The impulse response of the ZOH device is

$$h_{\text{zoh}}(t) = \begin{cases} 1, & 0 \le t < T, \\ 0, & \text{otherwise.} \end{cases} \tag{3.33}$$

The impulse response (3.33) is shown in Figure 3.11; the response $\hat{x}(t)$ to the input sequence $x(nT)$ is shown in Figure 3.12. The latter figure also shows, in a dashed line, the ideal waveform $x(t)$, which would be obtained by a Shannon reconstructor.

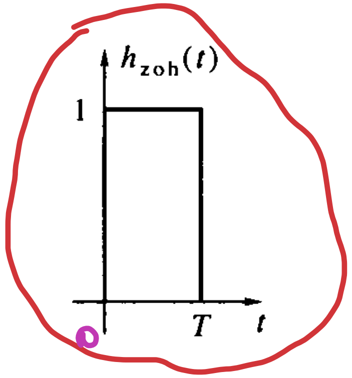


Figure 3.11 Impulse response of a zero-order hold.

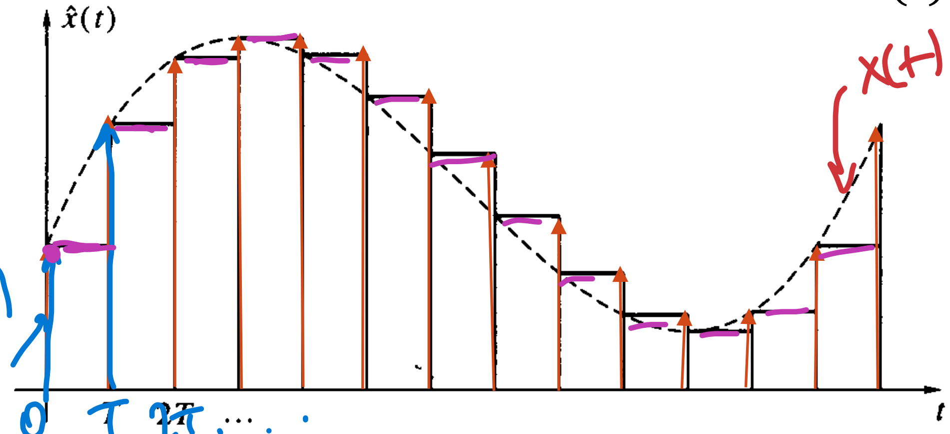


Figure 3.12 Time response of a zero-order hold. Staircase line: actual response $\hat{x}(t)$; dashed line: ideal response $x(t)$.

Zero-order hold (ZOH)

שחזור מסדר אפס

- השיחזור לפי שנון דורש אינסוף דגימות אחורה וקדימה, ולכן שיחזור שנון אינו מעשי.

$$\hat{x}(t) = x_s(t) * h(t)$$

n≠0 ∀ nT≤ t

n≤t/T

due to causality

- משחזר מעשי – יהיה סיבתי: $\hat{x}(t) = \sum_{n=-\infty}^{t/T} x(nT)h(t-nT)$
- במשחזר טוב $\hat{x}(t)$ דומה ל $x(t)$
- משחזר פשוט: ZOH $\hat{x}(t) = x[n], nT \le t < (n+1)T, n \in \mathbb{Z}$

Question: what is the frequency response of the $H\{h_{ZOH}(t)\}$?

The frequency response of the $H\{h_{ZOH}(t)\}$

To understand to what extent $\hat{x}(t)$ approximates $x(t)$, let us compute the frequency response of the ZOH. As we recall, the frequency response of the ideal (Shannon) reconstructor is a perfect rectangle on $-\pi/T \leq \omega \leq \pi/T$, with zero phase; see (3.25). By analyzing how the frequency response of the ZOH deviates from the perfect rectangle, we will understand the nature of distortions introduced by the ZOH. We have from (3.33)

Euler=$\sin(z)=\frac{e^{jz}-e^{-jz}}{2j}$

$$\hat{x}(t) = x_s(t) * h(t) \longrightarrow \hat{X}(j\omega) = X_s(j\omega) \cdot H(j\omega)$$

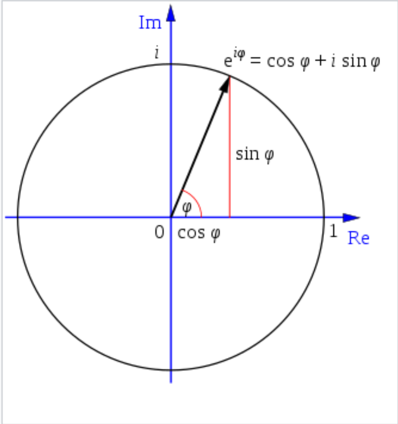
FT{h(t)} =

$$H(j\omega) = \int_0^T 1 \cdot e^{-j\omega t}\, dt = \frac{1-e^{-j\omega T}}{j\omega} = T \cdot e^{-j0.5\omega T} \cdot \frac{e^{j0.5\omega T} - e^{-j0.5\omega T}}{2 \cdot 0.5\, j\omega T} = T\, e^{-j0.5\omega T} \frac{\sin(0.5\omega T)}{0.5\omega T} =$$

$$= T\, e^{-j0.5\omega T} \frac{\sin(\pi\omega T/\pi 2)}{\pi\omega T/\pi 2} = T \cdot \operatorname{sinc}\left(\frac{\omega}{\omega_s}\right) \cdot e^{-j0.5\omega T} = T\, sinc\left(\frac{\omega T}{2\pi}\right) e^{-j0.5\omega T}$$

$\omega_s = \frac{2\pi}{T}$

LEONHARD EULER — HISTORICAL NOTE



A geometric interpretation of Euler's formula

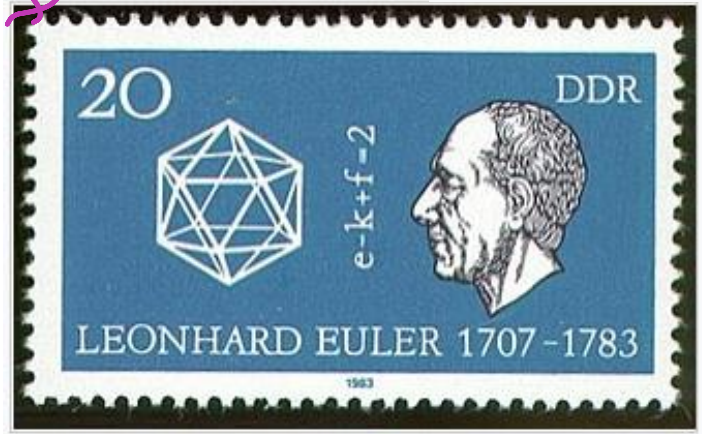
Leonhard Euler

Portrait by Jakob Emanuel Handmann (1753)

Born	15 April 1707 Basel, Switzerland
Died	18 September 1783 (aged 76) [OS: 7 September 1783] Saint Petersburg, Russian Empire
Alma mater	University of Basel (MPhil)

Saint Petersburg

1957 Soviet Union stamp commemorating the 250th birthday of Euler. The text says: 250 years from the birth of the great mathematician, academician Leonhard Euler.



Stamp of the former German Democratic Republic honoring Euler on the 200th anniversary of his death. Across the centre it shows his polyhedral formula, in English written as "$v - e + f = 2$".

Euler

RECONSTRUCTION

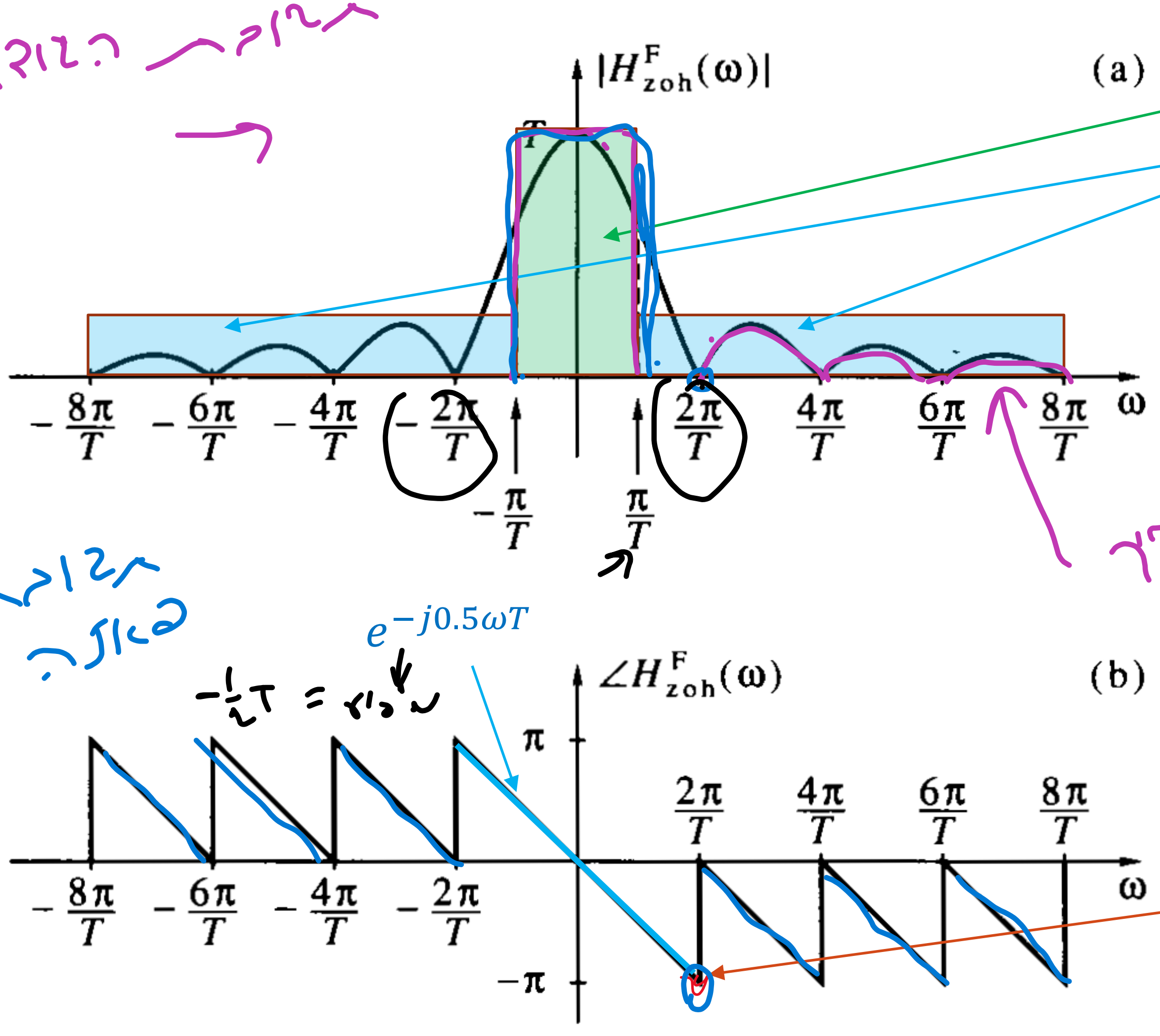


Figure 3.13 Frequency response of a zero-order hold: (a) magnitude; (b) phase.

- בתחום $|\omega| \le \omega_s/2$ הגבר קטן או שווה ל T
- בתחום $|\omega| > \omega_s/2$ הגבר שונה מ 0
- תדרים גבוהים – שינוי חד בזמן (מדרגות)
- משמעות : שכפולים לא מתאפסים – רק מונחתים
- פאזה – סינק ב ± 1

DIGITAL-TO-ANALOG CONVERSION

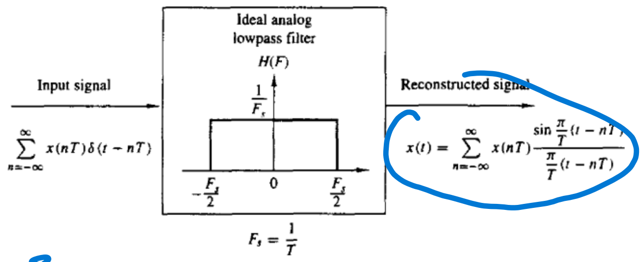


Figure 9.18 Signal reconstruction viewed as a filtering process.

MAGNITUDE AND PHASE COMPARISON TO IDEAL LOW-PASS FILTER

1. הגבר: קבוע ושווה ל T בתחום $|\omega| \le \omega_s/2$.

2. הגבר: שווה ל 0 בתחום $|\omega| > \frac{\omega_s}{2}$.

3. חלקים בגבולות ← שינוי חד בהגבר (מדרגה).

4. משמעות: פיגונים אינם מעגלים אלא רק מותחים.

5. פאזה מאחרת? ליניארית | פס שיהוי 0.5T - בתחום

$|\omega| \le \omega_s/2$

MAGNITUDE AND PHASE COMPARISON TO IDEAL LOW-PASS FILTER

The magnitude and phase responses of $H^{\mathrm{F}}_{\mathrm{zoh}}(\omega)$ are shown in Figure 3.13. We observe the following differences with respect to the ideal low-pass filter:

1. The magnitude response at low frequencies is not flat, but decays gradually. Furthermore, it decreases to zero at $\omega = \pm 2\pi/T$, rather than at $\pm\pi/T$.
2. The magnitude response has nonvanishing ripple at high frequencies, so the reconstructed signal $\hat{x}(t)$ has undesired high-frequency energy. In the time domain, the high-frequency energy is apparent in the staircaselike form of the output created by the hold operation.
3. The phase of the response is not zero, but piecewise linear, with slope $-0.5T$.

FULL RECONSTRUCTION FROM THE STAIRCASE SIGNAL

- Is it possible to reconstruct the $X(j\omega)$ from $\hat{X}(j\omega)$?

$$\hat{X}(j\omega) \overset{?}{=} X_s(j\omega) \cdot H(j\omega) \cdot G(j\omega)$$

- From ideal sampling and reconstruction by sinc we will request $G(j\omega) \cdot H(j\omega) = T \cdot \Pi\left(\frac{\omega}{\omega_s}\right)$

we found before that $H(j\omega) = T\text{sinc}\left(\frac{\omega T}{2\pi}\right) e^{-j0.5\omega T}$

$1/\omega_s$

and will obtain $G(j\omega) = \dfrac{\Pi\left(\frac{\omega}{\omega_s}\right)}{\text{sinc}\left(\frac{\omega}{\omega_s}\right)} e^{j0.5\omega T}$

$\frac{\sin(x)}{x}$ *correction*

not causal

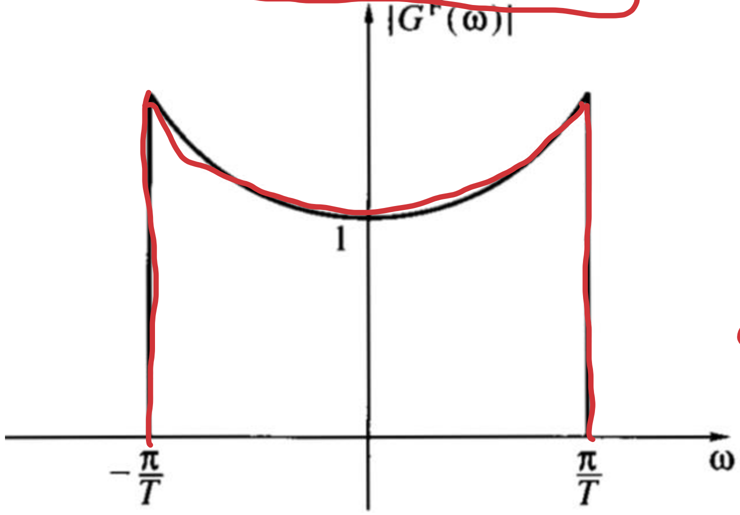


Figure 3.14 Magnitude response of an ideal reconstruction filter at the output of a ZOH.

PHYSICAL SAMPLING RECONSTRUCTION

Physical reconstruction is implemented using a device called *digital-to-analog converter*, or D/A. A D/A converter approximates zero-order hold operation. It accepts a discrete-time signal $x[n]$ in a form of a sequence of binary numbers. Each binary number is held fixed by a data register for a period of T seconds (the sampling interval). The binary number at the register's output is converted to a voltage waveform $\hat{x}(t)$. The voltage is approximately proportional to the present value of $x[n]$, and remains fixed for T seconds. When the next binary number $x[n+1]$ appears at the register's output, $\hat{x}(t)$ changes accordingly. The result is approximately the staircase waveform shown in Figure 3.12. Figure 3.16 depicts this sequence of operations schematically.

A/D converter =

D/A converter =

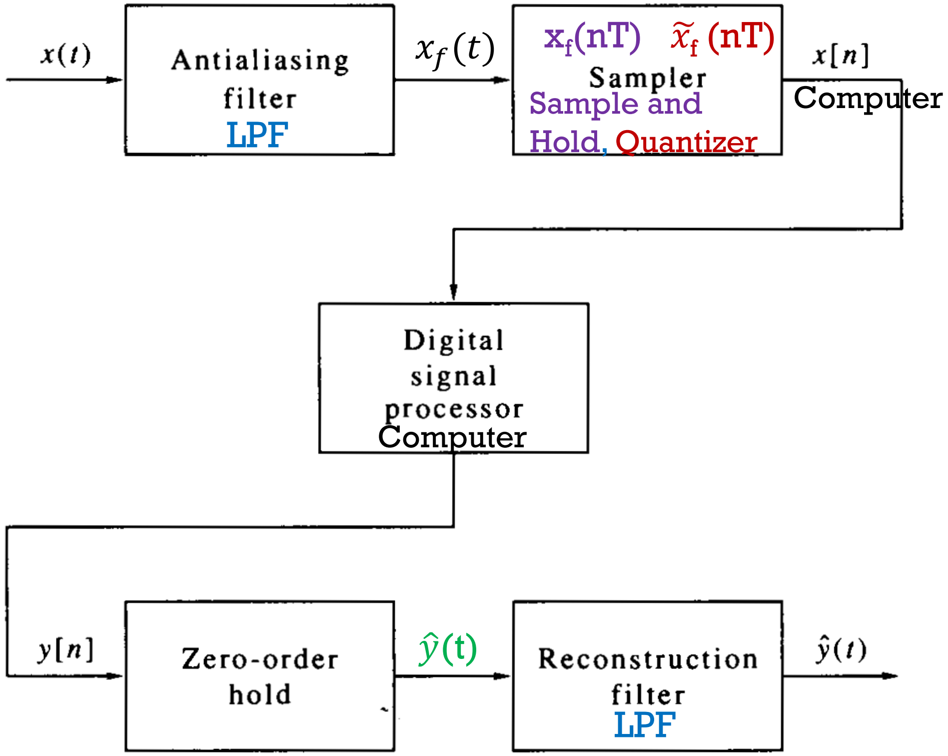


Figure 3.15 A typical digital signal processing system.

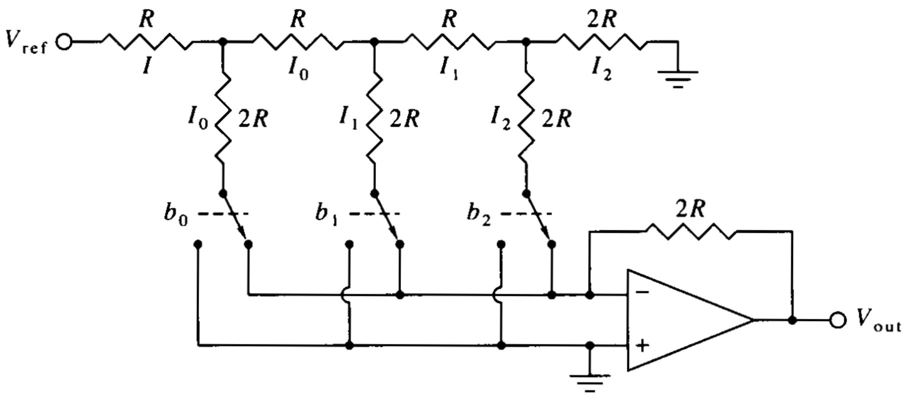


Figure 3.17 Schematic diagram of a digital-to-analog converter.

PHYSICAL SAMPLING RECONSTRUCTION

Physical reconstruction is implemented using a device called *digital-to-analog converter*, or D/A. A D/A converter approximates zero-order hold operation. It accepts a discrete-time signal $x[n]$ in a form of a sequence of binary numbers. Each binary number is held fixed by a data register for a period of T seconds (the sampling interval). The binary number at the register's output is converted to a voltage waveform $\hat{x}(t)$. The voltage is approximately proportional to the present value of $x[n]$, and remains fixed for T seconds. When the next binary number $x[n+1]$ appears at the register's output, $\hat{x}(t)$ changes accordingly. The result is approximately the staircase waveform shown in Figure 3.12. Figure 3.16 depicts this sequence of operations schematically.

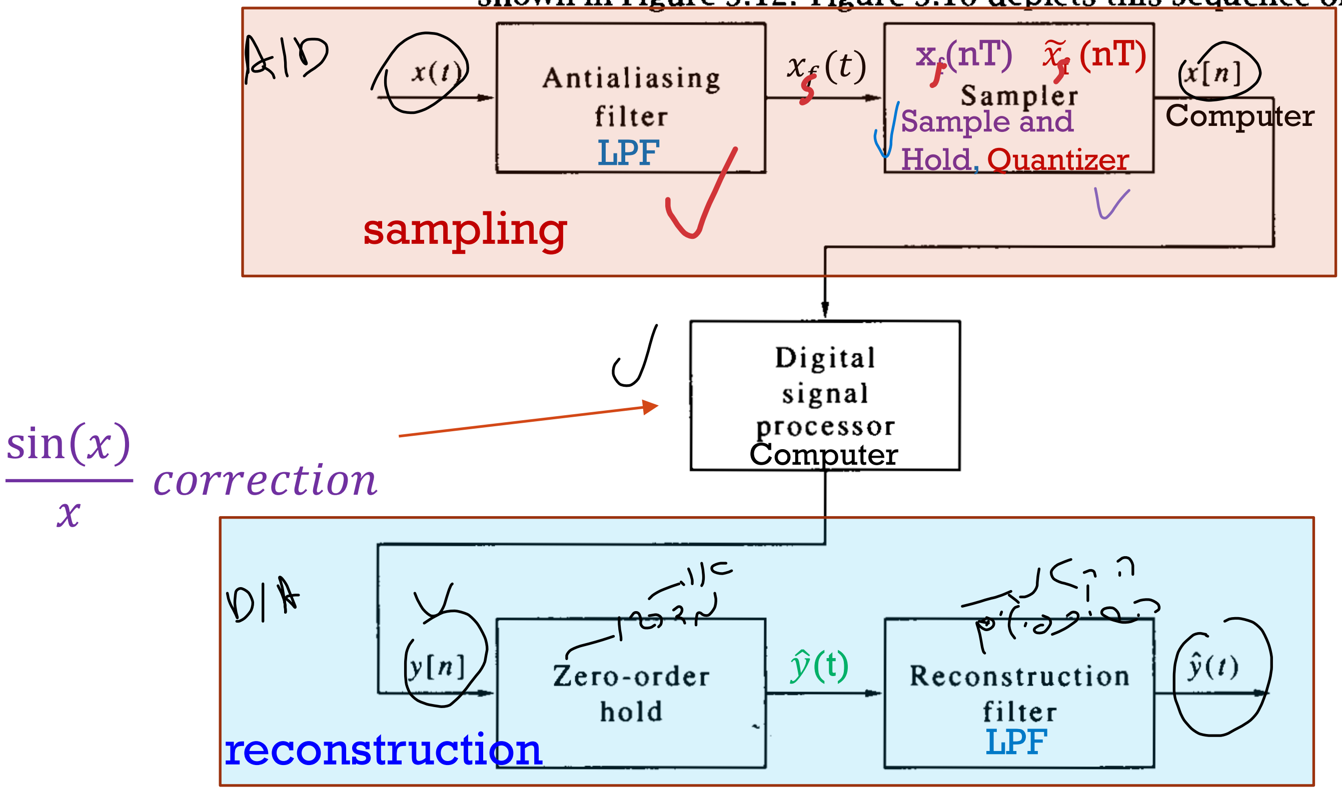


Figure 3.15 A typical digital signal processing system.

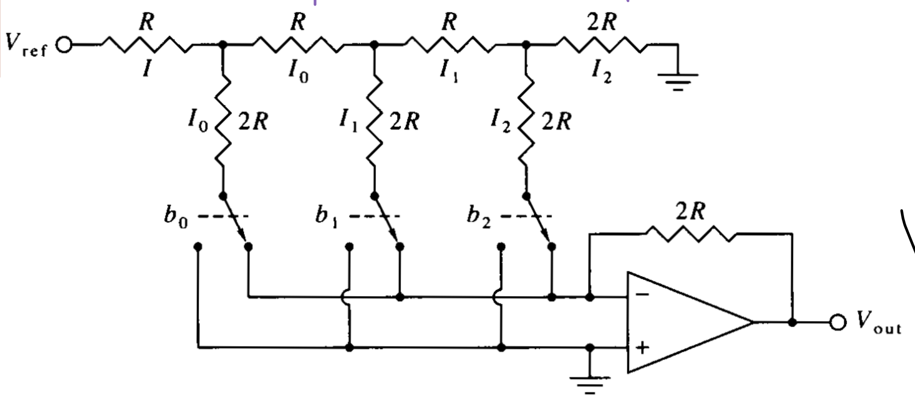


Figure 3.17 Schematic diagram of a digital-to-analog converter.

PRACTICAL RECONSTRUCTION

In practice, the sampling of an analog signal is performed by a sample-and-hold (S/H) circuit. The sampled signal is then quantized and converted to digital form. Usually, the S/H is integrated into the A/D converter.

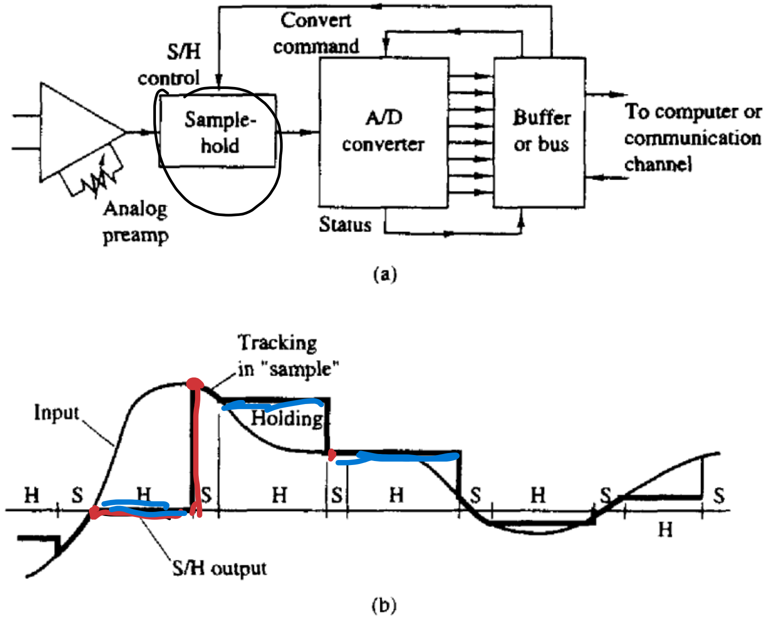


Figure 9.6 (a) Block diagram of basic elements of an A/D converter; (b) time-domain response of an ideal S/H circuit.

ותגובת התדר שלו היא:

$$H_{zoh}^{F}(\omega)=\int_{0}^{T}e^{-j\omega t}dt=\frac{1-e^{-j\omega T}}{j\omega}$$

$$=\frac{\sin(\omega T/2)}{\omega/2}e^{-j\omega T/2}$$

$$=T\,\mathrm{sinc}\left(\frac{\omega T}{2\pi}\right)e^{-j\omega T/2}$$

ציורים המתארים את $h_{zoh}(t)$, דוגמת שחזור ותגובת התדר מופיעים להלן:

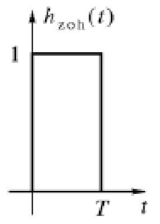


Figure 3.11 Impulse response of a zero-order hold.

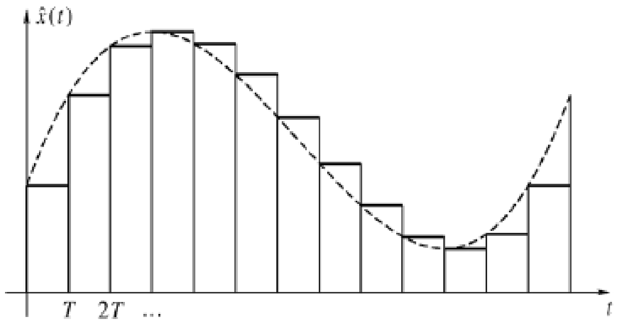


Figure 3.12 Time response of a zero-order hold. Staircase line: actual response $\hat{x}(t)$; dashed line: ideal response $x(t)$.

האיורים לעיל מתוך ספרו של B. Porat

The Origins of the Sampling Theorem

Hans Dieter Lüke, Aachen University of Technology

ABSTRACT Fifty years ago the publications of Claude E. Shannon brought the sampling theorem to the broad attention of communication engineers. This article demonstrates how practicians, theoreticians, and mathematicians discovered the implications of the sampling theorem almost independent of one another.

In 1948 and 1949, Claude E. Shannon published the two revolutionary papers in which he founded the information theory [1, 2]. In [1] the sampling theorem is formulated as "Theorem 13":

Let f(t) contain no frequencies over W. Then

$$f(t) = \sum_{-\infty}^{\infty} X_n \frac{\sin \pi(2Wt - n)}{\pi(2Wt - n)}$$

where

$$X_n = f\left(\frac{n}{2W}\right).$$

It was not until these papers were published that the theorem known as "Shannon's sampling theorem" became common property among communication engineers, although Shannon himself writes in [2] that

This is a fact which is common knowledge in the communication art.

A few lines further on, however, he adds:

... but in spite of its evident importance [it] seems not to have appeared explicitly in the literature of communication theory.

The following analysis takes the above statement as its starting point. It will become apparent that mathematicians, practicians, and theoreticians in communication engineering came across the implications of the sampling theorem almost independent of one another, and that the links between them did not emerge until later stages of this development.

The Practicians

In communication engineering, the first experiments with time-division multiplexing (TDM) in telephony led to the questions of how and how often it is necessary to sample a continuous-time signal.

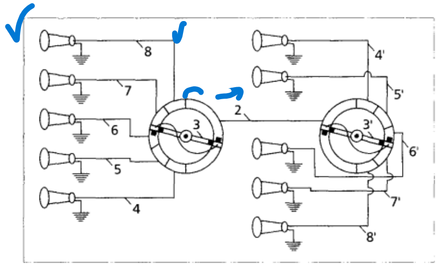
The attempt to transmit more than one signal simultaneously over a single wire began shortly after the early commercial successes with telegraphy in the 1840s. The first proposals for TDM using synchronously rotating commutators derive from F. C. Bakewell (1848), A. V. Newton (1851), and M. B. Farmer (1853). Technically more accomplished methods were then developed by B. Meyer (1870), J. M. E. Baudot (to 1874), as well as P. Lacour and P. B. Delany (1878) [3, 4]. It is significant not only that methods were used in which complete telegraphic signals from different transmitters were placed in chronological order (e.g., Baudot), but that certain systems were also equipped with fast rotating commutators which were able to transmit at least two samples of each elementary signal (e.g., Delany). This technique makes additional synchronization between transmitter and sampler unnecessary. One of these fast rotating commutators, the "distributor" of the telegraphy system by F. J. Patten (around 1891), was used for the first demonstration of TDM of telephone signals. The inventor's name was Willard M. Miner. He had his method patented in 1903 following many years of preliminary experiments [5]. Figures 1 and 2 from [6] show the circuit diagram and the "Patten Distributor" which was used. Miner determined the required sampling rate experimentally [6]:

It will be understood, then, that the apparatus devised by Mr. Miner, while in its general form the same as that heretofore used for multiplex telegraphy — or telephony for that matter — such apparatus is run at a much greater speed so as to bring the frequency of the closures of connection upon the several branches or sub-circuits up to a rate approximating in greater or less degree the rate of the vibrations of the overtones characterizing speech. A rate of closure of 1,000 or 2,000 per second will not answer the purpose, but as the rate increases and passes beyond 3,000, improved results become apparent, and are markedly better when a rate of 3,500 or 3,600 per second is reached; the best results being obtained with a rate of about 4,300 per second.

Miner thus assumed that the sampling rate would coincide approximately with the upper frequency components of speech. In actuality, his telephone apparatus will have had a cutoff frequency of barely more than 2 kHz, which fulfills the requirements of the sampling theorem.

Since a theoretical clarification of the sampling process was not forthcoming, pronouncements concerning the sampling rate in publications as well as patent applications for TDM of speech signals remained similarly vague right up until the 1930s. For example, L. von Kramolin in 1923 writes in a patent on TDM:

... therefore it is possible to work with a switching speed which lies beyond the limits of audibility, whereby the switching noise in the individual telephones is avoided and noiseless communication is possible.



■ **Figure 1.** *Circuit diagram.*

Editorial Liaison: J. O'Reilly

0163-6804/99/$10.00 © 1999 IEEE

Telephone French prediction for the year 2000

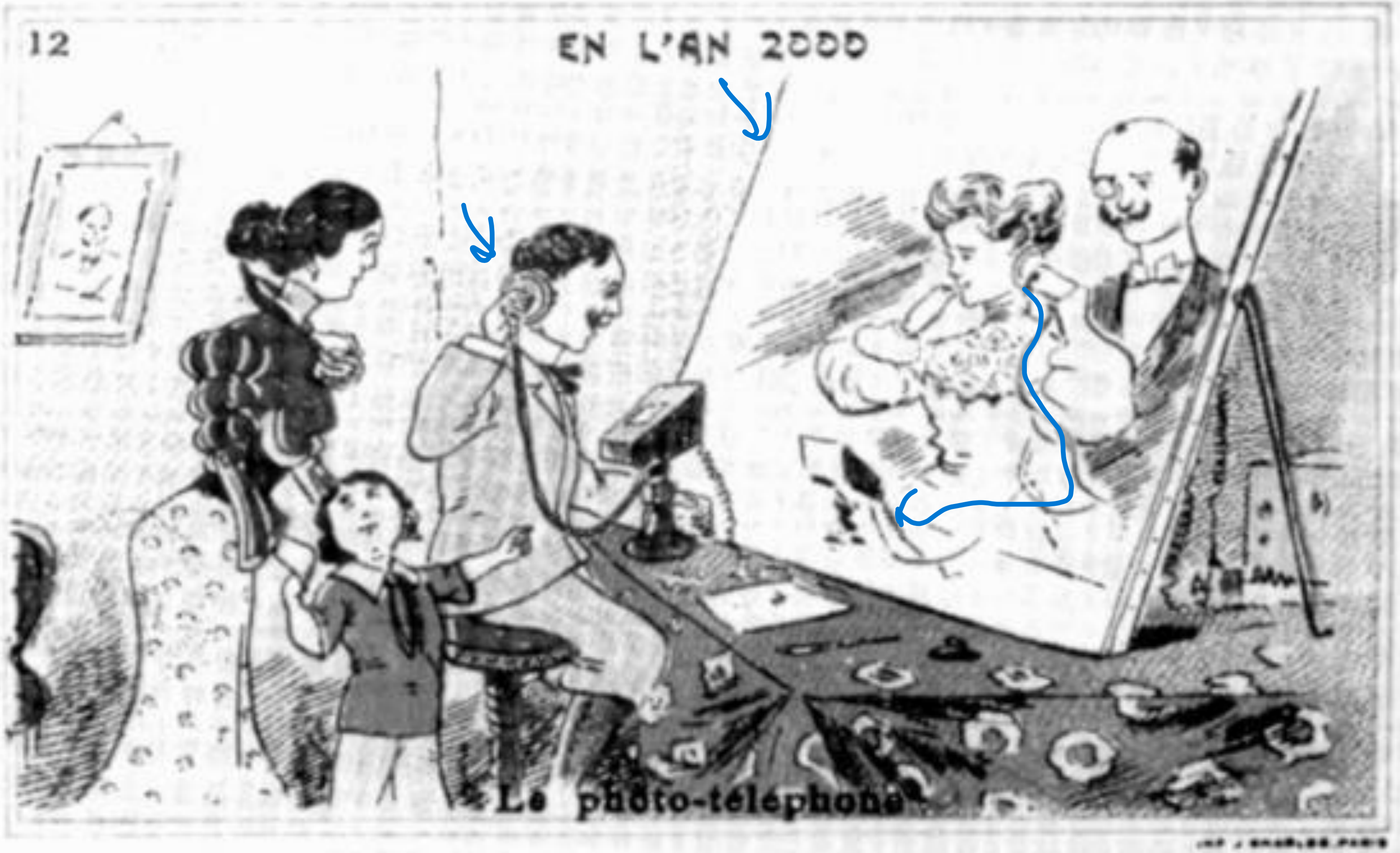


Fig. 17.2 A French prediction for the year 2000.

Le photo-telephone

SURVEY: RECONSTRUCTION OF BAND-LIMITED SIGNALS

- EasyPolls:

z(t)=[x(t)p(t)*h(t), with h(0)=1 and h(nT)=0 for all |n|>0. Can x(t) be reconstructed from z(t)? Assume x(t) is band-limited to pi/T.

- ✓ Yes
- No
- Sometimes, depending on the h(t)

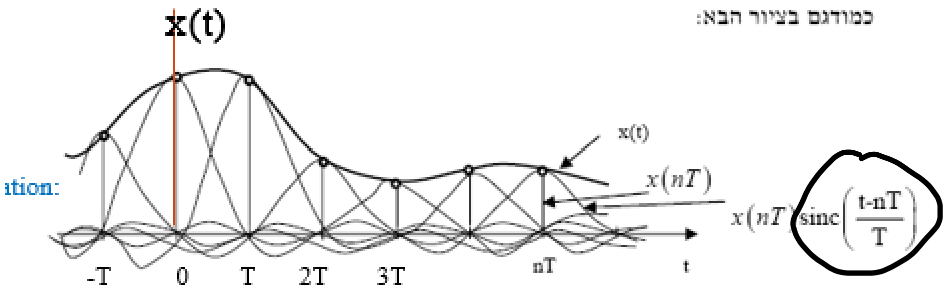
results vote

תשובה: כן

Shannon

שאלה: ... האם ניתן לשחזר את x(t) מתוך z(t)?

כמודגם בציור הבא:

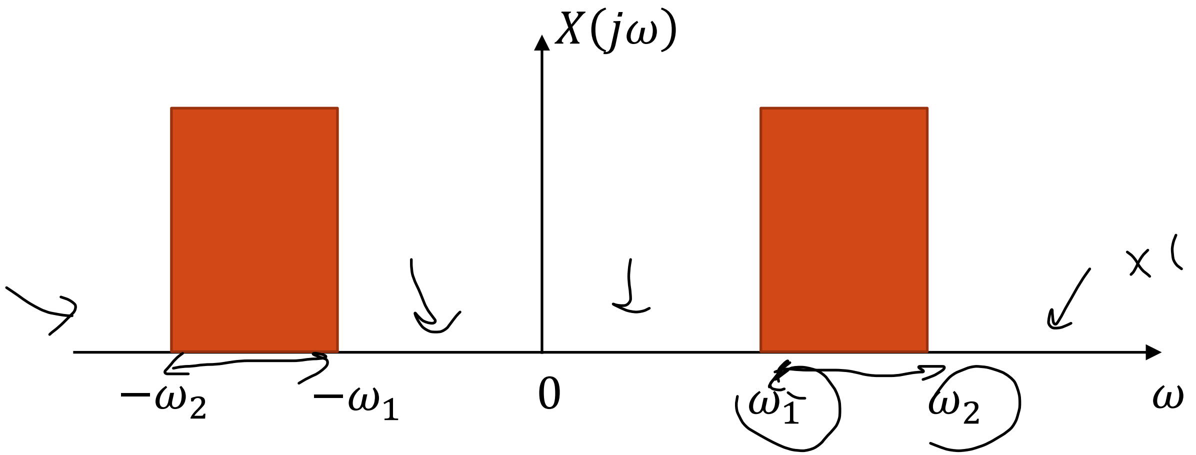


SAMPLING BAND-PASS SIGNALS דגימת אותות חסומי סרט

- אות חסום סרט (bandpass signal) הוא אות מוגבל סרט, ללא אנרגיה סביב DCתדר 0, כלומר

$$X(j\omega) = 0 \;\; \forall|\omega| \le \omega_1, |\omega| \ge \omega_2$$
$$0 < \omega_1 < \omega_2$$

Example:
modulated communication signals



- תנאי נייקויסט לאות זה $\omega_s \ge 2\omega_2$

Nyquist condition for such a signal

Can be sampled for smaller than Nyquist frequency: 1) minimizing memory 2) simple implementation etc

SAMPLING BAND-PASS SIGNALS דגימת אותות

חסומי סרט

- נניח בשלב זה שהתדר המקסימלי הוא כפולה שלמה של רוחב הסרט: $\omega_2 = L(\omega_2 - \omega_1), L \in \mathbb{N}$

Natural number =
whole and positive

- נדגום בתדר כפול $\omega_s = 2(\omega_2 - \omega_1) = 2\omega_2/L$

מרוחב הסרט

Here instead of twice the maximal frequency we will sample the twice bandwidth

כלומר $T = \frac{2\pi}{\omega_s} = \frac{\pi L}{\omega_2}$

פי L נמוך מתדר נייקוויסט!

הספקטרום של האות הדגום במקרה זה:

$$X_s(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j\omega - jk\omega_s) = \frac{1}{T}\sum_{x=-\infty}^{\infty} X[j\omega - jk2(\omega_2 - \omega_1)]$$

כל שיכפול שונה מ0 כאשר $\omega_1 < |\omega - 2k(\omega_2 - \omega_1)| < \omega_2$

SAMPLING BAND-PASS SIGNALS – GRAPHICAL

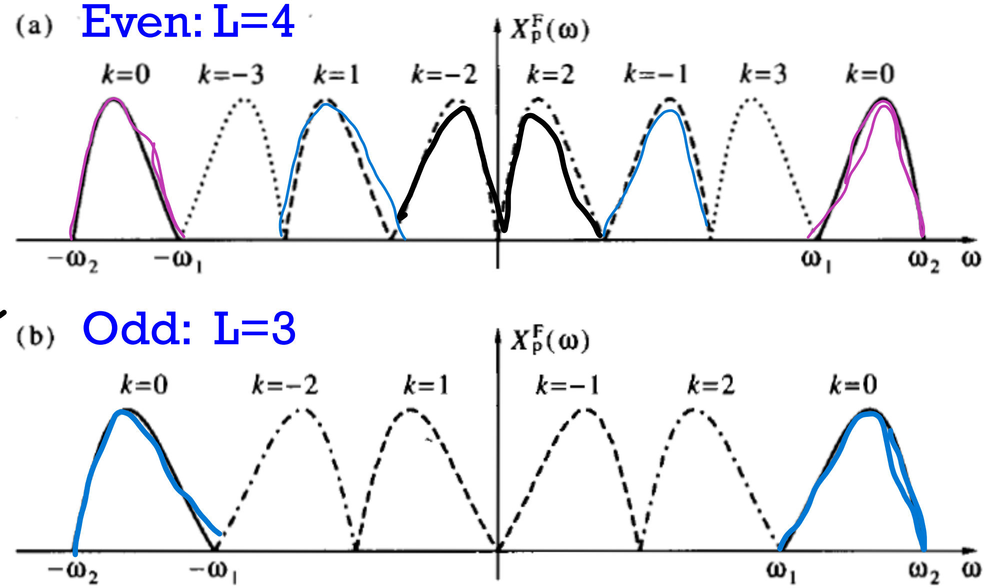


Figure 3.26 Sampling of a band-pass signal below the Nyquist rate when $(\omega_2 - \omega_1)$ is an integer multiple of the bandwidth: (a) $L = 4$; (b) $L = 3$.

This is illustrated in Figure 3.26 for even L (4 in this case) and odd L (3 in this case). As we see, the replicas corresponding to different values of k *do not overlap*. In particular, the replicas corresponding to $k = \pm L/2$ if L is even, or $k = \pm(L-1)/2$ if L is odd, appear in the interval

$$-(\omega_2 - \omega_1) \le \omega \le \omega_2 - \omega_1,$$

which corresponds to the interval $[-\pi, \pi]$ in the θ domain. The conclusion is that the sampled signal *is not aliased*, despite being sampled at a rate smaller than Nyquist's critical rate.

הספקטרום של האות הדגום במקרה זה:

$$X_s(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X(j\omega - jk\omega_s) = \frac{1}{T}\sum_{x=-\infty}^{\infty} X[j\omega - jk2(\omega_2 - \omega_1)]$$

כל שיכפול שונה מ0 כאשר $\omega_1 < |\omega - 2k(\omega_{2n} - \omega_1)| < \omega_2$

השיכפולים לכן לא יחפפו

SAMPLING BAND-PASS SIGNALS BELOW NYQUIST RATE: $\omega_2 \neq L \cdot (\omega_2 - \omega_1)$

<u>תרגיל:</u>

מה עושים כאשר $\omega_2 \neq L \cdot (\omega_2 - \omega_1)$?

נבחר $\omega_0 < \omega_1$ כך שיתקיים $\omega_2 = L \cdot (\omega_2 - \omega_0)$ ונחזור על התהליך

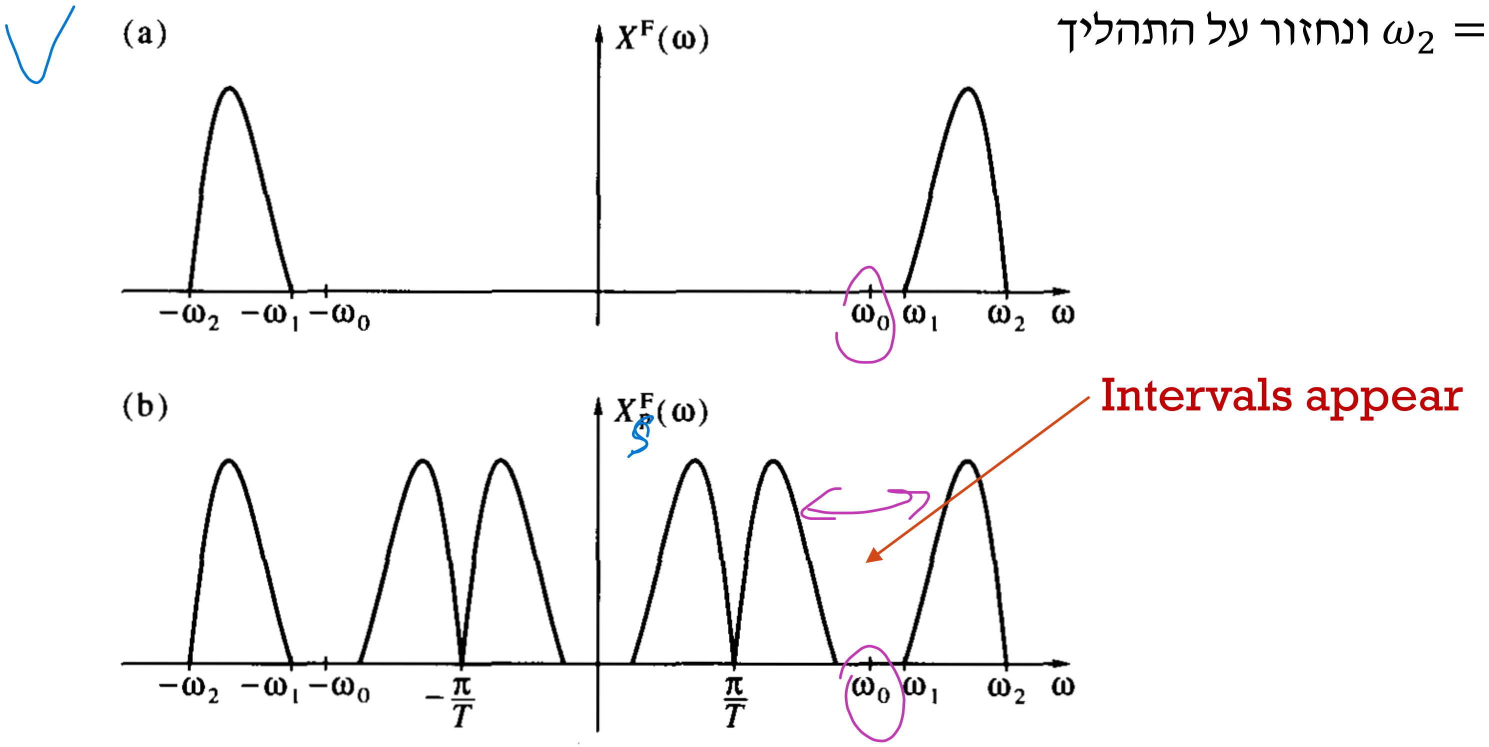


Figure 3.27 Sampling of a band-pass signal below the Nyquist rate in a general case: (a) Fourier transform of the continuous-time signal; (b) Fourier transform of the sampled signal.

EXAMPLE: RADIO

Example 3.12 Radio receivers, since the invention of the *superheterodyne*, convert their high-frequency input signal to an intermediate frequency, or IF. The great advantage of the superheterodyne is that the IF frequency is constant for the entire range of frequencies of the input signal. This makes it easy to amplify the signal and control its bandwidth; hence most of the amplification is done at the IF stage. The IF signal is, like the high-frequency input signal, modulated by the information signal, which is usually low frequency. Traditionally, the IF output is demodulated and low-pass filtered to provide the information signal.

Modern digital communication receivers often rely on digital processing of the information. At the time of writing, there is a growing interest in *direct IF sampling*, as an alternative to the procedure of demodulation followed by low-pass filtering and sampling. Direct IF sampling has the potential of eliminating costly and space-consuming analog hardware. For example, suppose that the IF frequency is 1 MHz and the information bandwidth is 38.4 kHz. In this case we have

$$L = \lfloor 1019.2/38.4 \rfloor = 26,$$

and the sampling frequency is

$$f_{\text{sam}} = \frac{2 \times 1019.2}{26} = 78.4\,\text{kHz}.$$

EXAMPLE: RECONSTRUCTION WITH BANDPASS FILTER (BPF)

The sample-and-hold time (or the conversion time for a flash A/D) should be about 0.1 microsecond or less in this case. □

When sampling a band-pass signal, there is seldom a need to reconstruct it in the pass band (usually either there is no need for reconstruction at all, or reconstruction is needed in the base band). If band-pass reconstruction is needed, it can be performed by passing the sampled signal through an ideal band-pass filter with frequency response

$$H^{\mathrm{F}}(\omega) = \begin{cases} T, & \omega_0 \le |\omega| \le \omega_2, \\ 0, & \text{otherwise}, \end{cases} \tag{3.56}$$

see Figure 3.28. The computation of the corresponding impulse response $h(t)$ is discussed in Problem 3.34.

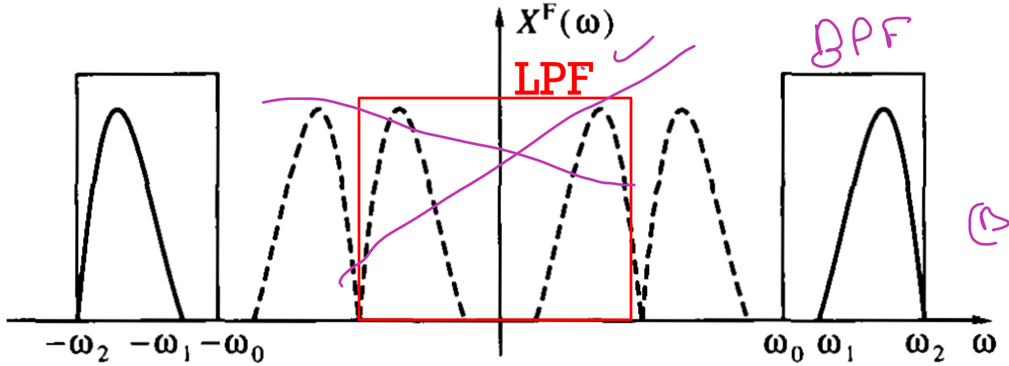
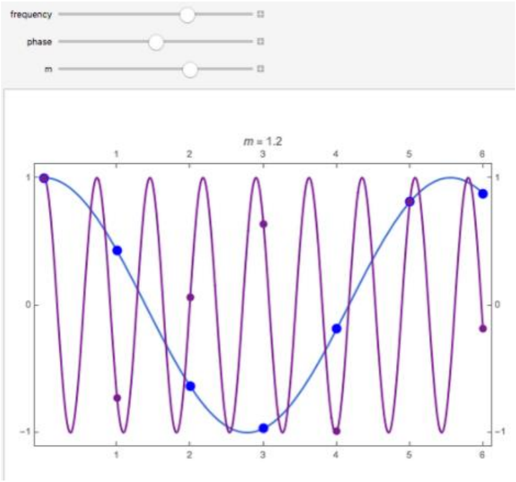
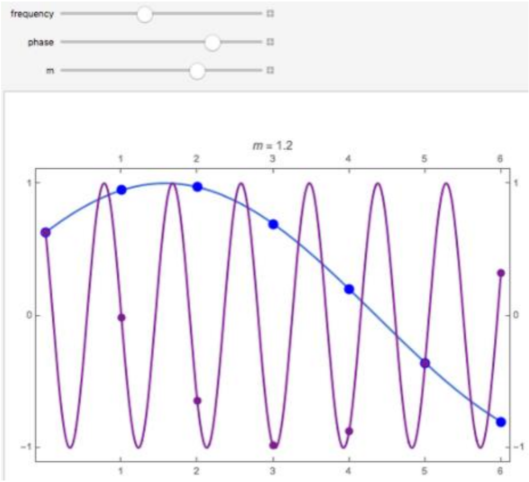


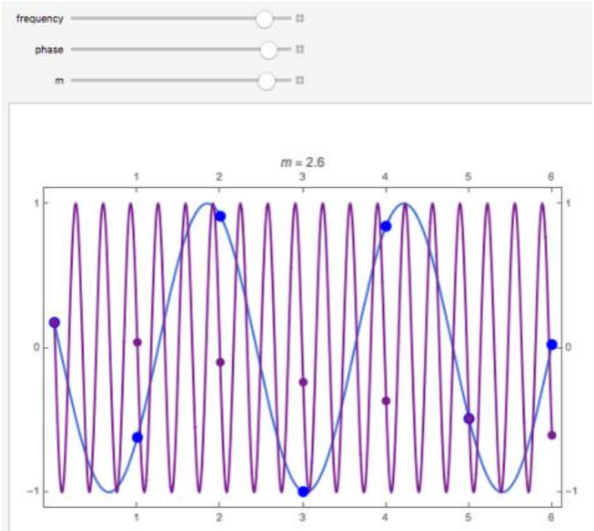
Figure 3.28 Reconstruction of a band-pass signal by an ideal band-pass filter (replicas shown by dashed lines will be eliminated by the filter).

ALIASING IN TIME SERIES ANALYSIS

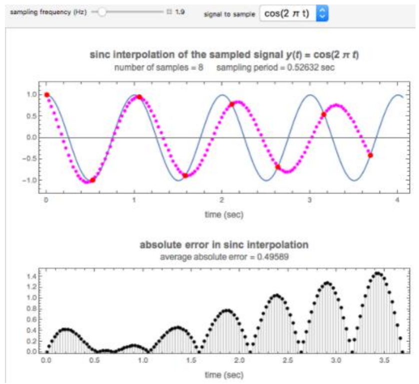
Given a power spectrum (a plot of power vs. frequency), aliasing is a false translation of power falling in some frequency range ($-f_c$, f_c) outside the range. **Aliasing** is caused by discrete sampling below the Nyquist frequency. It can be minimized by either increasing the underlying sampling rate or (if that is not practical or possible) pre-filtering the signal to suppress high-frequency components.





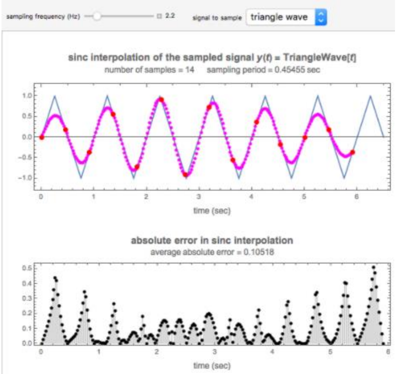


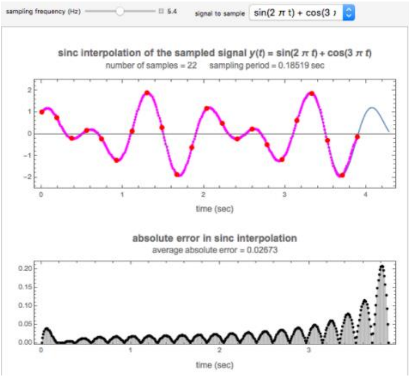
The relationship between two sinusoidal signals $a_t = \cos(2\pi f + \phi)$ and $b_t = \cos(2\pi(f + m) + \phi)$ is shown for $|f| \le 0.5$, $|\phi| \le \pi/2$, and $|m| \le 3$. The signals are assumed to be observed at times $t = 0, 1, \ldots, 6$ and the observed points are indicated. When m is an integer the points on the curve coincide and the signals are said to be aliased. Considering all frequencies in the range $|f| \le f_m$, the largest value of f_m so that all signals may be identified from the observed points is called the Nyquist frequency. It is well known that $f_m = 0.5$.



SINC INTERPOLATION FOR SIGNAL RECONSTRUCTION

The function $\text{sinc}(x)$ is defined by $\text{sinc}(x) = \sin(x)/x$ for $x \neq 0$, with $\text{sinc}(0) = 1$. The sinc interpolation formula is defined as $x(t) = \sum_{n=-\infty}^{\infty} x_n \, \text{sinc}\left(\frac{\pi}{T}(t - nT)\right)$, where T is the sampling period used to determine x_n from the original signal, and $x(t)$ is the reconstructed signal. The above formula represents a linear convolution between the sequence x_n and scaled and shifted samples of the sinc function. In this Demonstration, a limited number of samples x_n are generated, and the above sum is carried out for N samples, labeled from $k = 0$ to $k = N - 1$. Due to the shifting of the sinc function by integer multiples of T, this results in $x(t)$ having the exact value of a sample located at a multiple of T. This can be seen by observing that the absolute error is always zero at times which are integer multiples of T, in other words at the sample locations. In this implementation, the sinc function is sampled at a much higher rate than the sampling frequency used for the original function, in order to produce a smoother plotted result.





sinc interpolation formula reconstructs a continuous signal from some of its samples. The formula provides exact reconstructions for signals that are bandlimited and whose samples were obtained using the required Nyquist sampling frequency, to eliminate aliasing in the reconstruction of the signal.

SURVEY: RECONSTRUCTION OF BAND-LIMITED SIGNALS

- EasyPolls:

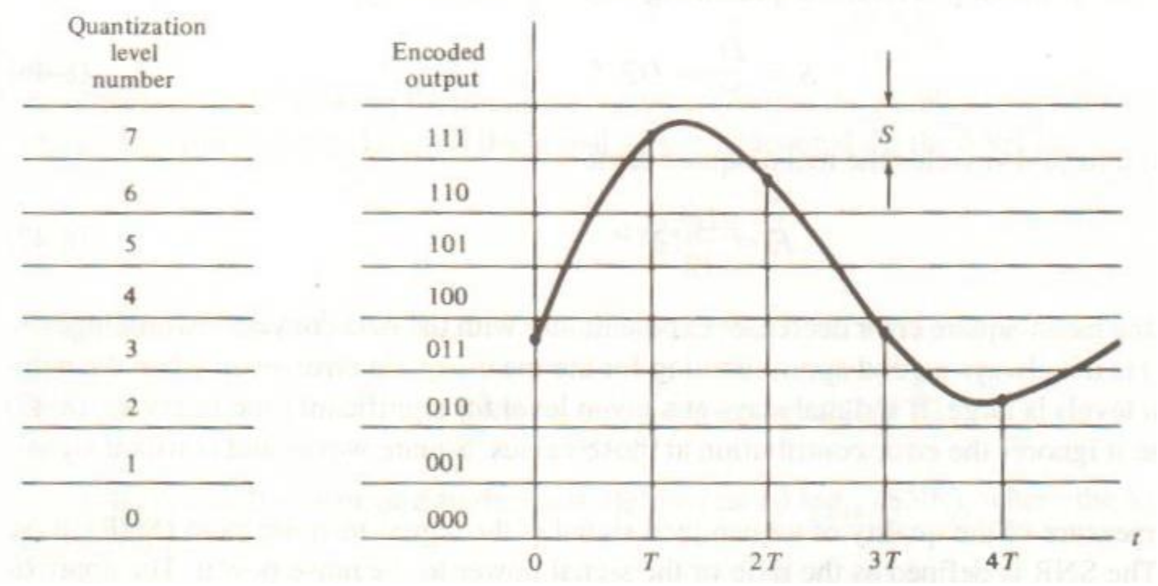
A real signal in the range 4-5 kHz can be sampled at f_s=2kHz with no aliasing. Will this sampling frequency change if the signal was complex, satisfying X(jw)=0, w<0?

- Yes, f_s should be higher
- Yes, f_s can be lower
- No, f_s is the same

results vote

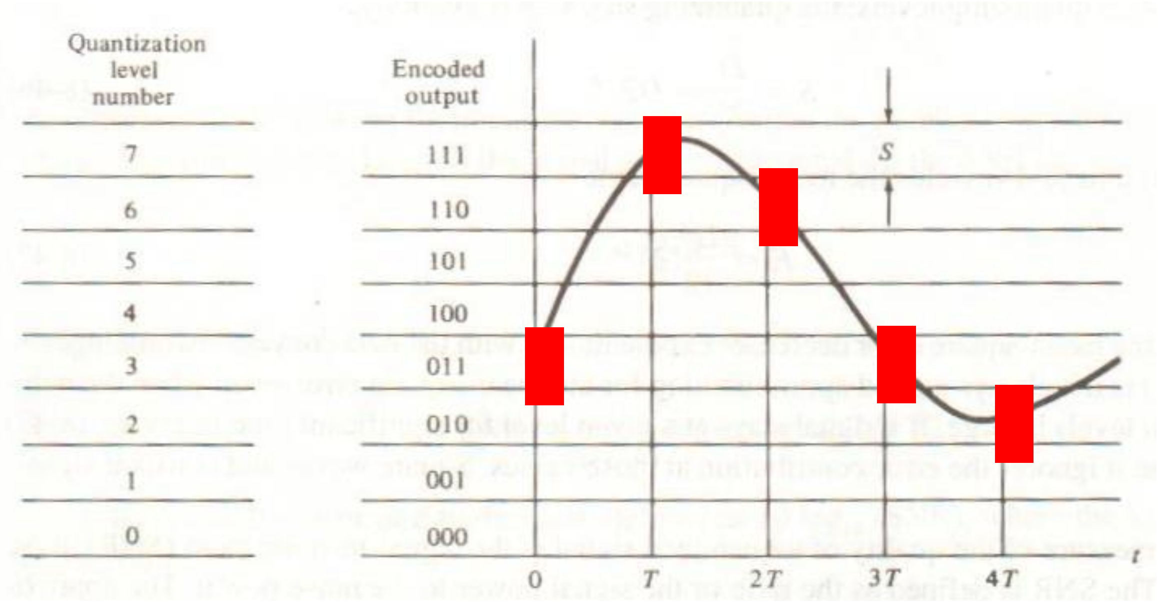
ENCODING

- The computer will store the data in n bits
- This means that we should have 2^n levels of quantization
  - In this example we use 3 bits for 8 levels



QUANTIZATION

- When we encode, we remove information about where in the quantization range the sample fell
  - So, encoding and quantization happen basically simultaneously
- Sample value is replaced by mean of quantization range
  - This is a limit on the amount of error caused by the A/D process

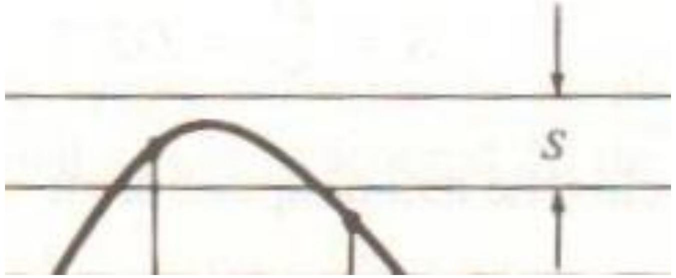


ERROR CAUSED BY QUANTIZATION

- s is the width of the quantization range
- Average squared error due to quantization
  - Integrate probability of each value times squared error at that value
- Ultimately, error depends only on quantization range

$$E = \int_{x_0-\frac{S}{2}}^{x_0+\frac{S}{2}} P(x)(x-x_0)^2 dx = \int_{x_0-\frac{S}{2}}^{x_0+\frac{S}{2}} \frac{1}{S}(x-x_0)^2 dx$$

$$= \frac{(x-x_0)^3}{3S}\Bigg|_{x_0-\frac{S}{2}}^{x_0+\frac{S}{2}} = \frac{S^2}{12}$$



ERROR IN TERMS OF THE DYNAMIC RANGE AND WORD LENGTH

- Dynamic range, D
  - the difference between the maximum and minimum values that can be represented by the converter
- Word length, n
  - the number of bits used in encoding
- Error decreases exponentially with word length

$$E = \frac{S^2}{12} = \frac{1}{12}\left(\frac{D}{2^n}\right)^2 = \frac{D^2}{12}2^{-2n}$$

EXPRESSING QUANTIZATION ERROR AS A SIGNAL-TO-NOISE RATIO (SNR)

- Noise
  - Squared difference between input signal and output signal
- Signal: size of output
  - For large word length, input and output size are the same
  - P_s, power of the input signal
- Results are independent of input signal
- Quantizer range should be as small as possible to accommodate signal size
- Reduce SNR by 6dB for each bit of word length

$$SNR = \frac{P_s}{E} = \frac{P_s}{\left(D^2/12\right)2^{-2n}} = 12 P_s D^{-2} 2^{2n}$$

$$SNR_{dB} = 10\log_{10} SNR = 10.8 + 10\log_{10} P_s - 20\log_{10} D + 6n$$