LECTURE 3 - OPTICAL MODES AND LOSSES

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Fundamentals of Fiber Optics Communication 377-2-5060

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OUTLINE

- Mode analysis with EM theory
- Wave propagation
- Single-mode fibers
- Birefringence
- Losses

MAXWELL'S EQUATIONS

Like all electromagnetic phenomena, propagation of optical fields in fibers is governed by Maxwell's equations. For a nonconducting medium without free charges:

Faraday's law $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1)$$

Ampere-Maxwell law $$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} \quad (2)$$

Gauss law $$\nabla \cdot \vec{D} = 0 \quad (3)$$

Gauss's law for magnetism $$\nabla \cdot \vec{B} = 0 \quad (4)$$

where E is the electric field vector, D is the electric displacement field vector, H is the magnetic field vector and B is the magnetic flux density vector.

MAXWELL'S EQUATIONS

- D and B are related to the field vectors and are defined as

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \tag{5}$$

$$\vec{B} = \mu_0 \left(\vec{H} + \vec{M} \right) \tag{6}$$

where ε_0 and μ_0 are the electric permittivity and magnetic permeability of vacuum, respectively, P is the polarization and M is the magnetization.

- For optical fibers $M = 0$ because of the nonmagnetic nature of silica glass.
- Evaluation of the electric polarization P requires a microscopic quantum-mechanical approach.

MAXWELL'S EQUATIONS

Although such an approach is essential when the optical frequency is near a medium resonance, a phenomenological relation between P and E can be used far from medium resonances.

This is the case for optical fibers in the wavelength region 0.5-2 μm, a range that covers the low loss region of optical fibers that is of interest for fiber-optic communication systems.

In general, the relation between P and E can be nonlinear. Here, in discussion of fiber modes, the nonlinear effects in optical fibers will be ignored.

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E - GENERAL CASE

The polarization of the material is defined as

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \tag{7}$$

where χ is the susceptibility which is a measure of the polarizability of the material.

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{EE} + \chi^{(3)} \mathbf{EEE} + \cdots \right) = \mathbf{P}_{\mathrm{L}} + \mathbf{P}_{\mathrm{NL}}^{(2)} + \mathbf{P}_{\mathrm{NL}}^{(2)} + \cdots \tag{8}$$

where $\chi^{(1)}$ is the linear susceptibility and $\chi^{(2)}$ and $\chi^{(3)}$ in the second and third order nonlinear susceptibility. $\chi^{(j)}$ is a tensor of $j + 1$ rank.

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E - GENERAL CASE

- If the intensity of the applied field is small the response is linear, as shown in (a).
- If the intensities are increased, the response of the material will become nonlinear (symmetric material - (b)).
- If no symmetry center is present in the crystal, the symmetry rule no longer holds and it follows a potential energy function according to (c).

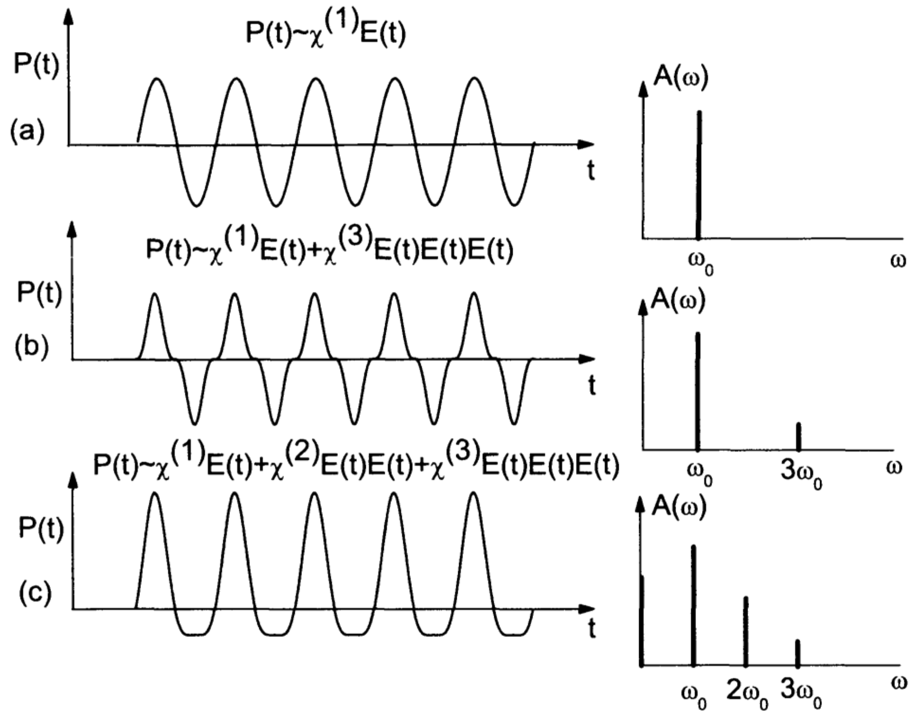


Figure 1: Potential energy function for linear and nonlinear media and the corresponding Fourier transforms [1].

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E - GENERAL CASE

The linear susceptibility $\chi^{(1)}$ is a tensor, consisting of $3^1 \times 3$ elements.

$$\begin{bmatrix} P_{\mathrm{L}x}(\omega) \\ P_{\mathrm{L}y}(\omega) \\ P_{\mathrm{L}z}(\omega) \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \chi_{xx}^{(1)}(\omega) & \chi_{xy}^{(1)}(\omega) & \chi_{xz}^{(1)}(\omega) \\ \chi_{yx}^{(1)}(\omega) & \chi_{yy}^{(1)}(\omega) & \chi_{yz}^{(1)}(\omega) \\ \chi_{zx}^{(1)}(\omega) & \chi_{zy}^{(1)}(\omega) & \chi_{xz}^{(1)}(\omega) \end{bmatrix} \begin{bmatrix} E_x(\omega) \\ E_y(\omega) \\ E_z(\omega) \end{bmatrix}$$

It can be expressed as a summation over the distinct components as well

$$P_{\mathrm{L}i}(\omega) = \varepsilon_0 \sum_j \chi_{ij}^{(1)}(\omega) E_j(\omega) \tag{9}$$

where $i, j = x, y, z$.

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E - GENERAL CASE

The second order nonlinear susceptibility $\chi^{(2)}$ consists of $3^2 \times 3$ elements and is a tensor of third rank.

$$\begin{bmatrix} P_{\mathrm{L}x}(\omega) \\ P_{\mathrm{L}y}(\omega) \\ P_{\mathrm{L}z}(\omega) \end{bmatrix} = \varepsilon_0 \begin{bmatrix} \chi^{(2)}_{xxx}(\omega) & \chi^{(2)}_{xxy}(\omega) & \cdots & \chi^{(2)}_{xzz}(\omega) \\ \chi^{(2)}_{yxx}(\omega) & \chi^{(2)}_{yxy}(\omega) & \cdots & \chi^{(2)}_{yzz}(\omega) \\ \chi^{(2)}_{zxx}(\omega) & \chi^{(2)}_{zxy}(\omega) & \cdots & \chi^{(2)}_{xzz}(\omega) \end{bmatrix} \begin{bmatrix} E_x(\omega_1) & E_x(\omega_2) \\ E_x(\omega_1) & E_y(\omega_2) \\ E_x(\omega_1) & E_z(\omega_2) \\ E_y(\omega_1) & E_x(\omega_2) \\ E_y(\omega_1) & E_y(\omega_2) \\ E_y(\omega_1) & E_z(\omega_2) \\ E_z(\omega_1) & E_x(\omega_2) \\ E_z(\omega_1) & E_y(\omega_2) \\ E_z(\omega_1) & E_z(\omega_2) \end{bmatrix}$$

It can be expressed as a summation over the distinct components as well

$$P^{(2)}_{\mathrm{NL}i}(\omega = \omega_1 + \omega_2) = \varepsilon_0 \sum_{jk} \chi^{(2)}_{ijk}(\omega_1, \omega_2) E_j(\omega_1) E_k(\omega_2) \tag{10}$$

where $i, j, k = x, y, z$.

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E

Relation between *P* and *E*

$$P(r,t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(r, t - t')E(r, t')\, \mathrm{d}t'$$

where $\chi^{(1)}$ is the linear susceptibility which is in general a second-rank tensor but a scalar for an isotropic medium such as silica glass.

Dimensionless proportionality constant, electric susceptibility χ, indicates the degree of polarization of a dielectric material in response to an applied electric field. The greater the electric susceptibility χ, the greater the ability of a material to polarize in response to the field, and thereby reduce the total electric field E inside the material (and store energy). It is in this way that the electric susceptibility χ influences the electric permittivity ε of the material.

RELATION BETWEEN ELECTRIC POLARIZATION P AND ELECTRIC FIELD VECTOR E

Note: Optical fibers become slightly birefringent because of unintentional variations in the core shape or in local strain.

Equation in the previous slide assumes a spatially local response. However, it includes the delayed nature of the temporal response, a feature that has important implications for optical fiber communications through chromatic dispersion.

FREQUENCY-DEPENDENT DIELECTRIC CONSTANT $\varepsilon_r(r,\omega)$

Taking the curl of Eq. (2), Eq. (5) and Eq. (6), we obtain the wave equation:

$$\nabla \times \nabla \times \vec{E} = -\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \vec{P}}{\partial t^2} \tag{11}$$

where $c = \sqrt{\varepsilon_0 \mu_0}$. By introducing the Fourier transform:

$$\tilde{E}(r,\omega) = \int_{-\infty}^{\infty} \vec{E}(\vec{r},t) e^{j\omega t}\, \mathrm{d}t \tag{12}$$

as well as a similar relation for $\vec{P}(\vec{r},t)$, and by using Eq. (11) can be written in the frequency domain as:

$$\nabla \times \nabla \times \tilde{E} = -\varepsilon_r(\vec{r},\omega)\frac{\omega^2}{c^2}\tilde{E} \tag{13}$$

FREQUENCY-DEPENDENT DIELECTRIC CONSTANT $\varepsilon_r(r,\omega)$

where ε_r is the frequency-dependent dielectric constant which is defined as:

$$\varepsilon_r(r,\omega) = 1 + \tilde{\chi}(r,\omega) \quad (14)$$

where $\tilde{\chi}$ is the Fourier transform of χ.

REFRACTIVE INDEX n AND ABSORPTION COEFFICIENT α

In general, $\varepsilon_r(r,\omega)$ is complex. Its real and imaginary parts are related to the refractive index n and the absorption coefficient α by the definition:

$$\varepsilon_r = \left(n + \frac{j\alpha c}{2\omega}\right)^2 \tag{15}$$

By using Eqs. (14) and (15), n and α are related to $\tilde{\chi}$ as:

$$n = \sqrt{1 + \Re\{\tilde{\chi}\}} \tag{16}$$

$$\alpha = \left(\frac{\omega}{nc}\right)\Im\{\tilde{\chi}\} \tag{17}$$

Both n and α are frequency dependent. The frequency dependence of n is referred to as chromatic dispersion or simply as material dispersion.

REFRACTIVE INDEX n AND ABSORPTION COEFFICIENT α

Note: Fiber dispersion is shown to limit the performance of fiber-optic communication systems in a fundamental way.

To solve Eq. (13), two simplifications can be made:

1) The term ε_r can be taken to be real and replaced by n^2 because of low optical losses in silica fibers.

2) Since $n(r, \omega)$ is independent of the spatial coordinate r in both the core and the cladding of a step-index fiber, one can use the identity:

$$\nabla \times \nabla \times \tilde{E} = \nabla(\nabla \cdot \tilde{E}) - \nabla^2 \tilde{E} = -\nabla^2 \tilde{E} \tag{18}$$

REFRACTIVE INDEX n AND ABSORPTION COEFFICIENT α

By using Eq. (18) in Eq. (13), we obtain

$$\nabla^2 \tilde{E} + n^2(\omega){k_0}^2 \tilde{E} = 0 \tag{19}$$

where the free-space wave number k_0 is defined as:

$$k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda} \tag{20}$$

and λ is the wavelength in vacuum.

FIBER MODES

An optical mode refers to a specific solution of the wave equation (Eq. (19)) that satisfies the appropriate boundary conditions and has the property that its spatial distribution does not change with propagation. The fiber modes can be classified as guided modes and leaky guided modes.

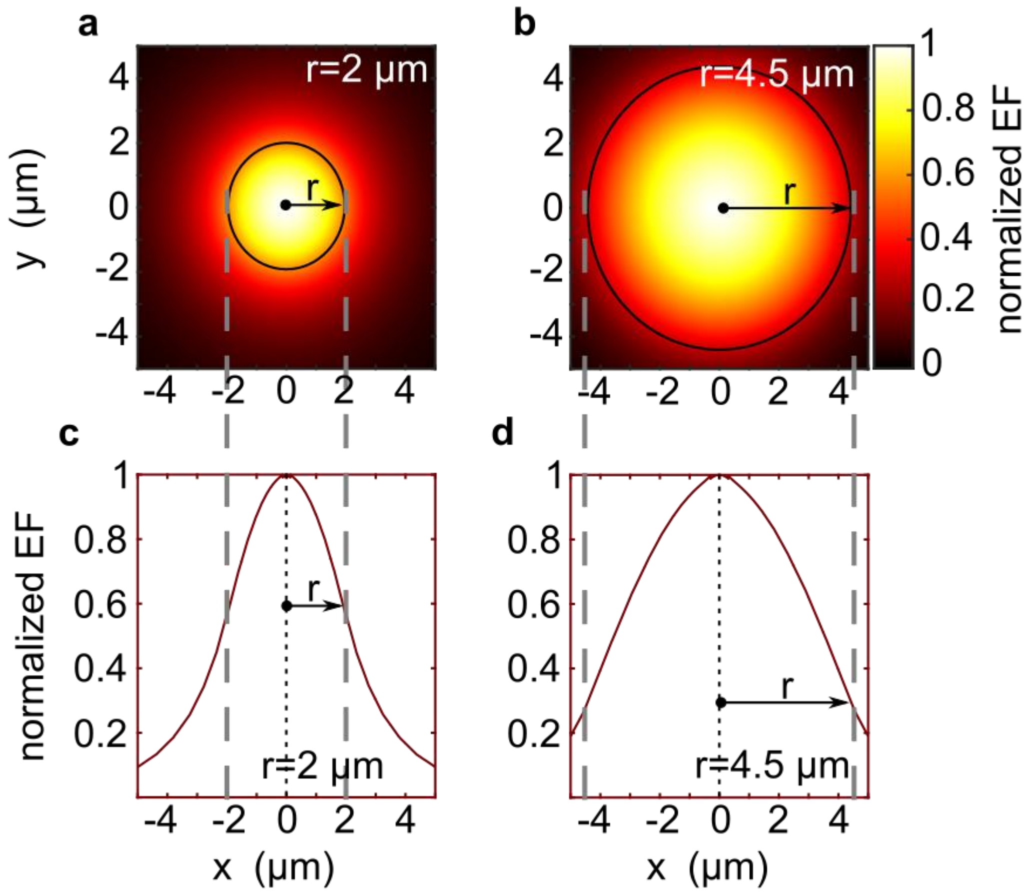


Figure 2: (a) Leaky guided mode. (b) Guided fiber mode. From [Karabchevsky et al ACS Photonics 2018]

FIBER MODES

Note: The signal transmission in fiber-optic communication systems takes place through the guided modes only. Here, we study the guided modes of a step-index fiber.

To take advantage of the cylindrical symmetry, Eq. (19) is written in the cylindrical coordinates ρ, ϕ, and z as:

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2 {k_0}^2 E_z = 0 \qquad (21)$$

where for a step-index fiber of core radius a, the refractive index n is of the form:

$$n = \begin{cases} n_1, & \rho \leq a \\ n_2, & \rho > a \end{cases} \qquad (22)$$

FIBER MODES

For simplicity of notation, the tilde over $\tilde{E}$ has been dropped and the frequency dependence of all variables is implicitly understood. Equation (21) is written for the axial component E_z of the electric field vector.

Similar equations can be written for the other five components of E and H. However, it is not necessary to solve all six equations since only two components out of six are independent. It is customary to choose E_z and H_z as the independent components and obtain E_ρ, E_ϕ, H_ρ and H_ϕ in terms of them.

FIBER MODES

Equation (21) is easily solved by using the method of separation of variables and writing E_z as:

$$E_z(\rho,\phi,z) = F(\rho)\Phi(\phi)Z(z) \tag{23}$$

By using Eq. (23) in Eq. (21), we obtain the three ordinary differential equations:

$$\frac{\partial^2 Z}{\partial z^2} + {\beta_m}^2 Z = 0 \tag{24a}$$

$$\frac{\partial^2 \Phi}{\partial \phi^2} + m^2 \Phi = 0 \tag{24b}$$

$$\frac{\partial^2 F}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial F}{\partial \rho} + \left(n^2 {k_0}^2 - {\beta_m}^2 - \frac{m^2}{\rho^2}\right) = 0 \tag{24c}$$

FIBER MODES

- Equation (24a) has a solution of the form $Z = \exp(j\beta_m z)$, where β_m is the propagation constant.
- Similarly, Equation (24b) has a solution $\Phi = \exp(jm\phi)$, but the constant m is restricted to take only integer values since the field must be periodic in ϕ with a period of 2π.
- Equation (24c) is the well-known differential equation satisfied by the Bessel functions. Its general solution in the core and cladding regions can be written as under assumption that the diameter of the cladding is infinite:

$$F(\rho) = \begin{cases} AJ_m(p\rho) + A'Y_m(p\rho), & \rho \leq a \\ CK_m(q\rho) + C'I_m(q\rho), & \rho > a \end{cases} \qquad (25)$$

FIBER MODES

where $p_m{}^2 = \beta_1{}^2 - \beta_{z,m}{}^2$ and $q_m{}^2 = \beta_{z,m}{}^2 - \beta_2{}^2$ represent equivalent transversal propagation constants in the core and cladding, respectively, with $\beta_i = n_i\omega/c = n_i k_0$. $\beta_{z,m}$ is the propagation constant in the z-direction.

J_m and Y_m are the first and the second kind of Bessel functions of the m^{th} order, and K_m and I_m are the first and the second kind of modified Bessel functions of the m^{th} order.

Note: two mode indices are the amplitude maxima of the standing wave patterns in the azimuthal and the radial directions, respectively.

BESSEL FUNCTIONS

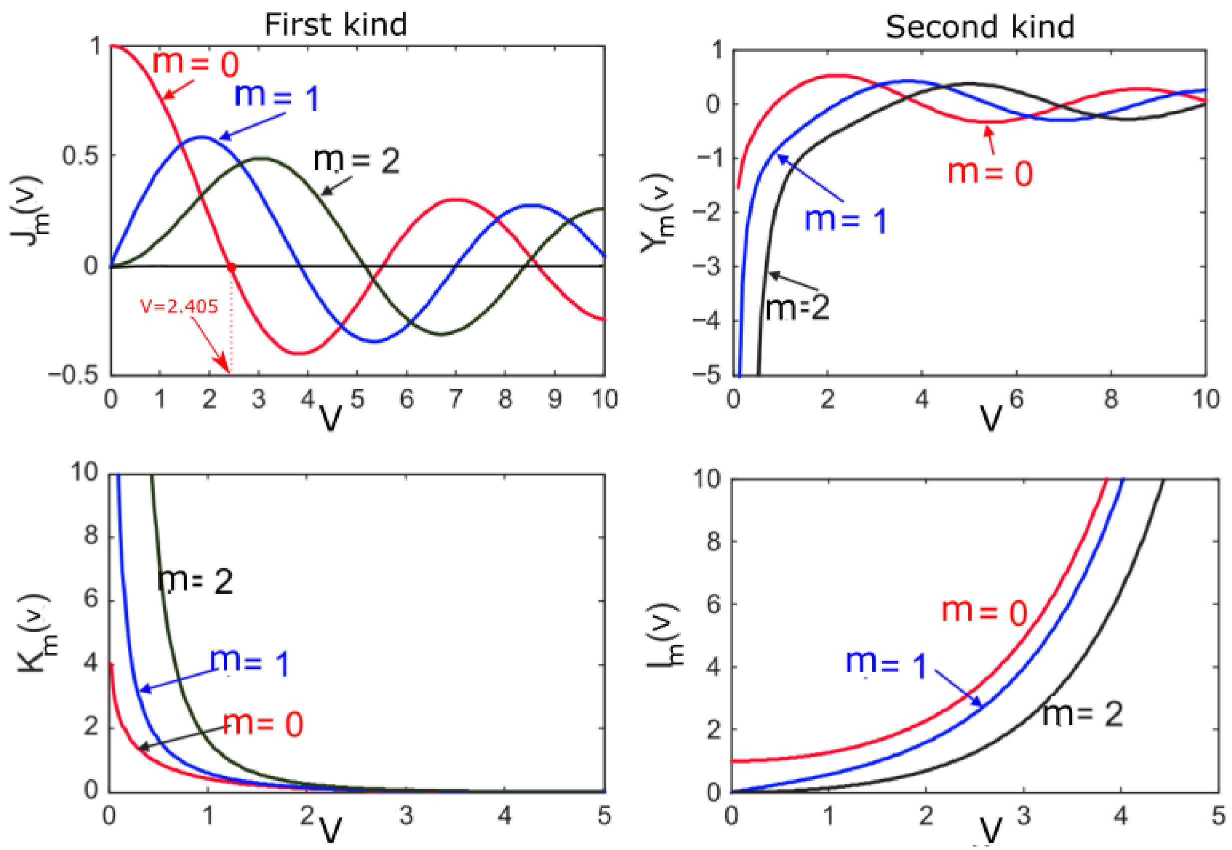


Figure 3: Bessel function (top) and modified Bessel functions (bottom) [2].

H.W.

Solution of the wave equation:

1) Detail the solution of the wave equation to obtain $Z(z)$.

FIBER MODES

A, A', C, and C' are constants that need to be defined using appropriate boundary conditions:

- First boundary condition - The field amplitude of a guided mode should be finite at the center of the core $\rho = 0$. Since the special function $Y_m(0) = -\infty$, one must set $A' = 0$ to ensure that $E_z(0)$ has a finite value.
- Second boundary condition - The field amplitude of a guided mode should be zero far away from the core ($\rho = \infty$). Since $I_m(\infty) \neq 0$, one must set $C' = 0$ to ensure that $E_z(\infty) = 0$.

Consider $A' = C' = 0$, the solution of Eq. (21) is:

$$E_z = \begin{cases} A j_m(p\rho)\exp(jm\phi)\exp(j\beta z), & \rho \leq a \\ C j_m(q\rho)\exp(jm\phi)\exp(j\beta z), & \rho > a \end{cases} \quad (26)$$

FIBER MODES

- Mathematically, $K_m(q_m\rho) \propto \exp(-q_m\rho)$, for $q, p > 0$, so that $K_m(q_m\rho)$ represents an exponential decay of optical field over ρ in the fiber cladding.
- For a propagation mode, $q_m > 0$ is required to ensure that energy does not leak through the cladding. In the fiber core, the Bessel function $J_m(p_m\rho)$ oscillates as shown in the next frame. This represents a standing-wave pattern in the core over the radius direction.
- For a propagating mode, $p_m \geq 0$ is required to ensure this standing-wave pattern in the fiber core. Mode index or effective index $\bar{n}, n_1 < \bar{n} < n_2, \bar{n} = \beta/k_0$.

Please note that based on the definitions of ${p_m}^2 = {\beta_1}^2 - {\beta_{z,m}}^2$ and ${q_m}^2 = {\beta_{z,m}}^2 - {\beta_2}^2$, the requirement of $q_m > 0$ and $p_m \geq 0$ is equivalent to ${\beta_2}^2 < {\beta_{z,m}}^2 \leq {\beta_1}^2$ or $n_2/n_1 < {\beta_{z,m}}^2/\beta_1 \leq 1$. This is indeed equivalent to the mode condition derived by the ray optics. In general, it may have multiple solutions in descending numerical order denoting by $\beta_{m,n}$.

CHARACTERISTICS OF PROPAGATION MODES IN THE FIBER

- Transverse electric-field mode (TE mode): $E_z = 0$
- Transverse magnetic-field mode (TM mode): $H_z = 0$
- Hybrid mode (HE_{mn} or EH_{mn} mode): $E_z \neq 0$ and $H_z \neq 0$
- V-number is an important parameter of a fiber, which is defined as:

$$V = a\sqrt{{p_m}^2 + {q_m}^2} \tag{27}$$

since

$${p_m}^2 = {\beta_1}^2 - {\beta_{z,m}}^2 = \left(\frac{2\pi n_1}{\lambda}\right)^2 - {\beta_{z,m}}^2 \tag{28}$$

and

$${q_m}^2 = {\beta_{z,m}}^2 - {\beta_2}^2 = {\beta_{z,m}}^2 - \left(\frac{2\pi n_1}{\lambda}\right)^2 \tag{29}$$

CHARACTERISTICS OF PROPAGATION MODES IN THE FIBER

V-number can be expressed as:

$$V = a\sqrt{{p_m}^2 + {q_m}^2} = \frac{2\pi a}{\lambda}\sqrt{{n_1}^2 - {n_2}^2} \qquad (30)$$

Relation between guided modes m and V-number

$$m \approx \frac{V^2}{2}$$

- In a multimode fiber, the number of guided modes can be on the order of several hundreds.
- A short optical pulse is injected into a fiber and the optical energy is carried by many different modes.
- Different modes have different propagation constants $\beta_{z,m}$ in the longitudinal direction and they will arrive at the output of the fiber in different times.
- The short optical pulse at the input will become a broad pulse at the output.

CHARACTERISTICS OF PROPAGATION MODES IN THE FIBER

- In optical communications systems, this introduces signal waveform distortions and bandwidth limitations.

To conclude: the single-mode fiber is required in high-speed long distance optical systems. The lowest-order propagation mode is HE_{11}, whereas the next lowest modes are TE_{01} and TM_{01}. ($m = 0$ and $n = 1$; m and n describe the electric field intensity profile. There are $2m$ field maxima around the fiber core circumference and n field maxima along the fiber core radial direction.)

LINEARLY POLARIZED MODES

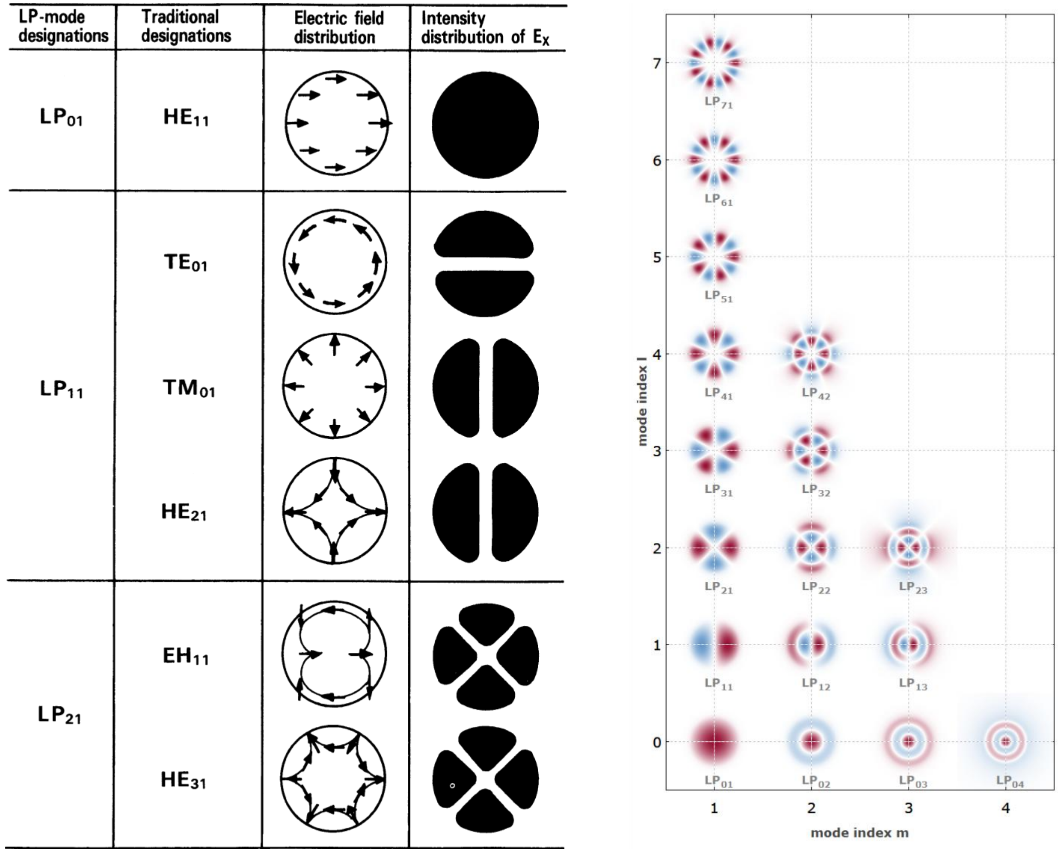


Figure 4: Linearly polarized modes.

CHARACTERISTICS OF PROPAGATION MODES IN THE FIBER

TE_{01} and TM_{01} have the same cutoff conditions:

- $q_{01} = 0$ so that these two modes radiate in the cladding.
- $J_0(p_{01}a) = 0$ so that the field amplitude at core-cladding interface ($\rho = a$) is zero.

Under the first condition ($q_0 = 0$), the cutoff V-number $V = a\sqrt{p_{01}{}^2 + q_{01}{}^2} = aU_{01}$.

Under the second condition, we can find $J_0(p_{01}a) = J_0(V) = 0$, which implies that $V = 2.405$ as the first root of $J_0(V) = 0$.

Therefore, the single-mode condition is:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_1{}^2 - n_2{}^2} < 2.405 \qquad (31)$$

FIBER MODES

Solutions of the wave equation

$$E_{z,m}(\rho,\phi,z)=\begin{cases}AJ_m(p_m\rho)\exp(jm\phi)\exp(j\beta_{z,m}z), & \rho\le 0\\ CK_m(q_m\rho)\exp(jm\phi)\exp(j\beta_{z,m}z), & \rho>0\end{cases} \quad (32)$$

$$H_{z,m}(\rho,\phi,z)=\begin{cases}BJ_m(p_m\rho)\exp(jm\phi)\exp(j\beta_{z,m}z), & \rho\le a\\ DK_m(q_m\rho)\exp(jm\phi)\exp(j\beta_{z,m}z), & \rho>a\end{cases} \quad (33)$$

H.W.

Solution of the wave equation:

1) Express the radial components E_ρ and H_ρ.

2) Express the angular components E_ϕ and H_ϕ.

FIBER MODES

The other four components E_ρ, E_ϕ, H_ρ and H_ϕ can be expressed in terms of E_z and H_z by using Maxwell's equations. In the core region, we obtain:

$$E_\rho = \frac{j}{p^2}\left(\beta\frac{\partial E_z}{\partial\rho} + \mu_0\frac{\omega}{\rho}\frac{\partial H_z}{\partial\phi}\right) \tag{34}$$

$$E_\rho = \frac{j}{p^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial\phi} - \mu_0\omega\frac{\partial H_z}{\partial\rho}\right) \tag{35}$$

$$H_\rho = \frac{j}{p^2}\left(\beta\frac{\partial H_z}{\partial\rho} - \varepsilon_0 n^2\frac{\omega}{\rho}\frac{\partial E_z}{\partial\phi}\right) \tag{36}$$

$$H_\phi = \frac{j}{p^2}\left(\frac{\beta}{\rho}\frac{\partial H_z}{\partial\phi} + \varepsilon_0 n^2\omega\frac{\partial E_z}{\partial\rho}\right) \tag{37}$$

These equations can be used in the cladding region after replacing p^2 by $-q^2$.

FIBER MODES

Equations (43)-(48) express the electromagnetic field in the core and cladding regions of an optical fiber in terms of four constants A, B, C, and D.

These constants are determined by applying the boundary condition that the tangential components of E and H be continuous across the core-cladding interface.

By requiring the continuity of E_z, H_z, E_ϕ and H_ϕ at $\rho = a$, we obtain a set of four homogeneous equations satisfied by A, B, C, and D. These equations have a nontrivial solution only if the determinant of the coefficient matrix vanishes.

FIBER MODES

After considerable algebraic details, this condition leads to the following eigenvalue equation:

$$\left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{K'_m(qa)}{pK_m(qa)}\right]\left[\frac{J'_m(pa)}{pJ_m(pa)} + \frac{{n_2}^2}{{n_1}^2}\frac{K'_m(qa)}{pK_m(qa)}\right] = \frac{m^2}{a^2}\left(\frac{1}{p^2} + \frac{1}{q^2}\right)\left(\frac{1}{p^2} + \frac{{n_2}^2}{{n_1}^2}\frac{1}{q^2}\right) \quad (38)$$

where a prime indicates differentiation with respect to the argument.

- For a given set of the parameters k_0, a, n_1, n_2 the eigenvalue Eq. (38) can be solved numerically to determine the propagation constant β.
- In general, Equation (38) may have multiple solutions for each integer value of m.
- These solutions can be enumerated in ascending numerical order and denoted by β_{mn} for a given m $(n = 1,2, \ldots)$. Each value mn corresponds to one possible propagation mode of the optical field whose spatial distribution is obtained from Eqs. (32)-(37).

FIBER MODES

Since the field distribution does not change with propagation except for a phase factor and satisfies all boundary conditions, it is an optical mode of the fiber.

In general, both E_z and H_z are nonzero (except for $m = 0$) in contrast with the planar waveguides, for which one of them can be taken to be zero. Therefore, fiber modes are referred to as hybrid modes and are denoted by HE_{mn} or EH_{mn} depending on whether H_z or E_z dominates.

In the special case $m = 0$, HE_{0n} and EH_{0n} are also denoted by TE_{0n} and TM_{0n} respectively, since they correspond to transverse-electric ($E_z = 0$) and transverse-magnetic ($H_z = 0$) modes of propagation.

FIBER MODES

- A mode is uniquely determined by its propagation constant β.
- It is useful to introduce a quantity $\bar{n} = \beta/k_0$, called the mode index or effective index.
- Mode propagates with an effective refractive index $\bar{n}$ whose value lies in the range $n_1 > \bar{n} > n_2$.
- A mode ceases to be guided when $\bar{n} \leq n_2$. This can be understood by noting that the optical field of guided modes decays exponentially inside the cladding layer since:

$$K_m(q\rho) = \left(\frac{\pi}{2q\rho}\right)^{1/2} \exp(-q\rho), \qquad q\rho \gg 1 \tag{39}$$

When $\bar{n} \leq n_2$, $q^2 \leq 0$ and the exponential decay does not occur. The mode is said to reach cutoff when q becomes zero or when $\bar{n} = n_2$. $p = k_0\sqrt{{n_1}^2 - {n_2}^2}$ when $q = 0$.

H.W.

Cutoff:

Explain the cutoff condition of the guiding in optical fibers in terms of the optical fields.

GUIDED MODES

A parameter that plays an important role in determining the cutoff condition is defined as:

$$V = k_0 a\sqrt{{n_1}^2 - {n_2}^2} \approx \left(\frac{2\pi}{\lambda}\right) a n_1 \sqrt{2\Delta} \tag{40}$$

which is the normalized frequency $V \propto \omega$ or simply the V-number.

It is also useful to introduce a normalized propagation constant b as:

$$b = \frac{\frac{\beta}{k_0} - n_2}{n_1 - n_2} = \frac{\bar{n} - n_2}{n_1 - n_2} \tag{41}$$

GUIDED MODES

- The figure in the next slide shows a plot of b as a function of V for a few low-order fiber modes obtained by solving the eigenvalue equation (Eq. (38)).
- A fiber with a large value of V supports many modes. A rough estimate of the number of modes for such a multimode fiber is given by $V^2/2$. For example, a typical multimode fiber with $a = 25\ \mu\text{m}$ and $\Delta = 5 \cdot 10^{-3}$ has $V \simeq 18$ at $\lambda = 1.3\ \mu\text{m}$ and would support about 162 modes. However, $V = 5$ supports 7 modes.
- Below a certain value of V all modes except the HE_{11} mode reach cutoff. Such fibers support a single mode and are called single-mode fibers.

NORMALIZED PROPAGATION CONSTANT b

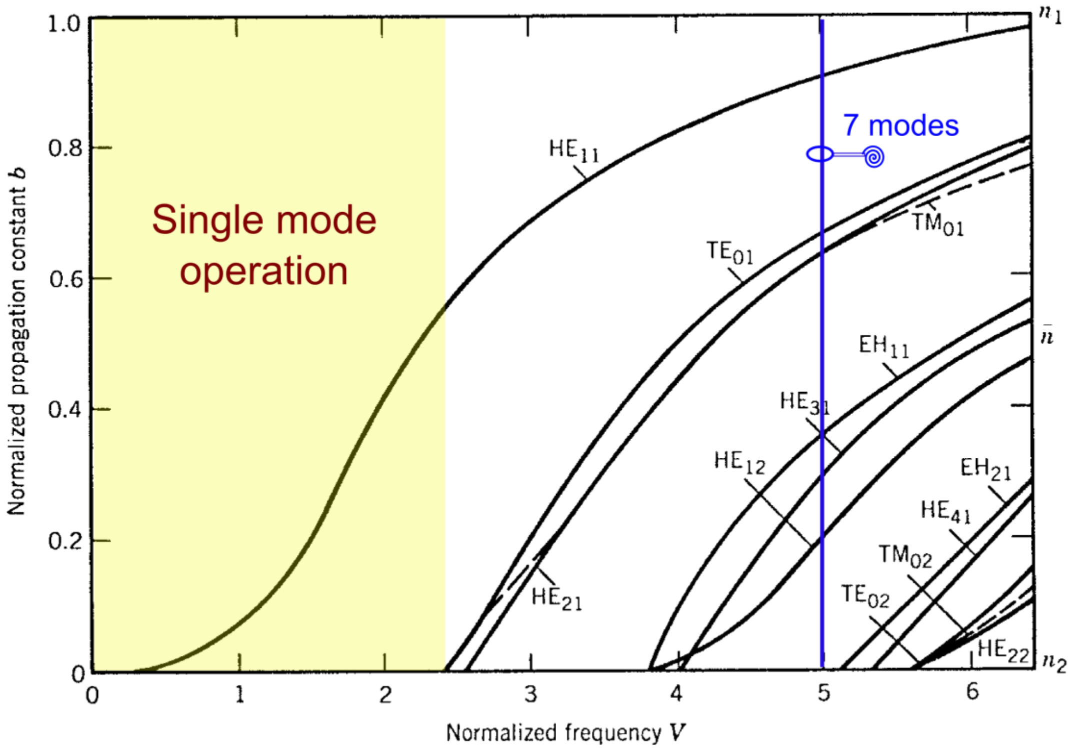


Figure 5: Normalized propagation constant b as a function of normalized frequency V for a few low-order fiber modes. The right scale shows the mode index n_{eff}.

SINGLE-MODE FIBERS

- Single-mode fibers are designed to support only the HE_{11} mode, also known as the fundamental mode of the fiber.
- All other modes are cut off at the operating wavelength.
- The V-number determines the number of modes supported by a fiber, as shown in the Fig. 5. The cutoff condition of various modes is also determined by V.
- The fundamental mode has no cutoff and is always supported by a fiber.

SINGLE-MODE CONDITION

The single-mode condition is determined by the value of V at which the TE_{01} and TM_{01} modes reach cutoff. The eigenvalue equations for these two modes can be obtained by setting $m = 0$ in Eq. (41) and are given by:

$$pJ_0(pa)K'_0(qa) + qJ'_0(pa)K_0(qa) = 0 \tag{42}$$

$$pn_2{}^2J_0(pa)K'_0(qa) + pn_1{}^2J'_0(pa)K_0(qa) = 0 \tag{43}$$

A mode reaches cutoff when $q = 0$. Since $pa = V$ when $q = 0$, the cutoff condition for both modes is simply given by $J_0(V) = 0$. The smallest value of V for which $J_0(V) = 0$ is 2.405. A fiber designed such that $V < 2.405$ supports only the fundamental HE_{11} mode. This is the single-mode condition.

SINGLE-MODE (SM) CONDITION

- Equation (40) can be used to estimate the core radius of the SM fiber used in Lightwave system.
- For the operating wavelength range 1.3-1.6 μm, the fiber is generally designed to become single mode for $\lambda > 1.2$ μm. By taking $\lambda > 1.2$ μm, $n_1 = 1.45$ and $\Delta = 5 \cdot 10^{-3}$, Eq. (40) shows that $V < 2.405$ for a core radius $a < 3.2$ μm.
- The required core radius can be increased to about 4 μm by decreasing Δ to $3 \cdot 10^{-3}$. Indeed, most telecommunication fibers are designed with $a \approx 4$ μm.

SINGLE-MODE FIBERS

The mode index $\bar{n}$ at the operating wavelength can be defined as:

$$\bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta) \qquad (44)$$

and by using Fig. 5, which provides b as a function of V for the HE_{11} mode. An analytic approximation for b is:

$$b(V) \approx \left(1.1428 - \frac{0.996}{V}\right)^2 \qquad (45)$$

and is accurate to within 0.2% for V in the range 1.5-2.5.

LINEARLY POLARIZED (LP) MODES

- The field distribution of the fundamental mode is obtained by using Eqs. (43)-(48). Hence, the HE_{11} mode is approximately linearly polarized for weakly guiding fibers. It is also denoted as LP_{01}, following an alternative terminology in which all fiber modes are assumed to be linearly polarized.
- A different notation LP_{mn} is sometimes used for weakly guiding fibers for which both E_z and H_z are nearly zero (LP stands for linearly polarized modes).

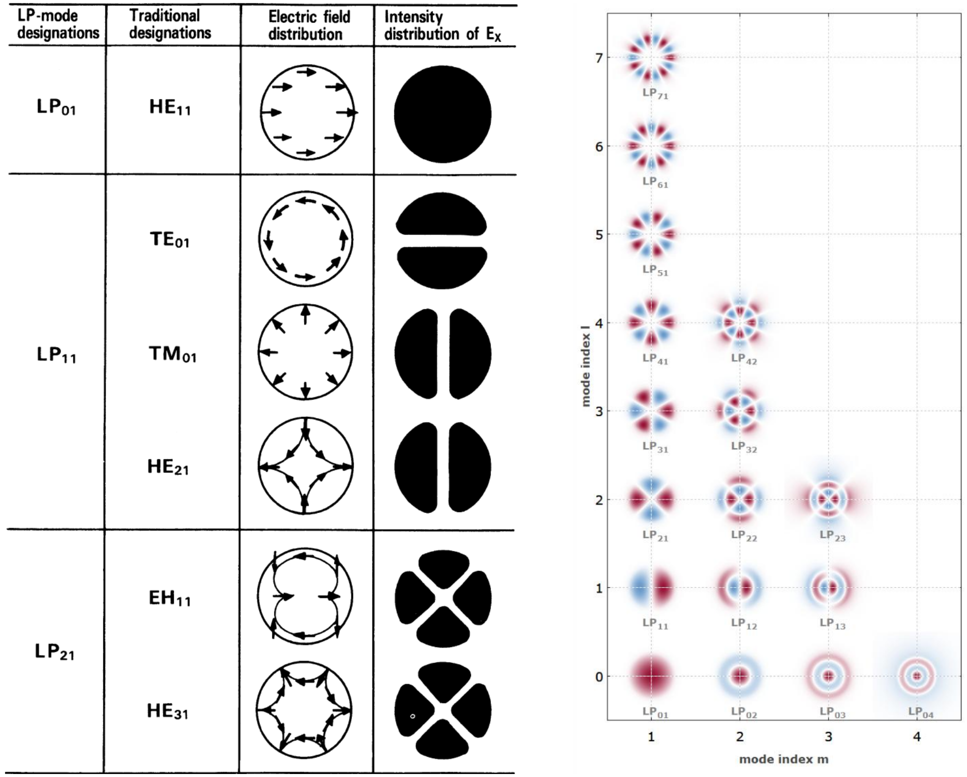


Figure 6: LP modes.

SINGLE-MODE FIBERS

One of the transverse components can be taken as zero for a linearly polarized mode. If we set $E_y = 0$, the E_x component of the electric field for the HE_{11} mode is given by:

$$E_x = E_0 \begin{cases} \left[\dfrac{J_0(p\rho)}{J_0(pa)}\right] \exp(j\beta z), & \rho \leq a \\ \left[\dfrac{K_0(q\rho)}{K_0(qa)}\right] \exp(j\beta z), & \rho > 0 \end{cases} \tag{46}$$

where E_0 is a constant related to the power carried by the mode. The dominant component of the corresponding magnetic field is given by $H_y = n_2(\varepsilon_0/\mu_0)^{1/2}E_x$.

This mode is linearly polarized along the x axis. The same fiber supports another mode linearly polarized along the y axis. In this sense a single-mode fiber actually supports two orthogonally polarized modes that are degenerate and have the same mode index.

SPOT SIZE

Since the field distribution is cumbersome to use in practice, it is often approximated by a Gaussian distribution of the form:

$$E_x = A\exp(-\rho^2/w^2)\exp(j\beta z) \tag{47}$$

where w is the field radius which also named the spot size.

It is determined by fitting the exact distribution to the Gaussian function or by following a variational procedure. The quality of fit is generally quite good for values of V around 2.

For $1.2 < V < 2.4$, the spot size w can be approximated as:

$$w/a \approx 0.65V + 1.619V^{-3/2} + 2.879V^{-6} \tag{48}$$

EFFECTIVE CORE AREA

The effective core area, defined as $A_{\text{eff}} = \pi w^2$, is an important parameter for optical fibers as it determines how tightly light is confined in the core.

The fraction of the power confined in the core is given by the confinement factor:

$$\Gamma = \frac{P_{\text{core}}}{P_{\text{total}}} = \frac{\int_0^a |E_x|^2 \rho \, \mathrm{d}\rho}{\int_0^\infty |E_x|^2 \rho \, \mathrm{d}\rho} = 1 - \exp\left(-\frac{2a^2}{w^2}\right) \tag{49}$$

- **For $V = 2$**, 75% of the power is confined in the core.
- **For $V = 1$**, 20% of the power is confined in the core.
- As a result, most telecommunication single-mode fibers are designed to operate in the range $2 < V < 2.4$.

SPOT SIZE

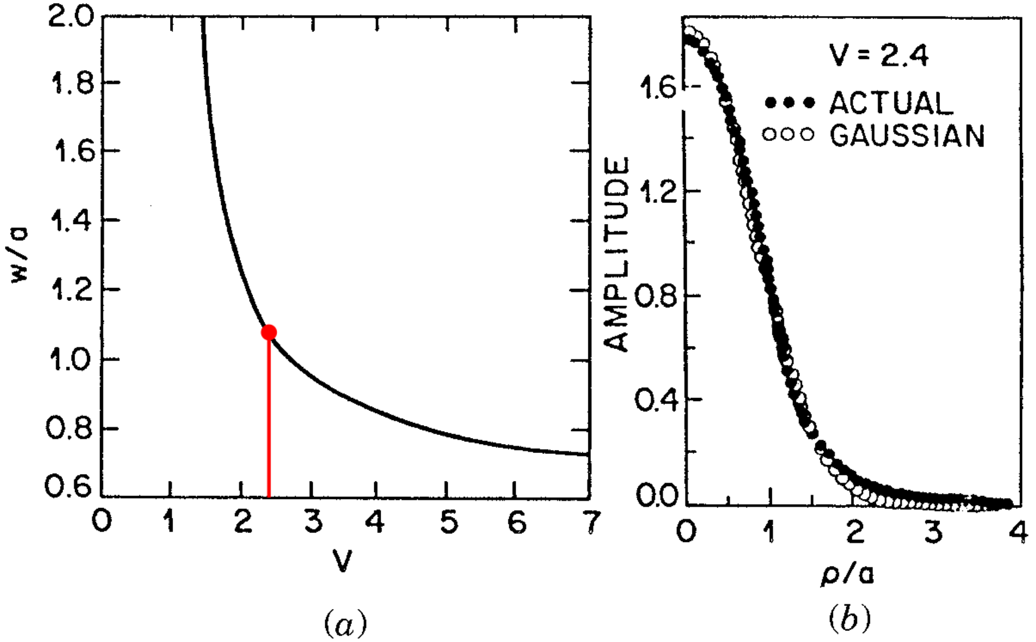


Figure 7: (a) Normalized spot size w/a as a function of the V parameter obtained by fitting the fundamental fiber mode to a Gaussian distribution. (b) quality of fit for $V = 2.4$ [3].

FIBER BIREFRINGENCE

The degenerate nature of the orthogonally polarized modes holds only for an ideal single-mode fiber with a perfectly cylindrical core of uniform diameter. Real fibers exhibit considerable variation in the shape of their core along the fiber length. They may also experience nonuniform stress such that the cylindrical symmetry of the fiber is broken. Degeneracy between the orthogonally polarized fiber modes is removed because of these factors, and the fiber acquires birefringence. The degree of modal birefringence is defined by:

$$B_m = |\tilde{n}_x - \tilde{n}_y| \tag{50}$$

where $\tilde{n}_x$ and $\tilde{n}_y$ are the mode indices for the orthogonally polarized fiber modes. Birefringence leads to a periodic power exchange between the two polarization components. The period, referred to as the **beat length**, is given by:

$$L_B = \lambda / B_m \tag{51}$$

STATE OF POLARIZATION IN A BIREFRINGENT FIBER

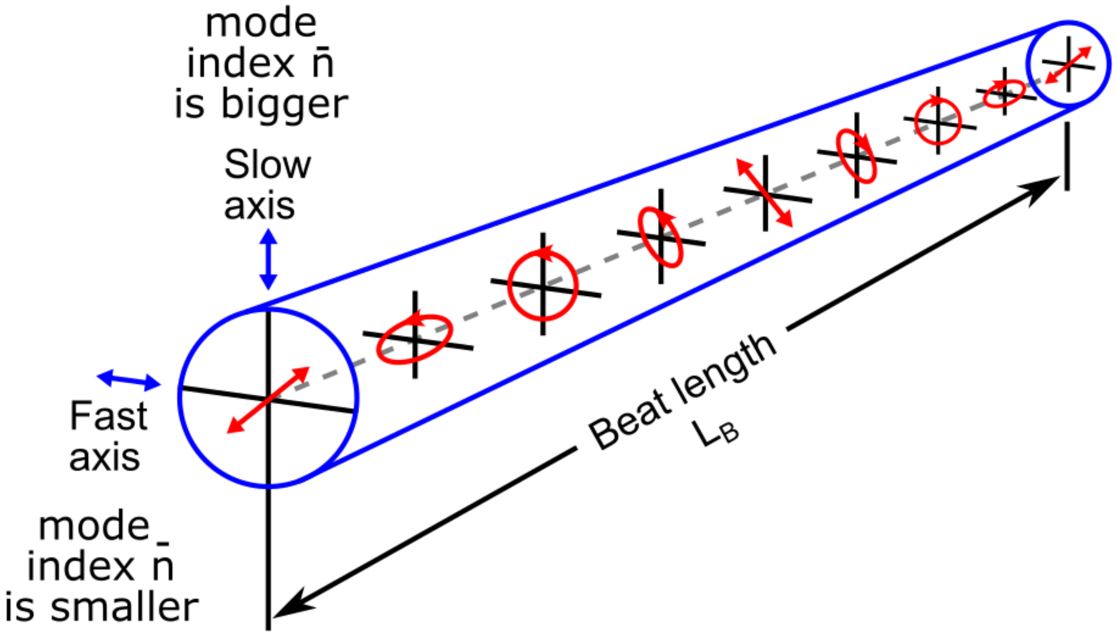


Figure 8: State of polarization in a birefringent fiber over one beat length. Input beam is linearly polarized at 45⁰ with respect to the slow and fast axes.

STATE OF POLARIZATION IN A BIREFRINGENT FIBER

- Linearly polarized light remains linearly polarized only when it is polarized along one of the principal axes. Otherwise, its state of polarization changes along the fiber length from linear to elliptical, and then back to linear, in a periodic manner over the length L_B. Figure 8 shows schematically such a periodic change for a fiber of constant birefringence B.
- The **fast axis** in the Figure corresponds to the axis along which the mode index is smaller. The other axis is called the **slow axis**.
- In conventional single-mode fibers, birefringence is not constant along the fiber but changes randomly, both in magnitude and direction, because of variations in the core shape (elliptical rather than circular) and the anisotropic stress acting on the core. As a result, light launched into the fiber with linear polarization quickly reaches a state of arbitrary polarization. Moreover, different frequency components of a pulse acquire different polarization states, resulting in pulse broadening.

STATE OF POLARIZATION IN A BIREFRINGENT FIBER

This phenomenon is called **polarization-mode dispersion (PMD)** and becomes a limiting factor for optical communication systems operating at high bit rates.

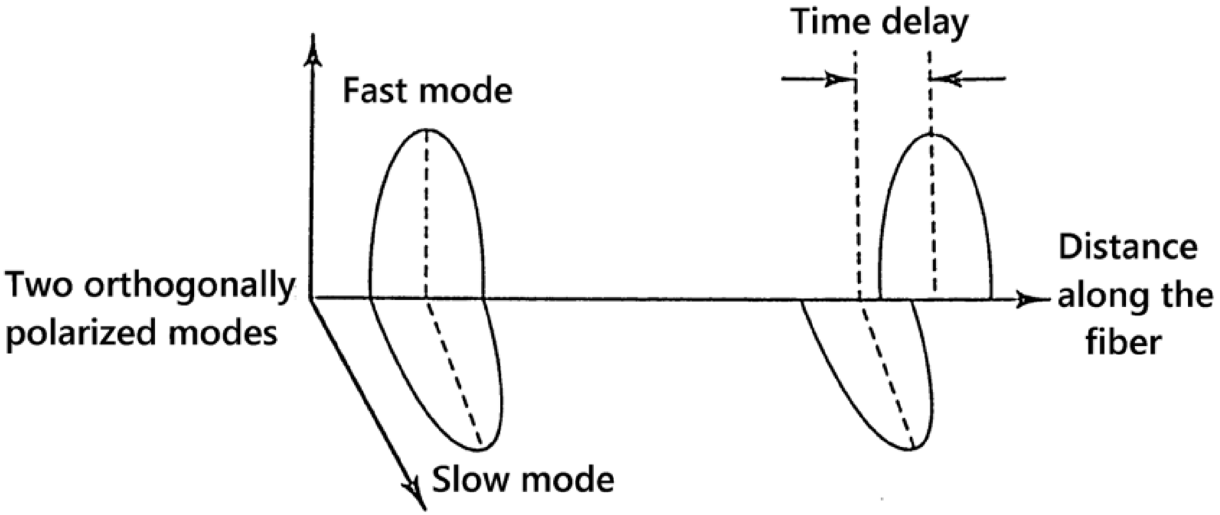


Figure 9: polarization-mode dispersion (PMD) in optical fiber.

POLARIZATION-MAINTAINING FIBER

It is possible to make fibers for which random fluctuations in the core shape and size are not the governing factor in determining the state of polarization. Such fibers are called polarization-maintaining fibers. A large amount of birefringence is introduced intentionally in these fibers through design modifications so that small random birefringence fluctuations do not affect the light polarization significantly. Typically, $B_m \approx 10^{-4}$ for such fibers.

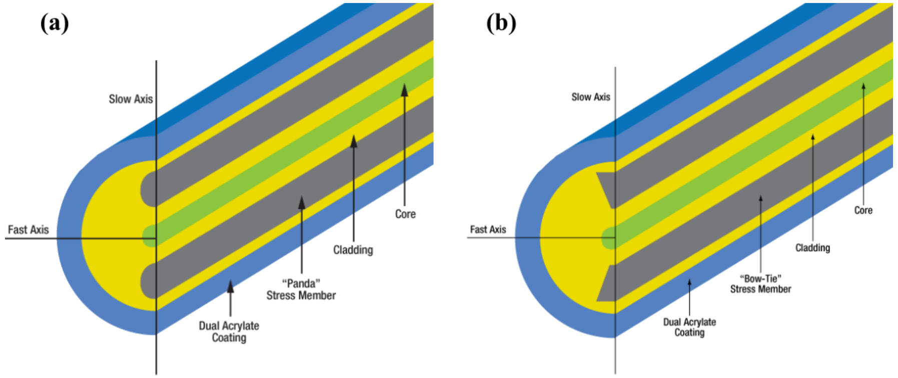


Figure 10: polarization-maintaining "Panda" and "Bow-Tie" fiber [from Thorlabs].

POLARIZING FIBER

It is also possible to make a polarizing fiber. In this fiber, two rods apply stress on the core, create a slow and a fast axis. The field in the slow axis has low attenuation while the field in the fast axis has high attenuation. Therefore, only the field in the slow axis will propagate without losses. This fiber has a narrow wavelength band (~150 nm).

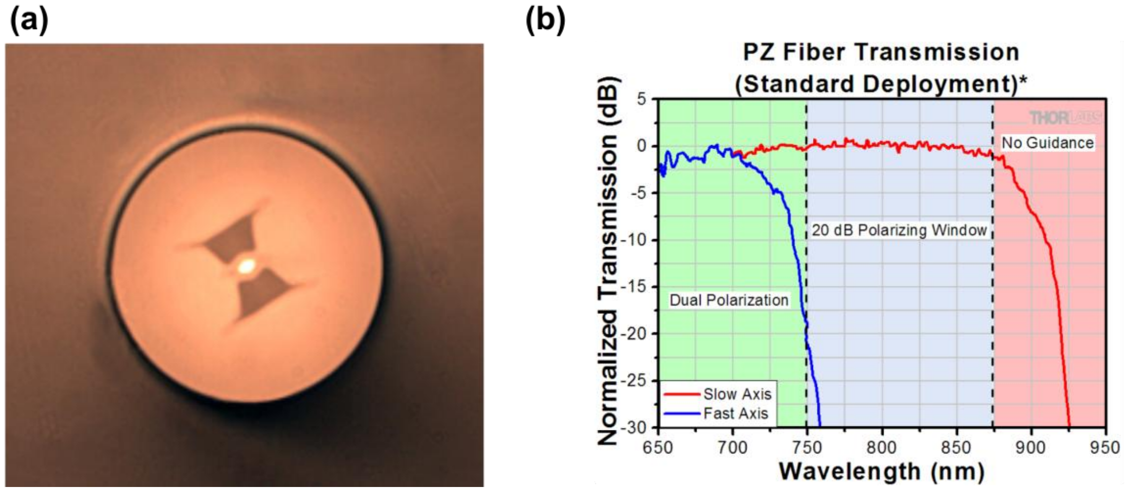


Figure 11: (a) cross-section and (b) transmission of a polarizing fiber [from Thorlabs].

THE AVERAGE OPTICAL POWER

Under general conditions, the changes in the average optical power P of a bit stream propagating inside an optical fiber are governed by Beer's law:

$$\frac{\partial P}{\partial z} = -\alpha P \tag{52}$$

where α is the attenuation coefficient.

Although denoted by the same symbol as the absorption coefficient, α in the Eq. (52) includes not only material absorption but also other sources of power attenuation. If P_{in} is the power launched at the input of a fiber of length L, the output power P_{out} from Eq. (52) is given by:

$$P_{\text{out}} = P_{\text{in}} \exp(-\alpha L) \tag{53}$$

ATTENUATION COEFFICIENT

The average optical power can be simply expressed as:

$$P(z) = P_0 \exp(-\alpha z) \tag{54}$$

where P_0 is the input optical power.

This attenuation coefficient α of an optical fiber can be obtained by measuring the input and the output optical power:

$$\alpha = -\frac{1}{L} \ln\left[\frac{P(L)}{P_0}\right] \tag{55}$$

where L is the fiber length and $P(L)$ is the optical power measured at the output of the fiber.

ATTENUATION COEFFICIENT

However, engineers like use decibel (dB) to describe fiber attenuation and use dB/km as the unit of attenuation coefficient. If we define dB as the attenuation coefficient which has the unit of dB/km, then the optical power level along the fiber length can be expressed as:

$$P(z) = P_0 \cdot 10^{-\frac{\alpha(\mathrm{dB/km})}{10}z} \tag{56}$$

Similar to Eq. (56), for a fiber of length L, dB/km can be estimated using

$$\alpha(\mathrm{dB/km}) = -\frac{10}{L}\log\left[\frac{P(L)}{P_0}\right] \tag{57}$$

ATTENUATION COEFFICIENT

Comparing Equations (55) and (57) the relationship between $\alpha(\mathrm{dB/km})$ and α can be found as:

$$\frac{\alpha(\mathrm{dB/km})}{\alpha} = \frac{10\log\left[\frac{P(L)}{P_0}\right]}{\ln\left[\frac{P(L)}{P_0}\right]} = 10\log(e) = 4.343 \qquad (58)$$

or simply, $\alpha(\mathrm{dB/km}) = 4.343\alpha$.

dB is a simpler parameter to use for evaluation of fiber loss. For example, for an 80 km long fiber with $\alpha = 0.25\,\mathrm{dB/km}$ attenuation coefficient, the total fiber loss can be easily found as 20 dB.

OPTICAL FIBER ATTENUATION

- Attenuation is a parameter of an optical fiber which determines how far an optical signal can be delivered at a detectable power level.
- There are several sources that contribute to fiber attenuation, such as **absorption** and **scattering**.

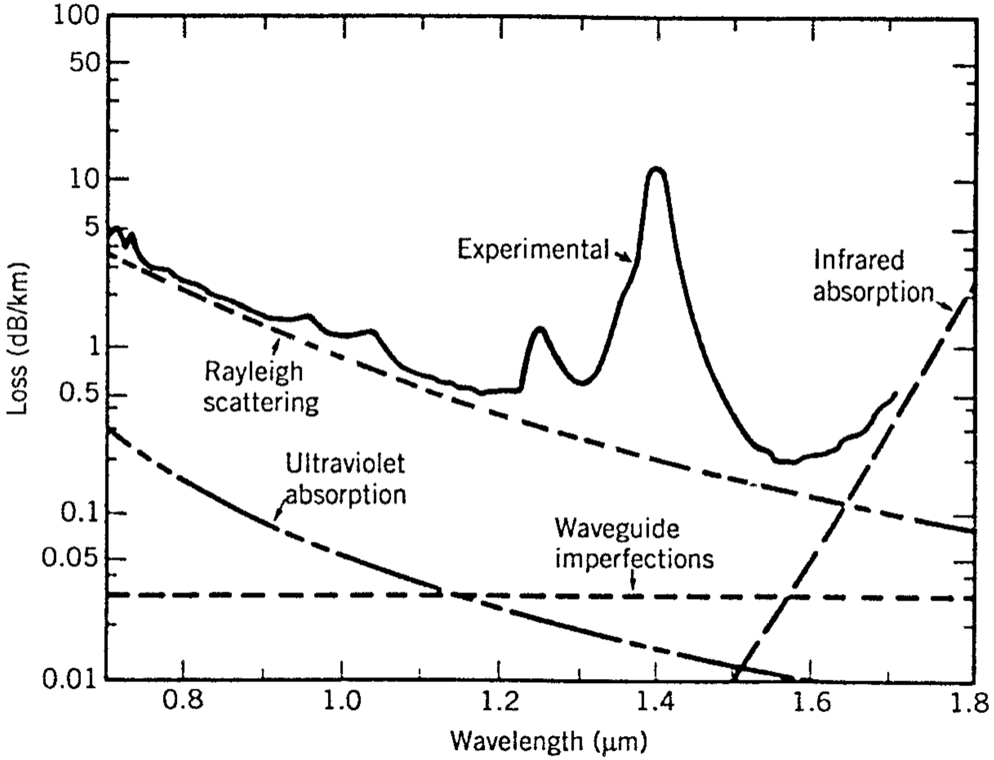


Figure 12: The loss spectrum $\alpha(\lambda)$ of a single-mode fiber made in 1979 with 9.4 μm core diameter and $\Delta = 1.9 \cdot 10^{-3}$ [3].

OPTICAL FIBER ATTENUATION

- The fiber exhibited a loss of about 0.2 dB/km in the wavelength region near 1.55 μm, the lowest value first realized in 1979. This value is close to the fundamental limit of about 0.16 dB/km for silica fibers.
- The loss spectrum exhibits a strong peak near 1.39 μm and several other smaller peaks.
- A secondary minimum is found to occur near 1.3 μm, where the fiber loss is below 0.5 dB/km. Since fiber dispersion is also minimum near 1.3 μm, this low-loss window was used for second-generation lightwave systems.

OPTICAL FIBER ATTENUATION

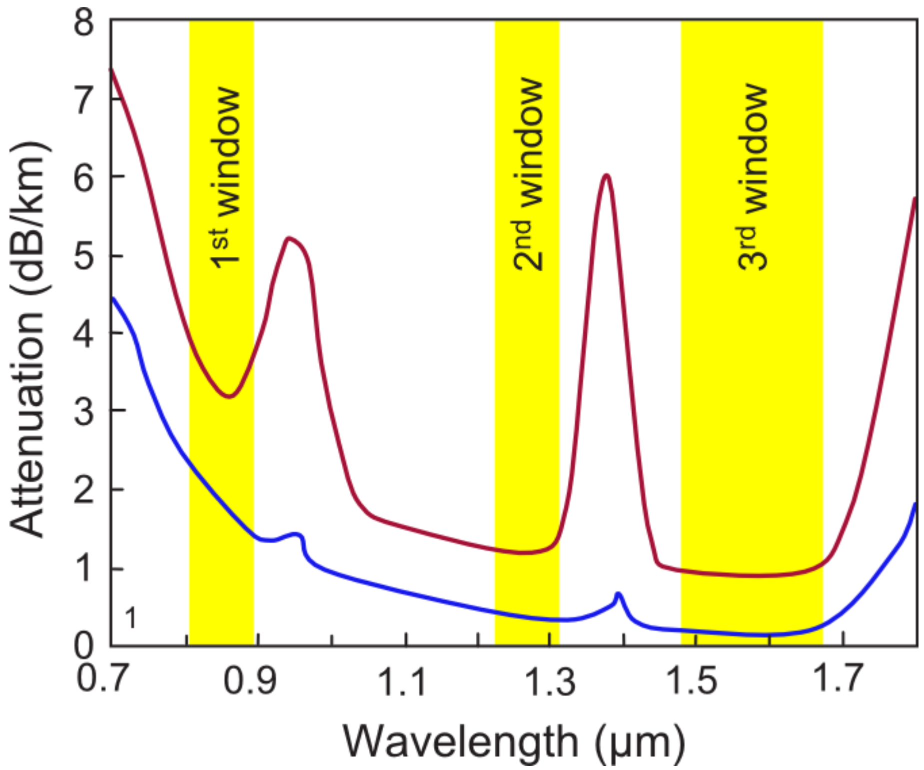


Figure 13: Attenuation of old (brown line) and new (blue line) silica fibers. The shaded regions indicate the three telecommunication wavelength windows.

Fiber losses are considerably higher for shorter wavelengths and exceed 5 dB/km in the visible region, making it unsuitable for long-haul transmission. Several factors contribute to overall losses; their relative contributions are also shown in the figure. The two most important among them are material absorption and Rayleigh scattering.

OPTICAL FIBER ATTENUATION

- The brown line shows the attenuation of old fibers that were made before the 1980s. In addition to strong water absorption peaks, the attenuation is generally higher than new fibers due to material impurity and waveguide scattering.
- Three wavelength windows, where optical attenuation has local minimums, have been used since the 1970s for optical communications in 850 nm, 1310 nm, and 1550 nm.
- In the early days of optical communication, the first wavelength window in 850 nm was used partly because of the availability of GaAs-based laser sources, which emit in that wavelength window.
- The advances in longer wavelength semiconductor lasers based on InGaAs and InGaAsP pushed optical communications toward the second and the third wavelength windows in 1310 nm and 1550 nm where optical losses are significantly reduced and optical systems can reach longer distances without regeneration.

MATERIAL ABSORPTION

Material absorption can be divided into two categories:

1) Intrinsic absorption which correspond to absorption by fused silica (material used to make fibers).

2) Extrinsic absorption which is related to losses caused by impurities within silica.

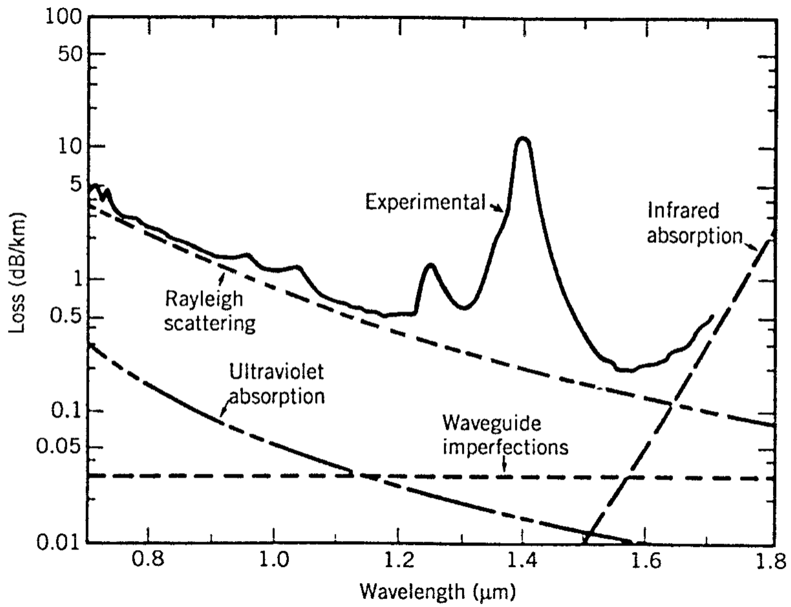
Any material absorbs at certain wavelengths corresponding to the electronic and vibrational resonances associated with specific molecules. For silica (SiO_2) molecules, electronic resonances occur in the ultraviolet region ($\lambda < 0.4\ \mu m$), whereas vibrational resonances occur in the infrared region ($\lambda > 7\ \mu m$). Because of the amorphous nature of fused silica, these resonances are in the form of absorption bands whose tails extend into the visible region.

Figure 12 shows that intrinsic material absorption for silica is less than 0.03 dB/km in the 1.3-1.6 μm wavelength window commonly used for lightwave systems.

MATERIAL ABSORPTION

Transition-metal impurities such as Fe, Cu, Co, Ni, Mn, and Cr absorb strongly in the wavelength range 0.6-1.6 μm. Their amount should be reduced to below 1 part per billion to obtain a loss level below 1 dB/km. Such high-purity silica can be obtained by using modern techniques.

- The main source of extrinsic absorption in state-of-the-art silica fibers is the presence of water vapors.
- OH ion harmonic and combination tones with silica produce absorption at the 1.39, 1.24, and 0.95 μm wavelengths.
- Even a concentration of 1 part per million can cause a loss of about 50 dB/km at 1.39 μm.
- The OH ion concentration is reduced to below 10^{-8} in modern fibers to lower the 1.39 μm peak below 1 dB.



MATERIAL ABSORPTION

- In a new kind of fiber, known as the dry fiber, the OH ion concentration is reduced to such low levels that the 1.39 μm peak almost disappears.
- Figure 14 shows the loss and dispersion of such a fiber (marketed under the trade name AllWave).
- Such fibers can be used to transmit wavelength division multiplexing (WDM) signals over the entire 1.30-1.65 μm wavelength range.

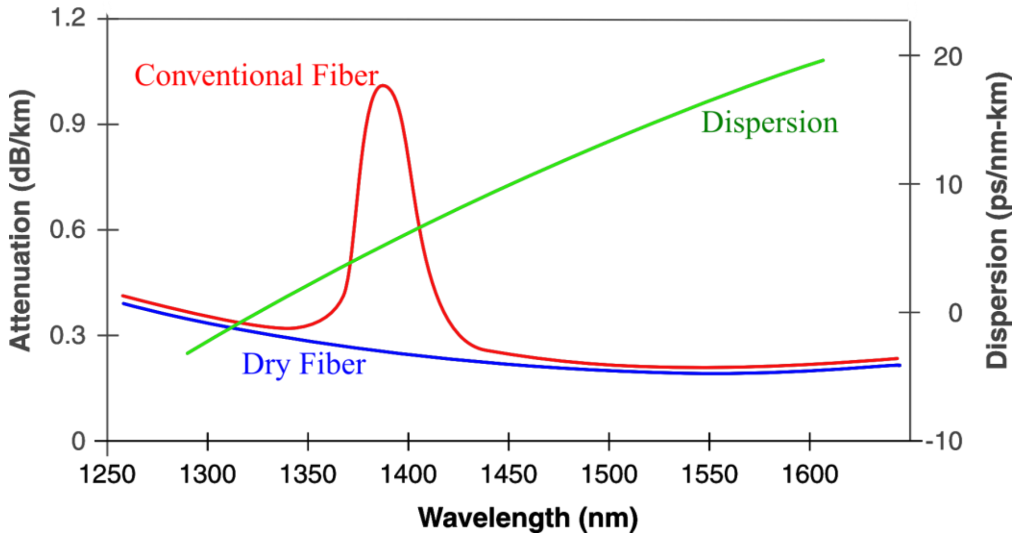


Figure 14: Loss and dispersion of the AllWave fiber. Loss of a conventional fiber is shown by the gray line for comparison [Courtesy Lucent Technologies].

FIBER REFLECTION AND SCATTERING

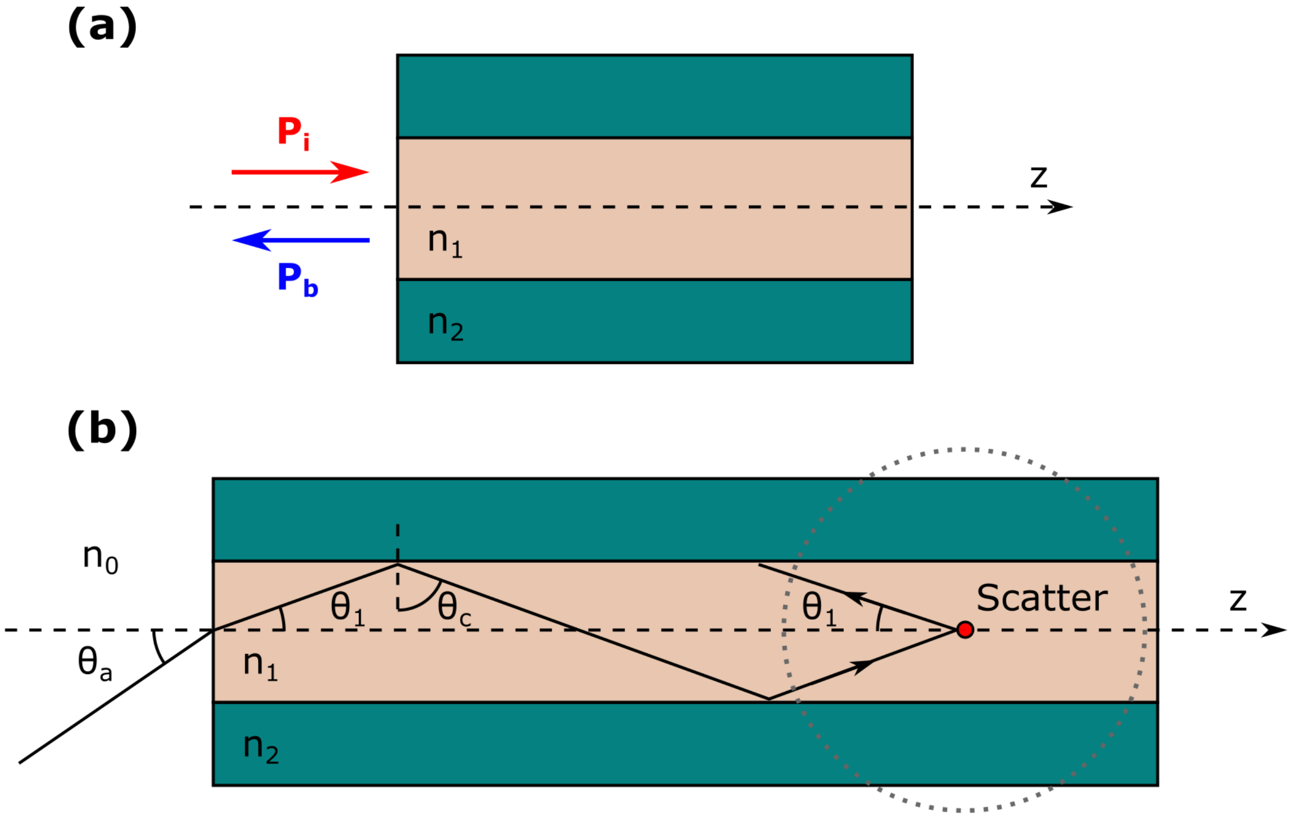
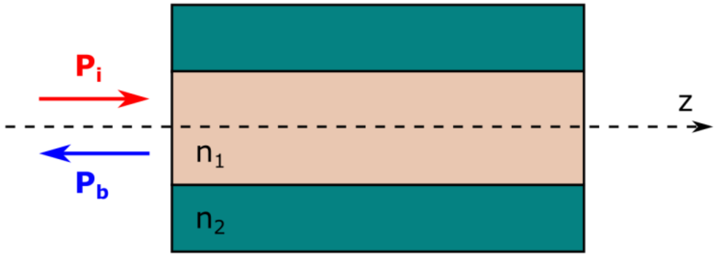


Figure 15: Illustrations of (a) fiber back-reflection and (b) scattering in fiber core.

BACK-REFLECTION

- When light exits the fiber some of the power is back-reflected from the end-surface of the fiber into the fiber core. In some devices the back reflection from the fiber end-surface can harm the device.



What can be done to reduce the Fresnel reflection back to the device?

- The end-surface of a fiber connector can be angle-cleaved to the fiber axis (angles of 5-15^0). This is usually referred to as APC (angle-polished connector). The light will back-reflected into the cladding and not back to the core.

BACK-REFLECTION

If the fiber has the core index $n_1 = 1.47$ and cladding index $n_2 = 1.467$ what is the minimum angle φ such that the Fresnel reflection by the fiber end-facet will not become the guided fiber mode?

- The angle has to be designed such that after reflection at the fiber end-surface, all these three light beams will not be coupled into fiber-guided mode in the backward propagation direction $\theta = \pi/2 - 2\varphi < \theta_c$. For the reflected light beam not to become the guided mode of the fiber, $\theta < \theta_c$ is required, where θ_c is the critical angle.
- Therefore, the (a) requirement for φ is $\varphi > \pi/4 - \theta_c/2$. (b) The beam has an angle θ_1 with respect to the surface normal of the fiber end-surface, which is related to φ by $\theta_1 = \pi/2 - \theta_c + \varphi$.

ANGLE POLISHED FIBER

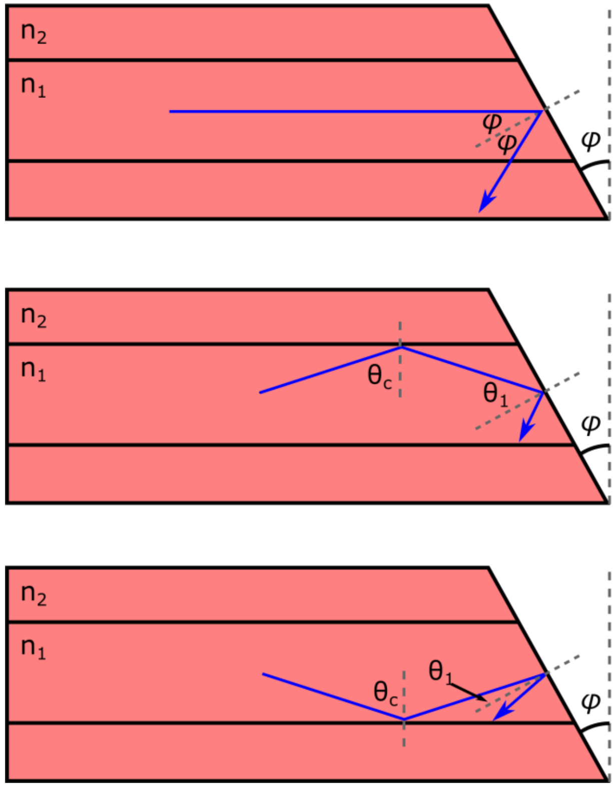
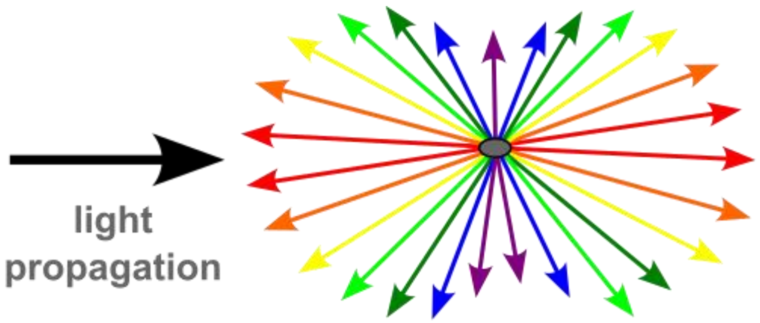


Figure 16: Illustration of an angle-polished fiber surface.

RAYLEIGH SCATTERING

- Rayleigh scattering is an elastic scattering of light a by particles with a size much smaller than the wavelength of the radiation.
- It is inversely proportional to the wavelength. A blue light is scattered much more than a red light as light propagates through air.
- Rayleigh scattering of sunlight in Earth's atmosphere causes diffuse sky radiation, which is the reason for the blue color of the daytime and twilight sky, as well as the yellowish to reddish hue of the low Sun.



RAYLEIGH SCATTERING

- Silica molecules move randomly in the molten state and freeze in place during fiber fabrication. Density fluctuations lead to random fluctuations of the refractive index on a scale smaller than the optical wavelength λ.
- Light scattering in such a medium is known as Rayleigh scattering. The scattering cross section varies as λ^{-4}. As a result, the intrinsic loss of silica fibers from Rayleigh scattering can be written as:

$$\alpha_R = C/\lambda^4 \qquad (59)$$

where the constant C is in the range 0.7-0.9 (dB/km)μm^4, depending on the constituents of the fiber core. These values of C correspond to $\alpha_R = 0.12 - 0.16$ dB /km at $\lambda = 1.55$ μm, indicating that fiber loss in Fig. 13 describing loss is dominated by Rayleigh scattering near this wavelength.

RAYLEIGH SCATTERING

The contribution of Rayleigh scattering can be reduced to below 0.01 dB/km for wavelengths longer than 3 μm.

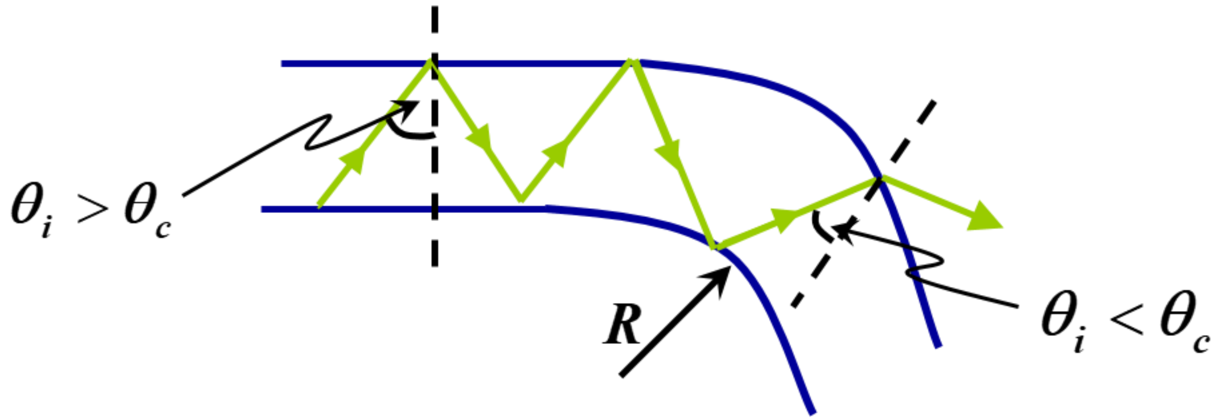
- Silica fibers cannot be used in this wavelength region, since infrared absorption begins to dominate the fiber loss beyond 1.6 μm.
- Fluorozirconate (ZrF_4) fibers have an intrinsic material absorption of about 0.01 dB/km near 2.55 μm and have the potential for exhibiting loss much smaller than that of silica fibers. However, State-of-the-art fluoride fibers exhibit a loss of about 1 dB/km because of extrinsic losses.
- Chalcogenide and polycrystalline fibers exhibit minimum loss in the far-infrared region near 10 μm. The theoretically predicted minimum value of fiber loss for such fibers is below 10^{-3} dB/km because of reduced Rayleigh scattering. However, practical loss levels remain higher than those of silica fibers.

WAVEGUIDE IMPERFECTIONS - CORE-CLADDING INTERFACE

- An ideal single-mode fiber with a perfect cylindrical geometry guides the optical mode without energy leakage into the cladding layer. In practice, imperfections at the core-cladding interface (e.g., random core-radius variations) can lead to additional losses which contribute to the net fiber loss. The physical process behind such losses is Mie scattering, occurring because of index inhomogeneities on a scale longer than the optical wavelength.
- Care is generally taken to ensure that the core radius does not vary significantly along the fiber length during manufacture. Such variations can be kept below 1%, and the resulting scattering loss is typically below 0.03 dB/km.

WAVEGUIDE IMPERFECTIONS - BENDS

Bends in the fiber constitute another source of scattering loss. Normally, a guided ray hits the core-cladding interface at an angle greater than the critical angle to experience total internal reflection. However, the angle decreases near a bend and may become smaller than the critical angle for tight bends. The ray would then escape out of the fiber.

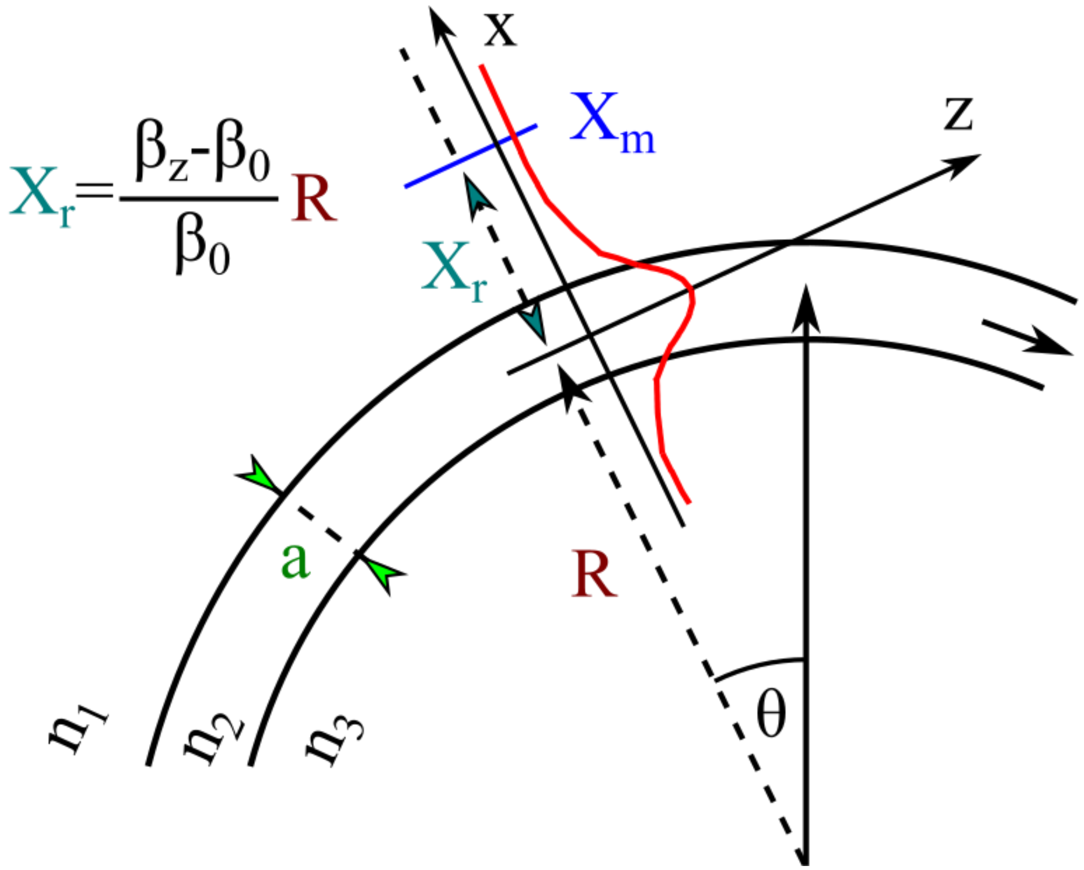


WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

β_z - the propagation constant.

β_0 - the propagation constant of unguided light in medium n_1.

R - the radius curvature.



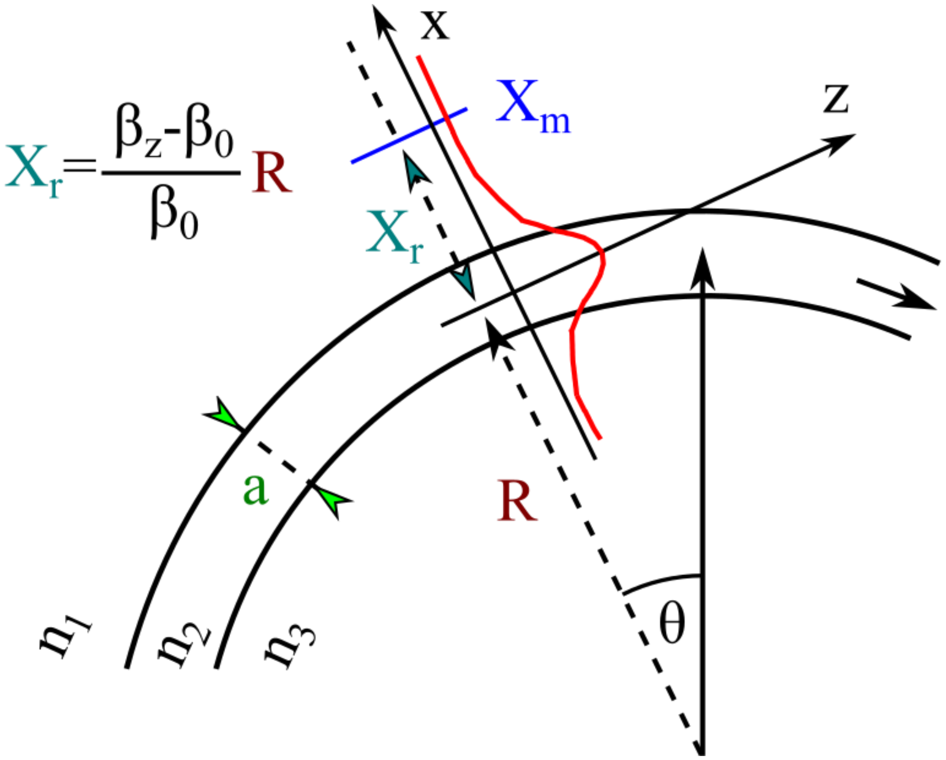
WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

For mode to exist, the angular velocity needs to be equal to preserve a wavefront. Therefore, the tangential phase velocity of the wave needs to be proportional to the distance from the center of the waveguide.

$$v = r\frac{d\theta}{dt} = x\frac{d\theta}{dt} = x\frac{\omega}{\beta x} = \frac{\omega}{\beta} \qquad (60)$$

The maximal velocity in n_1 is

$$v_1 = \frac{c}{n_1}$$



WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

For a certain distance X_m the velocity needs to be bigger than v_1 which is not physical. The photons will radiate into medium n_1.

$$X_m = R + X_r$$

In the center of the guiding layer (n_2):

$$v = \frac{\omega}{\beta_z} = R\frac{d\theta}{dt}$$

$$\boxed{\frac{d\theta}{dt} = \frac{\omega}{R\beta_z}} \tag{61}$$

In medium 1 (n_1):

$$\boxed{\frac{d\theta}{dt} = \frac{\omega}{\underbrace{(R + X_r)}_{X_m}\beta_0}} \tag{62}$$

WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

To preserve the wavefront, the angular velocities need to be equal.

$$\frac{\omega}{R\beta_z} = \frac{\omega}{(R + X_r)\beta_0} \quad \Rightarrow \quad R\beta_z = (R + X_r)\beta_0$$

$$X_m = \frac{\beta_z}{\beta_0} R \qquad X_r = \frac{\beta_z - \beta_0}{\beta_0} R$$

Till X_m, the wavefront is preserved.

Since $\beta_i = \beta_{\text{air}} n_i$, we get:

$$X_m = \frac{n_2}{n_1} R \qquad X_r = \frac{\Delta n}{n_1} R \tag{63}$$

For $x > X_m$ the radiation is slower while disappearing after the distance of Z_c.

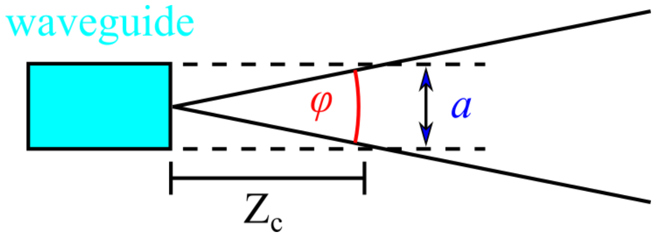
WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

$$\frac{dP(z)}{dz} = -\alpha P(z) \quad \Rightarrow \quad \boxed{\frac{P_{\text{loss}}}{Z_c} = \alpha P_{\text{total}}}$$

where P_{loss} is the power in the tail of the mode beyond X_r (the power lost by radiation within a length Z_c) and P_{total} is the total power.

Z_c can be calculated by analogy to the emission of photons from an abruptly terminated waveguide. It is the distance for which the light emitted into a medium from an abruptly terminated waveguide remains collimated.

$$\boxed{Z_c = \frac{a}{\varphi} = \frac{a^2}{2\lambda_1}}$$



when $\lambda_1 = \frac{\lambda_0}{n_1}$, a is the near-field beam width and φ is the far-field angle

WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

$$\alpha_R = \frac{P_{\text{loss}}}{P_{\text{total}}} = \frac{\int_{X_r}^{\infty} E^2(x)dx}{\int_{-\infty}^{\infty} E^2(x)dx} \cdot \frac{1}{Z_c}$$

Substitute E of the modes in the different regions:

1) Inside the waveguide:

$$E(x) = \sqrt{C_0}\cos(hx), \qquad -\frac{a}{2} \le x \le \frac{a}{2}$$

2) Outside the waveguide:

$$E(x) = \sqrt{C_0}\cos\left(\frac{ha}{2}\right)\exp\left[-\left(\frac{|x| - a/2}{q}\right)\right], \qquad -\frac{a}{2} \le x \le \frac{a}{2}$$

WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

$$P_{\text{loss}} = \int_{X_r}^{\infty} E^2(x)dx = C_0 \frac{q}{2} \cos^2\left(\frac{ha}{2}\right) \exp\left[-\frac{2}{q}\left(X_r - \frac{a}{2}\right)\right]$$

and

$$P_{\text{total}} = \int_{-\infty}^{\infty} E^2(x)dx = C_0 \left[\frac{a}{2} + \frac{1}{2h}\sin(hx) + q\cos^2\left(\frac{ha}{2}\right)\right]$$

$$\alpha_R = \frac{P_{\text{loss}}}{P_{\text{total}}} \cdot \frac{1}{Z_c} = \frac{\frac{q}{2}\cos^2\left(\frac{ha}{2}\right) \exp\left(-\overbrace{\frac{2}{q}\frac{\beta_z - \beta_0}{\beta_0}}^{C_2} R\right) 2\lambda_1 \exp\left(\frac{a}{q}\right)}{\left[\frac{a}{2} + \frac{1}{2h}\sin(hx) + q\cos^2\left(\frac{ha}{2}\right)\right] a^2}$$

$$\boxed{\alpha_R = C_1 \cdot \exp(-C_2 R)}$$

WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

$$\boxed{\alpha_R = C_1 \cdot \exp(-C_2 R)} \tag{64}$$

where C_1 and C_2 are constants that depend on the dimensions of the waveguide and on the shape of the mode.

Case	Index of refraction		Width a	C_1	C_2	R
	Waveguide	surrounding	[μm]	[dB/cm]	[cm^{-1}]	for α= 0.1 dB/cm
1	1.5	1.00	0.198	2.23×10^5	3.47×10^4	4.21 μm
2	1.5	1.485	1.04	9.03×10^3	1.46×10^2	0.78 μm
3	1.5	1.4985	1.18	4.69×10^2	0.814	10.4 cm

To conclude:

$$\boxed{\alpha = \alpha_{abs} + \alpha_{FR} + \alpha_R}$$

WAVEGUIDE IMPERFECTIONS - MACRO-BENDING

In the mode description, a part of the mode energy is scattered into the cladding layer.

$$\alpha_{\text{bend}} \propto \exp(-R/R_c) \tag{65}$$

where R is the radius of curvature of the fiber bend and $R_c = a/({n_1}^2 - {n_2}^2)$.

For single-mode fibers, $R_c = 0.2 - 0.4\ \mu\text{m}$ typically, and the bending loss is negligible (<0.01 dB/km) for bend radius $R > 5\ \text{mm}$. Since most macroscopic bends exceed $R = 5\ \text{mm}$, macro-bending losses are negligible in practice.

WAVEGUIDE IMPERFECTIONS - MICRO-BENDING

- A major source of fiber loss, particularly in cable form, is related to the random axial distortions that invariably occur during cabling when the fiber is pressed against a surface that is not perfectly smooth.
- Such losses are referred to as micro-bending losses and have been studied extensively. Microbends cause an increase in the fiber loss for both multimode and single-mode fibers and can result in an excessively large loss (~100 dB/km) if precautions are not taken to minimize them.

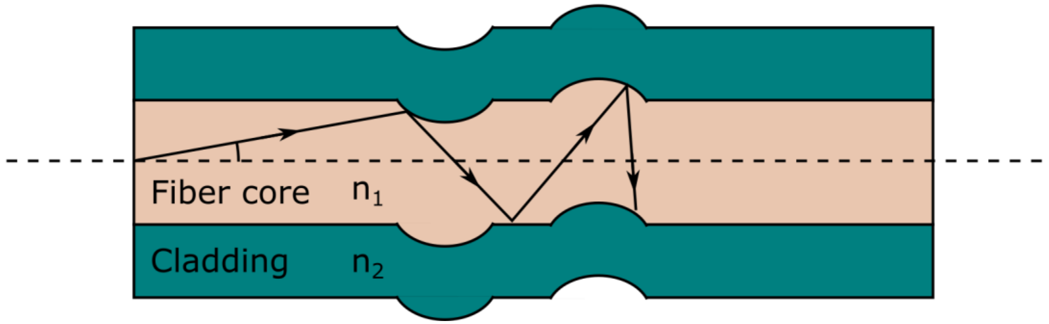


Figure 17: Illustration of micro-bending loss.

WAVEGUIDE IMPERFECTIONS - MICRO-BENDING

- For single-mode fibers, micro-bending losses can be minimized by choosing the V parameter as close to the cutoff value of 2.405 as possible so that mode energy is confined primarily to the core.
- In practice, the fiber is designed to have V in the range 2.0-2.4 at the operating wavelength.
- Many other sources of optical loss exist in a fiber cable. These are related to splices and connectors used in forming the fiber link and are often treated as a part of the cable loss; microbending losses can also be included in the total cable loss.

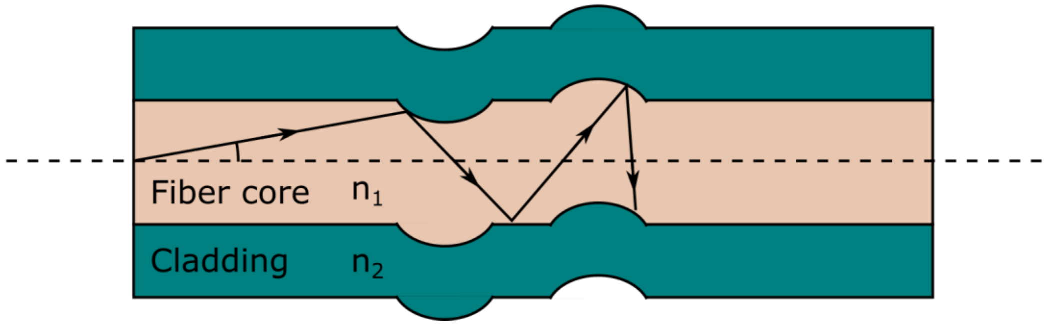


Figure 17: Illustration of micro-bending loss.

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