Modal Theory for Twisted Waveguides

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INTRODUCTION

Twisted waveguides are promising building blocks for broadband polarization rotation in integrated photonics. They may find applications in polarization-encoded telecommunications and quantum-optical systems.

In our work, we develop a rigorous modal theory for such waveguides.

Using covariant approach for expressing Maxwell's equations in helical reference frame

$$g^{kl}\nabla_k\nabla_l E^i - g^{ik}\nabla_k\left(\nabla_l E^l\right) + k^2\varepsilon E^i = 0$$

and defining an eigenmode of a twisted waveguide as a natural generalization of an eigenmode of a straight waveguide

$$E^i = e^i \exp(i\beta z), e^i = \{e_X, e_Y, e_Z\}$$

we derive the eigenmode equation which appears to be nonlinear with respect to the eigenvalue, i.e., propagation constant

$$[A - \beta^2 + 2i\alpha(\beta^{-1}B + \beta C) + \alpha^2 D]|e\rangle = 0, \qquad |e\rangle = \{e_X, e_Y\},$$

where $\alpha = \frac{2\pi}{L_{twist}}$ is the constant twist rate, A, B, C, D are the differential operators depending on transverse coordinates.

By analyzing the obtained equations, we establish fundamental properties of the eigenmodes and prove their orthogonality. Finally, we apply the modal approach to analyze polarization conversion in twisted waveguides.

METHODS

- We express Maxwell's equations in helical reference frame using **tensor formalism**.
- Obtained eigenmode equation is a nonlinear eigenvalue problem (NLEVP) which we solve within two methods: using **perturbation theory** and **Newton's Method**.
- We approximate the differential operators by sparse matrices using **Finite-Difference Method** on **Yee grid.**
- We simulate light propagation using **Eigenmode Expansion Method (EME)** generalized to the modes of a twisted waveguide.
- We verify results obtained within **EME** with the **Beam Propagation Method (BPM).**

RESULTS

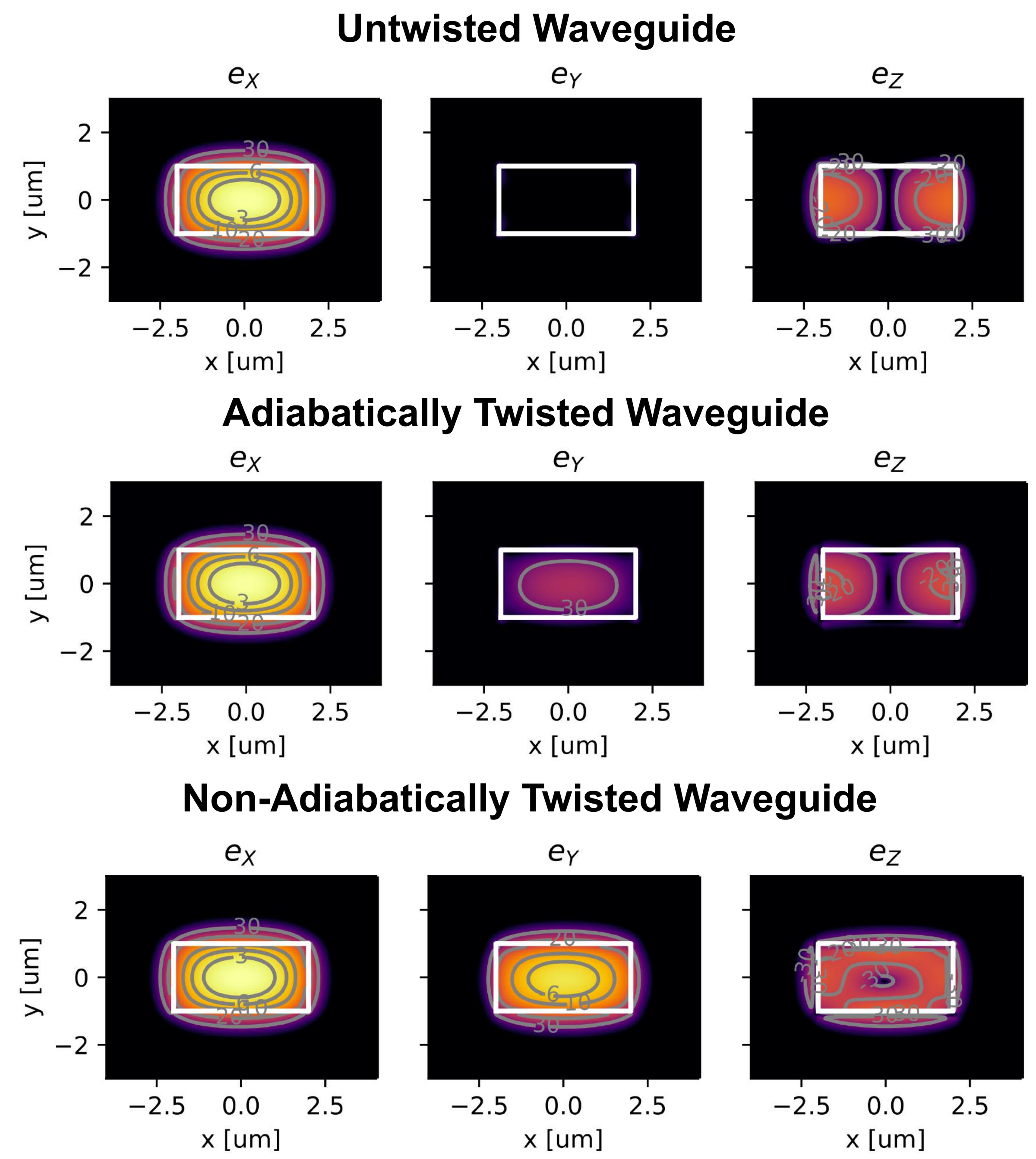


Fig. 1 Modal electric field of the quasi-TE mode

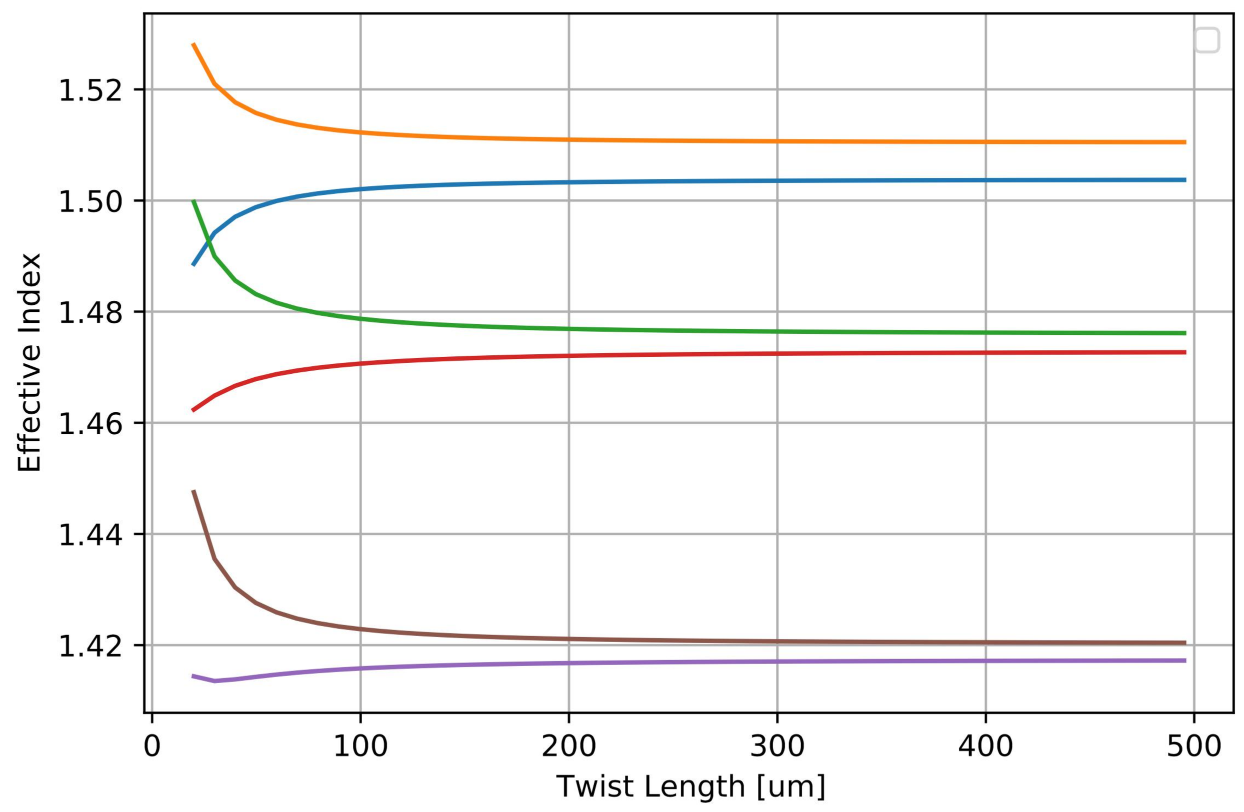


Fig. 2 Modal indices of the lowest order modes as a function of twist length

Fig. 3 Adiabatic polarization conversion in twisted waveguide with the twist length L = 1000 um

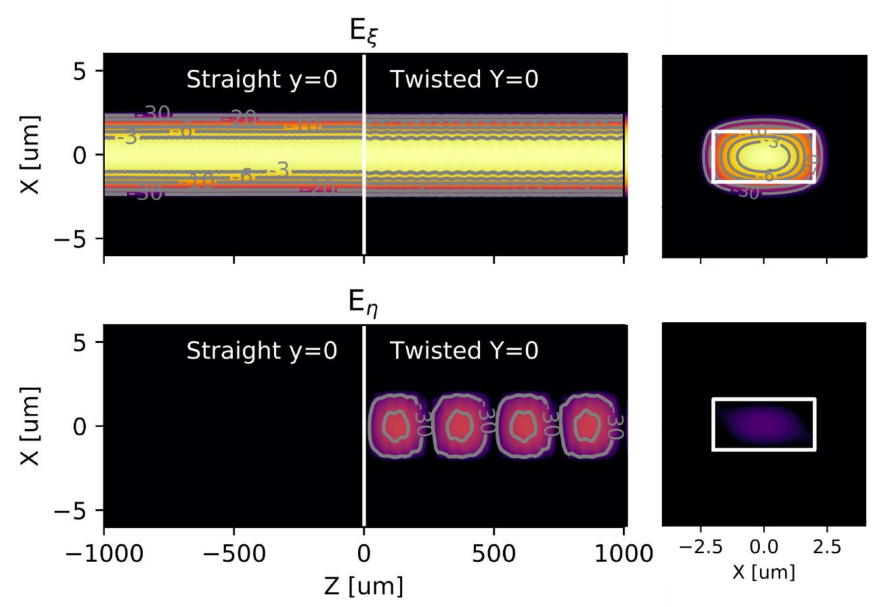
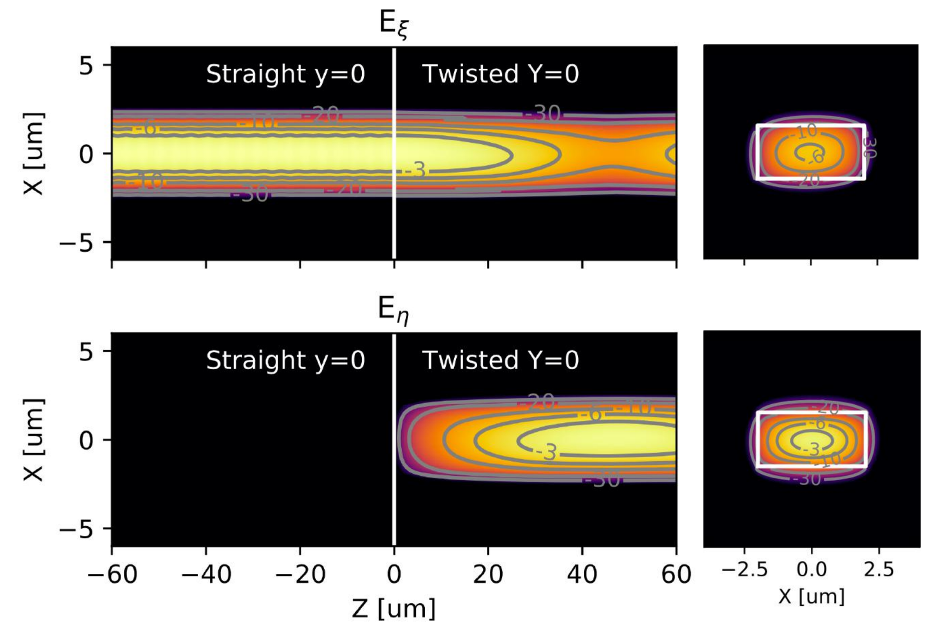


Fig. 4 Non-adiabatic propagation in rapidly twisted waveguide with the twist length L = 60 um



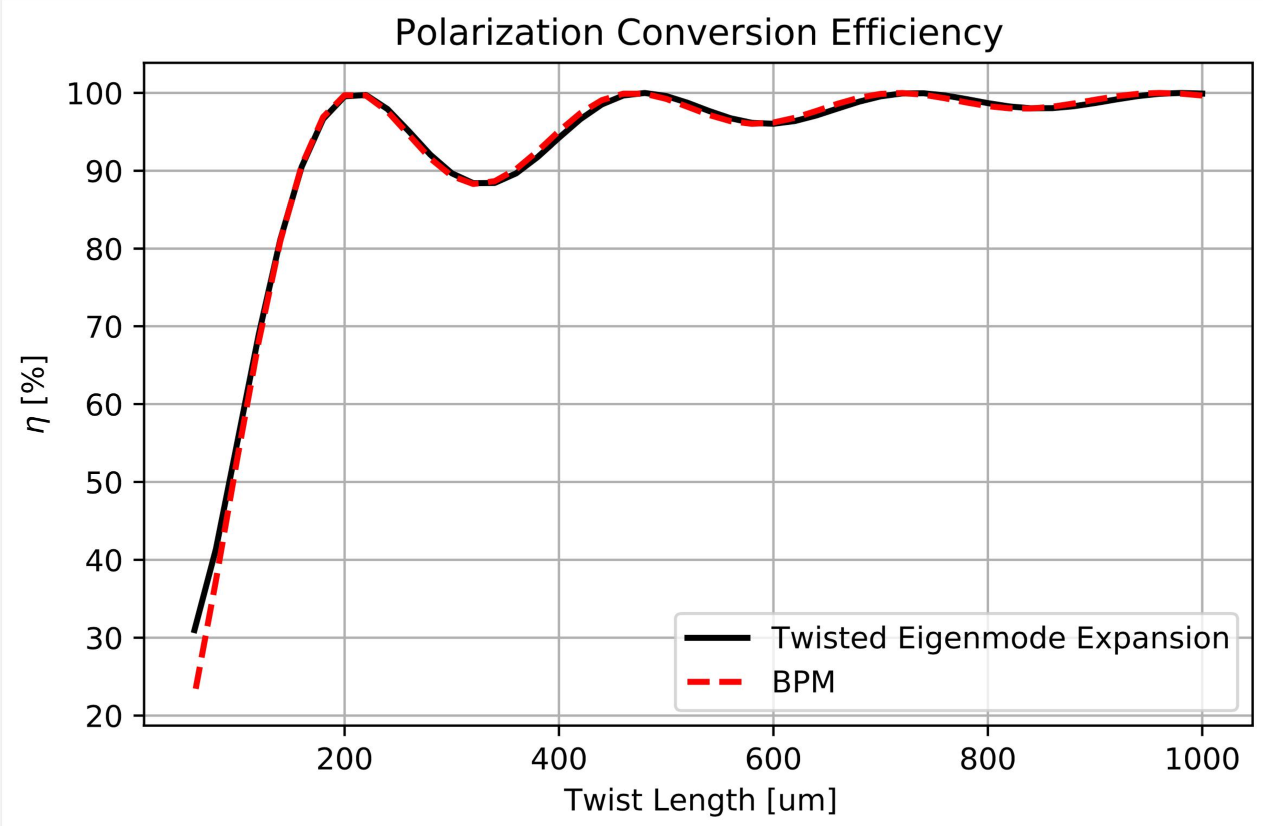


Fig. 5 Polarization conversion efficiency of the studied twisted waveguide as a function of twist length

CONCLUSIONS

1. We have rigorously defined an eigenmode of a twisted waveguide.
2. The eigenproblem for a twisted waveguide is a nonlinear eigenvalue problem.
3. We have solved the eigenproblem with a finite-difference mode solver which we specifically developed for this purpose.
4. By combining the mode solver with the eigenmode expansion method we obtained precise and efficient analysis tool for twisted waveguides.
5. In the framework of the modal approach, we intuitively explained the phenomenon of polarization rotation in twisted waveguides.

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