(x,z)$ (in the size of two wavelengths without time dependence) from a chosen mode. (3D graph)
i. In this section, describe shortly the steps to express the function of $H_y(x)$ for each layer for TM polarization (similar to the expression that we did in the lecture for TE polarization). The final formula is written at the end of this section. Follow the missing development steps according to the guidance in Appendix 1.

Appendix 1:

Start from Maxwell equation: $\nabla^2\vec{H} = \mu_0 {n_i}^2 \varepsilon_0 \frac{d^2\vec{H}}{dt^2}$

A. Assume: 1. In the waveguide, a harmonic wave is propagating in the z direction.

2. The waveguide is infinite in the y direction and the magnetic field doesn't change in this direction $\left(\frac{\partial\vec{H}}{\partial y} = 0\right)$.

Show that the propagating electric field can be written as: $\vec{H}(x,y,x) = \vec{H}(x)e^{-j(\omega t-\beta z)}$

B. Use the fact that this are TM modes**(?)**

$$E(x) = \left\{E_x(x), \underbrace{E_y(y)}_{0}, E_z(z)\right\}$$

$$H(x) = \left\{\underbrace{H_x(x)}_{0}, H_y(y), \underbrace{H_z(z)}_{0}\right\}$$

Show that:

$$\frac{\partial^2 H_y(x)}{\partial x^2} + \left({n_i}^2{k_0}^2 - \beta^2\right)H_y(x) = 0$$

C. **Show that** the general solution of the equations in each region is:

$$H_y(x) = A\exp(-qx) \qquad 0 \le x \le \infty$$

$$H_y(x) = B\cos(hx) + C\sin(hx) \qquad -t_g \le x < 0$$

$$H_y(x) = D\exp[p(x+t_g)] \qquad -\infty \le x < -t_g$$

D. Write equations for q, p and h.

E. Use the boundary conditions for the continuity of the magnetic field for x=0 and x=-t_g.

Show that:

1) $A = B$

2) $D = B\cos\left(h\cdot t_g\right) - C\sin\left(h\cdot t_g\right)$

F. Use the boundary conditions for the continuity of the electric field for x=0 and x=-t_g while

$$E_x(x) = \frac{\beta}{\omega {n_i}^2 \varepsilon_0} H_y \hat{x} \qquad E_z(x) = \frac{j}{\omega {n_i}^2 \varepsilon_0} \frac{\partial}{\partial x} H_y \hat{z}$$

Show that:

3) $-A \frac{q}{{n_3}^2} = C \frac{h}{{n_2}^2} \quad \rightarrow \quad C = -A \frac{q}{h} \left(\frac{n_2}{n_3}\right)^2$

4) $A \frac{h}{{n_2}^2} \sin(h \cdot t_g) + C \frac{h}{{n_2}^2} \cos(h \cdot t_g) = \frac{P}{{n_1}^2} [A\sin(h \cdot t_g) + C\cos(h \cdot t_g)]$

G. Use equations 3 and 4 and **show that:**

$$\tan(h \cdot t_g) = \frac{\frac{p}{{n_1}^2} + \frac{q}{{n_3}^2}}{\left[\frac{h}{{n_2}^2} - \frac{pq}{h}\left(\frac{n_2}{n_3 n_1}\right)^2\right]} = \frac{h{n_2}^2({n_3}^2 p + {n_1}^2 q)}{{n_3}^2 {n_1}^2 h^2 - {n_2}^4 qp} = \frac{h(p' + q')}{h^2 - q'p'}$$

Where

$$q' = q\left(\frac{n_2}{n_3}\right)^2 \qquad p' = p\left(\frac{n_2}{n_1}\right)^2$$

H. Use equations 1 and 2 and **show that:**

$$H_y(x) = A\exp(-qx) \qquad 0 \le x \le \infty$$

$$H_y(x) = A\left[\cos(hx) - \frac{q}{h}\left(\frac{n_2}{n_3}\right)^2 \sin(hx)\right] \qquad -t_g \le x < 0$$

$$H_y(x) = A\left[\cos(h \cdot t_g) + \frac{q}{h}\left(\frac{n_2}{n_3}\right)^2 \sin(h \cdot t_g)\right] \exp[p(x + t_g)] \qquad -\infty \le x < -t_g$$

Summary:

The final formula to the magnetic field for TM modes in the waveguide.

$$H_y(x) = C_m\exp(-qx) \qquad 0 \le x \le \infty$$

$$H_y(x) = C_m\left[\cos(hx) - \frac{q}{h}\left(\frac{n_2}{n_3}\right)^2 \sin(hx)\right] \qquad -t_g \le x < 0$$

$$H_y(x) = C_m\left[\cos(h \cdot t_g) + \frac{q}{h}\left(\frac{n_2}{n_3}\right)^2 \sin(h \cdot t_g)\right] \exp[p(x + t_g)] \qquad -\infty \le x < -t_g$$

Where (from developing the total power and normalizing. Don't need to calculate.)

$$C_m = 2\sqrt{\frac{\omega \varepsilon_0 P_{\text{total}}}{\beta t'_g}}$$

$$t'_g = \left(\frac{h^2 + q'^2}{h^2}\right)\left[\frac{h^2 + q^2}{n_3{}^2 q\left(h^2 + q'^2\right)} + \frac{h^2 + p^2}{n_1{}^2 p\left(h^2 + p'^2\right)} + \frac{t_g}{n_2{}^2}\right]$$

$$q' = q\left(\frac{n_2}{n_3}\right)^2 \quad p' = p\left(\frac{n_2}{n_1}\right)^2$$